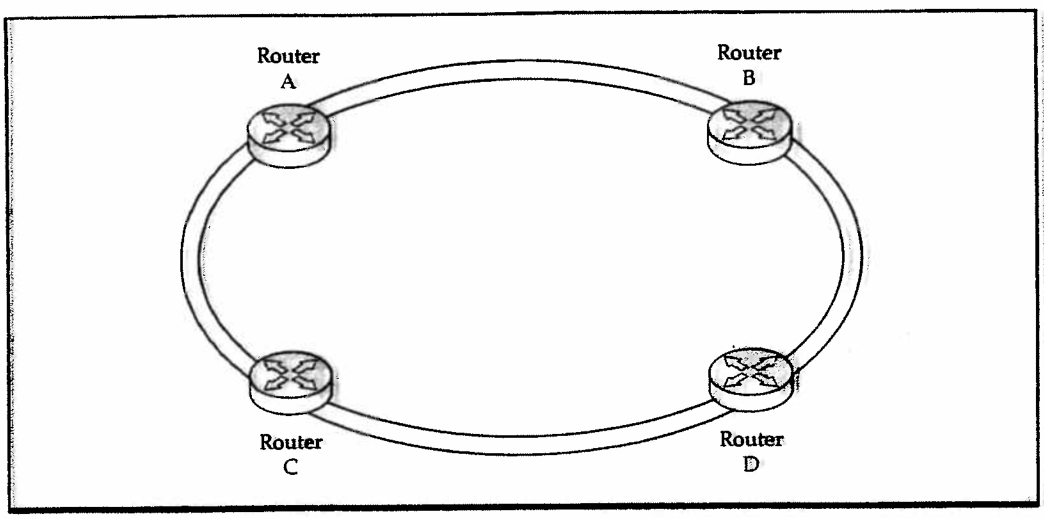
A STUDY OF RING NETWORKS



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25th Anniversary of

Okamura and Seymour,

"Multicommodity flows

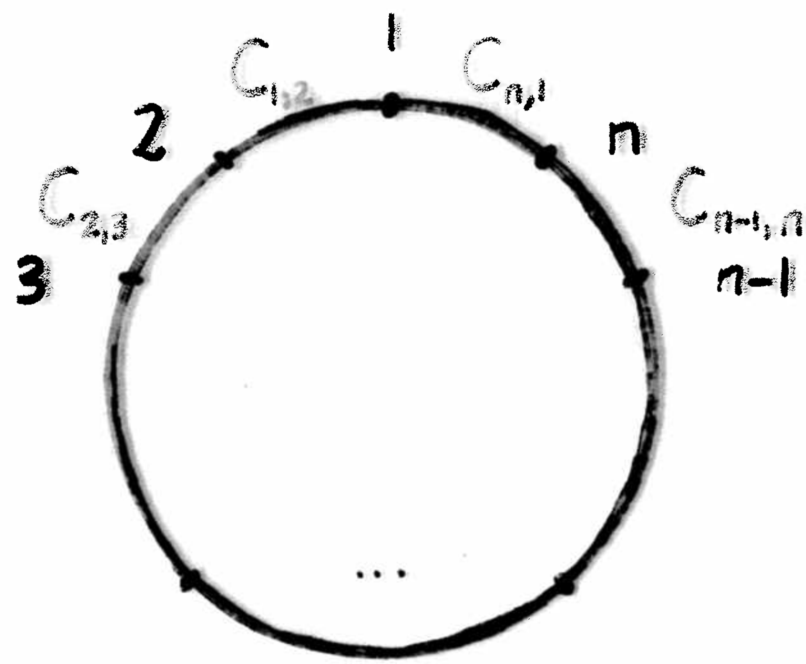
in planar graphs,"

