Homework 11

Due Tuesday November 29, at class

1. The data $x(n) = Ar^n + w(n)$, $n = 0, \ldots, N-1$, are observed where $w(n)$ is Gaussian white noise with variance σ^2 and $r > 0$ is known. Find the CRLB for A. Show that an efficient estimator exists and find its variance. What happens to the variance as $N \to \infty$ for various values of r?

2. If $x(n) = r^n + w(n)$, $n = 0, \ldots, N-1$, are observed, where $w(n)$ is Gaussian white noise with variance σ^2 (known) and $r > 0$ is to be estimated, find the CRLB. Does an efficient estimator exist and if so find its variance?

3. Consider the problem of line fitting to noisy observations:

$$x_n = A + Bn + w_n, \quad n = 1, \ldots, N$$

where $w_n \sim N(0, \sigma^2)$. In this case, the parameter is the vector $\boldsymbol{\theta} = [A\,B]^T$.

a. Determine the Fisher Information Matrix for $\boldsymbol{\theta}$ and the CRLBs for $\hat{A}$ and $\hat{B}$.

b. Compare the CRLB of $\hat{A}$ in this case, with the CRLB for $\hat{A}$ when B is known.