Homework 12

Due Thursday, November 7, at class
Linear Models for Poisson Data

Linear models of the form $\mathbf{s} = \mathbf{H}\boldsymbol{\theta}$ are difficult to apply in non-Gaussian situations. One case of particular interest is Poisson data. To apply a linear model, one approach is to first transform the Poisson data so it is *approximately* Gaussian and then fit the linear model to the transformed data.

Let $\mathbf{x} \sim \text{Poisson}(\boldsymbol{\lambda})$, where $\mathbf{x}$ and $\boldsymbol{\lambda}$ are $n \times 1$ dimensional vectors. We suppose that $\boldsymbol{\lambda}$ has a simple description in terms of a small number of parameters. Proceed as follows.

1. Show that, for large values of λ, a Poisson random variable $x \sim \text{Poisson}(\lambda)$ is approximately Gaussian distributed $\mathcal{N}(\lambda, \lambda)$.

2. Argue that $y = g(x)$ is also approximately Gaussian distributed, and show that $y \sim \mathcal{N}(g(\lambda), [g'(\lambda)]^2\lambda)$, where $g'(\lambda)$ is the derivative of g at λ.

3. Combine the results in steps 1 and 2 to argue that $y = \sqrt{x}$ is approximately Gaussian distributed with a variance that does not depend on λ.

4. Using the above results, now fit a linear model to the transformed data y and discuss its properties (e.g., distribution of the estimated parameters, optimality or lack of optimality, etc).