J. Combin. Theory, 1981

RINGS: Important in communications

SONET: Synchronous Optical Network

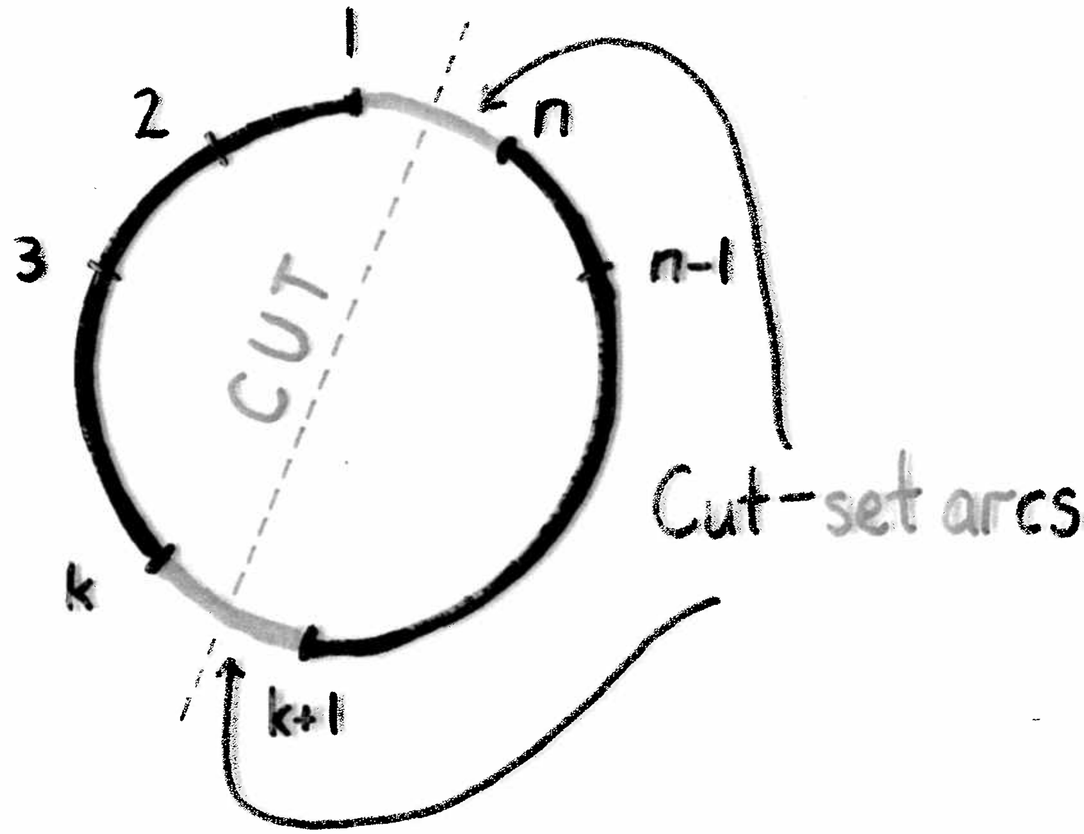
Model: Undirected network of n nodes



Edge capacity: Max. traffic in both directions

Rate $R_{i,j}$: Traffic demand from i to j

Okamura and Seymour's Result for Rings



Theorem: Assume routing.

Feasible flow iff total traffic across every cut ≤ capacity of cut-set arcs

Ex: $\sum_{i=1}^{k} \sum_{j=k+1}^{n} R_{i,j} + \sum_{i=k+1}^{n} \sum_{j=1}^{k} R_{i,j} \leq C_{k,k+1} + C_{n,1}$

Transportation or Manufacturing:

- Physical entity
- Transferred in original form
- Each commodity has unique destination
- Flow conservation

Communication:

- Information
- A processor may transmit a <u>function</u> of its gathered information to some collection of neighboring processors
- Same information may need to be sent to <u>multiple</u> destinations.
- Noise

Need different theory/tools.

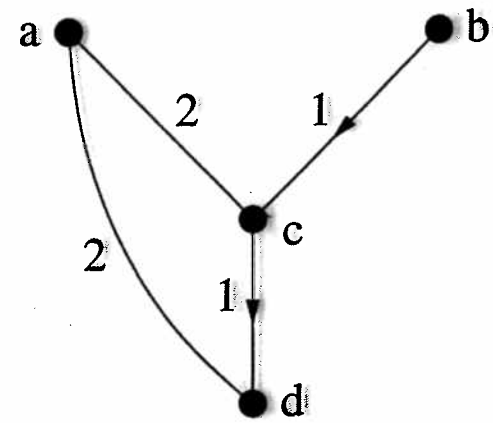
Two-Way Communication Channels

Shannon (1961)

Edges: Cables

Wireless Channels

etc.

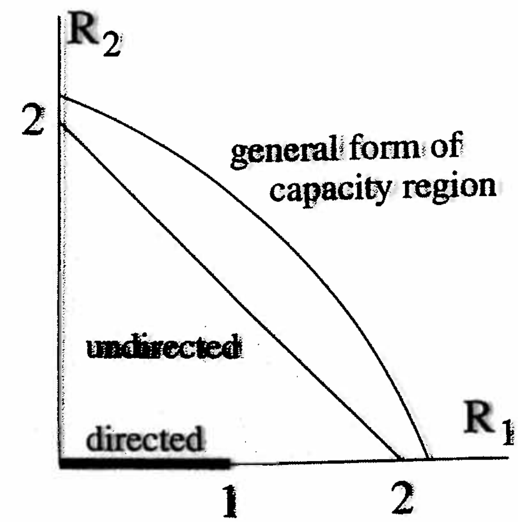


· Two-way channel (TWC)

Edge ac:

$P(y_a, y_c \mid x_a, x_c)$

(Noise permitted)



· Edge capacity region:

- Set of rate pairs (R_1, R_2) achievable with coding
- Convex since time-sharing permitted.

