

### HW 3: Covariance Matrices and Eigendecompositions

Due Thursday Sept. 10 at class

Consider a random observation vector

$$\mathbf{Y} = \mathbf{X} + \mathbf{W}.$$

$\mathbf{X}$  is a random signal given by

$$\mathbf{X} \sim \mathbf{H}\boldsymbol{\theta},$$

where  $\mathbf{H}$  is an  $N \times L$ ,  $L \ll N$ , deterministic signal matrix and  $\boldsymbol{\theta}$  is a random signal parameter distributed according to

$$\boldsymbol{\theta} \sim \mathcal{N}(\mathbf{0}, \sigma_{\theta}^2 \mathbf{I}_{L \times L}).$$

$\mathbf{W}$  is a random noise vector distributed according to

$$\mathbf{W} \sim \mathcal{N}(\mathbf{0}, \sigma_w^2 \mathbf{I}_{N \times N}).$$

- Give expressions for the covariance  $\mathbf{R}_{xx}$  of  $\mathbf{X}$  and  $\mathbf{R}_{yy}$  of  $\mathbf{Y}$  in terms of  $\mathbf{H}$ .
- Assuming the  $L$  columns of  $\mathbf{H}$  are orthonormal vectors, determine the eigenvalues and eigenvectors of  $\mathbf{R}_{yy}$ . How are the eigenvectors and eigenvalues related to  $\mathbf{H}$ ?
- Let's put the ideas above into action. Generate 1000 random sinusoids in noise and form the *sample covariance matrix*  $\mathbf{S}$  according to the Matlab code below:

```
M = 1000; N = 32; L = 2;
S = zeros(N,N);
for m=1:1000;
    x = cos(2*pi*(2*rand-1)*(L/N)*(0:(N-1))'+pi*rand);
    y = x + 0.5 * randn(N,1);
    S = S+y*y';
end;
S = S/M;
```

Use the built-in Matlab function `eig` to determine a small set of vectors that span the signal subspace.