Networks of TWCs: Kramer (2003)
Kramer and Savari (2003), (2004)

Bidirected (2D) Cut Set Bounds:
For a large class of networks of TWCs it is easy to optimize Cover and Thomas' cut set bounds.

- Applies to network coding
- Possible to show routing is throughput-optimal for multiple unicast sessions on rings.

TWO EXTENSIONS

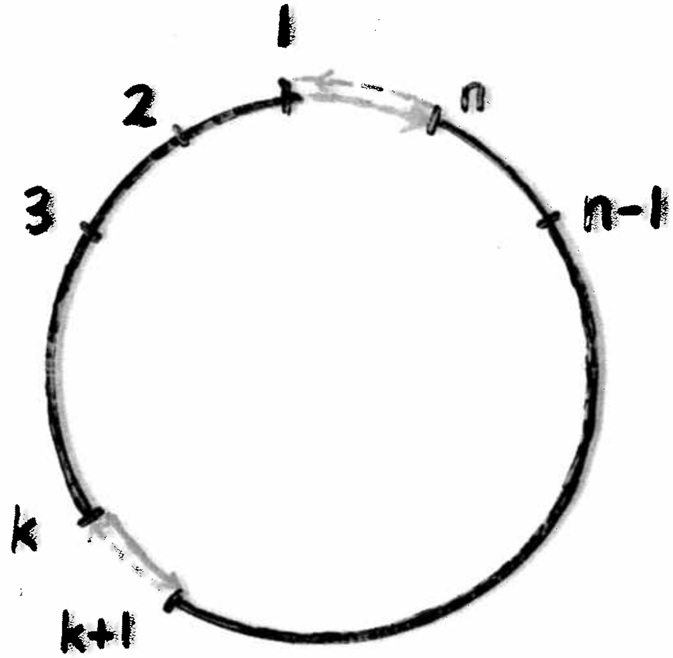
- Multiple broadcasts over undirected rings
- Multiple unicasts over bidirectional rings

(Focus of ISIT 2006 talk.)

2D Cut Set Bounds

Simplest case:

R_i: traffic from node i to every other node



For edge (i,j), let $r_{i,j}$ = traffic from i to j

Note: $r_{i,j} + r_{j,i} \leq C_{i,j}$

We have $R_1 + R_2 + \cdots + R_k \leq r_{1,n} + r_{k,k+1}$

and $R_{k+1} + R_{k+2} + \cdots + R_n \leq r_{n,1} + r_{k+1,k}$

Thus, $R_1 + R_2 + \cdots + R_n \leq C_{n,1} + C_{k,k+1}$

A similar bound holds for any pair of edges

By combining multiple 2D cut set bounds, can obtain a bound for any larger set of edge capacities

Reorder and relabel the n edge capacities by

$$\Gamma^{(1)} \leq \Gamma^{(2)} \leq \cdots \leq \Gamma^{(n)}.$$

It is possible to show

$$R_1 + \cdots + R_n \leq \min \left\{ \Gamma^{(1)} + \Gamma^{(2)}, \frac{\Gamma^{(1)} + \Gamma^{(2)} + \Gamma^{(3)}}{2}, \ldots, \frac{\Gamma^{(1)} + \Gamma^{(2)} + \cdots + \Gamma^{(n)}}{n-1} \right\}$$

Achieving the Bound with Routing

Suppose the tightest of the bounds is

$$R_1 + R_2 + \cdots + R_n \leq \frac{\Gamma^{(1)} + \Gamma^{(2)} + \cdots + \Gamma^{(i)}}{i-1}$$

Then
$$\Gamma^{(i)} \leq \frac{\Gamma^{(1)} + \Gamma^{(2)} + \cdots + \Gamma^{(i)}}{i-1} \leq \Gamma^{(i+1)}$$

Any feasible rate tuple is a convex combination of the rate tuples

$$(0, 0, 0, \cdots, 0)$$

$$\left(\frac{\Gamma^{(1)} + \Gamma^{(2)} + \cdots + \Gamma^{(i)}}{i-1}, 0, 0, \cdots, 0\right)$$

$$\left(0, \frac{\Gamma^{(1)} + \Gamma^{(2)} + \cdots + \Gamma^{(i)}}{i-1}, 0, \cdots, 0\right)$$

$$\vdots$$

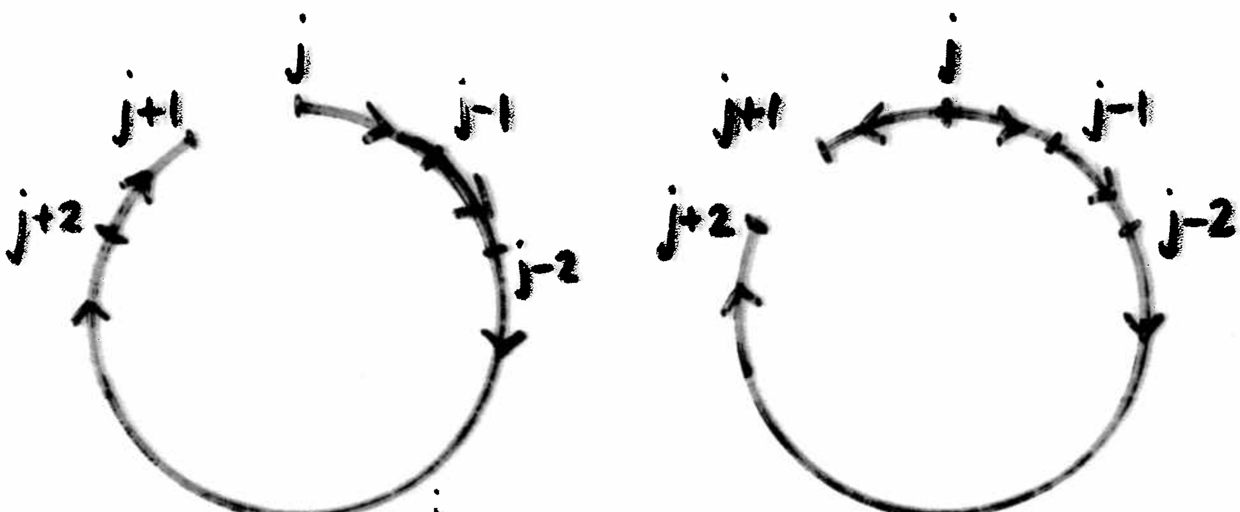
$$\left(0, 0, 0, \cdots, \frac{\Gamma^{(1)} + \Gamma^{(2)} + \cdots + \Gamma^{(i)}}{i-1}\right)$$

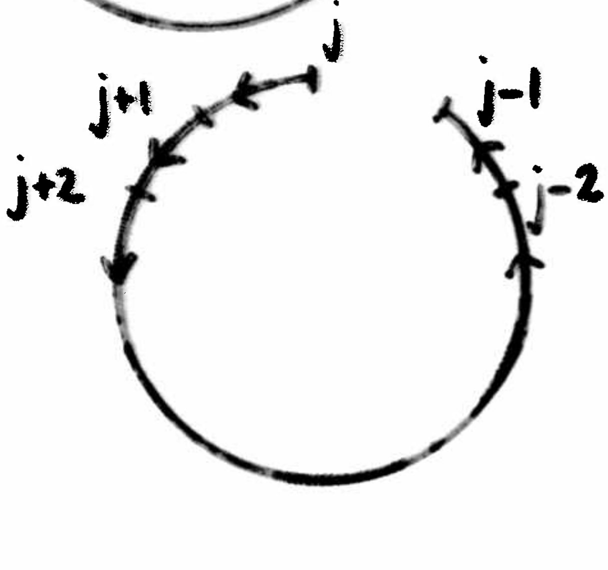
Concentrate on

$$R_j = \frac{\Gamma^{(1)} + \cdots + \Gamma^{(i)}}{i-1}$$

$$R_k = 0, \quad k \neq j$$

Node j can potentially transmit along n trees





Each case:
One edge missing

Let $f^{(i)}$ be the flow on the tree for which the edge corresponding to $\Gamma^{(i)}$ is missing. Then

$$\begin{aligned}
f^{(2)} + f^{(3)} + \cdots + f^{(n)} &\leq \Gamma^{(1)} \\
f^{(1)} + f^{(3)} + \cdots + f^{(n)} &\leq \Gamma^{(2)} \\
&\vdots \\
f^{(1)} + f^{(2)} + \cdots + f^{(n-1)} &\leq \Gamma^{(n)}
\end{aligned}$$

Solution:

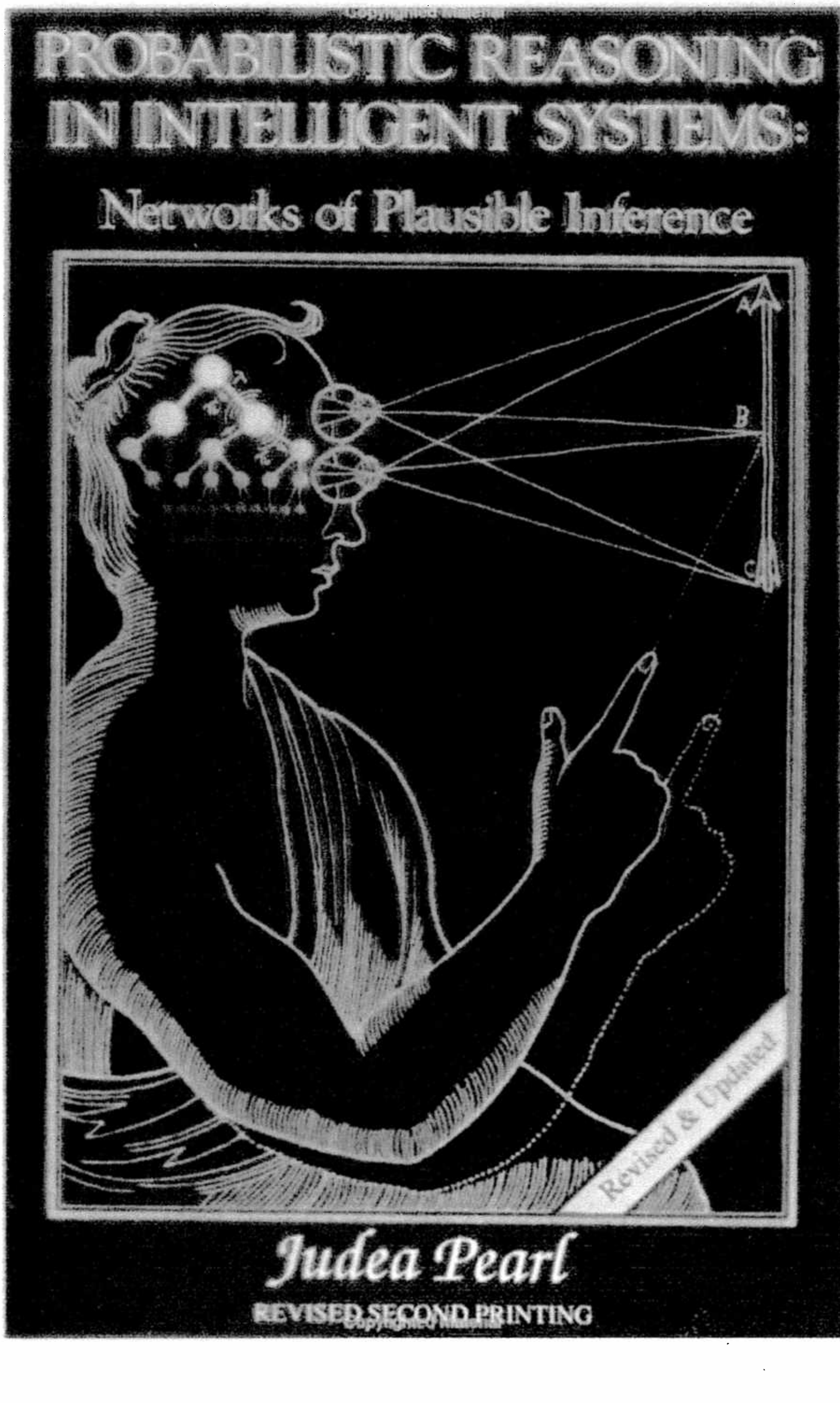
$$f^{(j)} = \begin{cases} \dfrac{\Gamma^{(1)} + \cdots + \Gamma^{(i)}}{i-1} - \Gamma^{(j)}, & j = 1, 2, \cdots, i \\ 0, & j = i+1, \cdots, n \end{cases}$$

attains the bound.

Focus of ISIT 2006 talk:

Multiple unicast sessions over bidirectional rings

- Analysis more intricate
- Appears to give new results even in the multicommodity flow setting
- 2D Cut Set Bounds do not provide all the bounds
  - Need PdE bounds and new extensions.


PROBABILISTIC REASONING
IN INTELLIGENT SYSTEMS:
Networks of Plausible Inference
A
B
C
Revised & Updated
Judea Pearl
REVISED SECOND PRINTING

CAUSALITY:

- Does the information sent along certain links capture the whole flow of information in the network?

- If so, the capacities of those links limit information rates.

- Calculus for generating rate bounds.

Ongoing Work:

General multicast problems.