Finding Globally Optimum Solutions in Antenna Optimization Problems

Aydin Babakhani*, Javad Lavaei, John C. Doyle, and Ali Hajimiri
California Institute of Technology
E-mail: aydin@caltech.edu

Introduction

During the last decade, the unprecedented increase in the affordable computational power has strongly supported the development of optimization techniques for designing antennas. Among these techniques, genetic algorithm [1] and particle swarm optimization [2] could be mentioned. Most of these techniques use physical dimensions of an antenna as the optimization variables, and require solving Maxwell's equations (numerically) at each optimization step. They are usually slow, unable to handle a large number of variables, and incapable of finding the globally optimum solutions. In this paper, we are proposing an antenna optimization technique that is orders of magnitude faster than the conventional schemes, can handle thousands of variables, and finds the globally optimum solutions for a broad range of antenna optimization problems. In the proposed scheme, termination impedances embedded on an antenna structure are used as the optimization variables. This is particularly useful in designing on-chip smart antennas, where thousands of transistors and variable passive elements can be employed to reconfigure an antenna. By varying these parasitic impedances, an antenna can vary its gain, band-width, pattern, and efficiency. The goal of this paper is to provide a systematic, numerically efficient approach for finding globally optimum solutions in designing smart antennas.

Description of the Problem

Let us consider the problem in Figure 1, where a dipole antenna with a width of w and a length of l is placed at a distance of h on top of a ground plane. Similar to the most of the conventional antenna optimization problems, it is assumed that the shape of this planar ground plane is the optimization variable. The ground plane has to be fit in a square area with a length of d (Figure 1). A fixed voltage source with an amplitude of 1V and a frequency of f drives the dipole antenna. The optimization goal is to maximize the received signal in the bore-sight (θ=0º), with a constraint that the received signal in one of the directions θ= -75º, -60º, -45º, -30º, -15º, 15º, 30º, 45º, 60º, 75º has to be zero (pattern null). Although this may seem to be a trivial optimization problem, it is worth mentioning that none of the existing antenna optimization techniques provides a globally optimum solution for this problem. In some of these conventional techniques, a solid ground plane is divided into many small squares and a locally optimum solution is found by removing a part of the ground plane. Unfortunately, in this case, the total number of possibilities increases exponentially with the number of metal squares. For a 10×10 mesh array, 2^{100} possible ground shapes exist! Each one of these 2^{100} ground shapes generates an antenna pattern that is unique. In order to find these patterns, a full electromagnetic simulation is required for every ground shape. This is obviously very time consuming and prevents us from finding a globally optimum solution.

In this paper, a different approach to finding a globally optimum solution is chosen. Instead of changing the physical geometry of the ground plane, the solid ground plane is converted to many small isolated metal squares (patches) and a lumped termination port

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is used to connect every two adjacent patches. The goal is to find *the globally optimum solution* by searching for *the best passive network* that connects these termination ports to each other and satisfies the constraints of the problem. The following several paragraphs describe this technique in more detail.

In order to capture the pattern at a specific angle (and with a known polarization), a receiving short dipole antenna is located at that angle in the far-field. A lumped differential port feeds this short receiving (sensing) dipole antenna. Lumped termination ports are also used to connect every two adjacent metal squares. In addition to these ports, a lumped port drives the main transmitting antenna (the dipole antenna in Figure 1). After this step, an equivalent circuit model for the problem is derived. This can be done by extracting the scattering parameters of the whole structure, as shown in Figure 2. In this Figure, ports 1 to z are the receiving short dipoles in the far-field, ports z to n are the termination ports connecting every two adjacent metal squares, and port n+1 is the transmitting (input) port driving the main dipole antenna. Figure 2 shows the Y-parameter matrix (Y_s) that is derived from the S-parameter data, by assuming that all of the termination ports are connected to a reference impedance (50Ω). Without loss of generality, let us assume that the receiving ports 1 to z (sensing ports) are all kept open. The goal is to find the Y-parameters (Y_p) of a passive network, connected to ports z+1 to n that makes the circuit satisfy any linear constraints on the voltages of the receiving ports 1 to z, as well as any linear constraint on the input impedance seen at port n+1 (transmitting port). This passive network also has to maximize a linear function of voltages at ports 1 to z. From Figure 2, it can be shown that,

$$\begin{bmatrix} I & i_{in} \end{bmatrix} = \begin{bmatrix} V_1 & V_2 & v_{in} \end{bmatrix} \cdot Y_s \tag{1}$$

$$\begin{bmatrix} I & i_{in} \end{bmatrix} = -\begin{bmatrix} V_1 & V_2 & v_{in} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & Y_p & 0 \\ 0 & 0 & -y_{in} \end{bmatrix} \tag{2}$$

where I is a vector representing the currents at ports 1 to n, i_{in} is the current at the transmitting port (port n+1), V_1 is a vector representing the voltages at ports 1 to z, V_2 is a vector representing the voltages at ports z+1 to n, v_{in} is the input voltage, Y_s is the Y-parameter matrix calculated from the scattering parameters of the electromagnetic simulation, Y_p is the Y-parameter matrix of the passive network, and y_{in} is the input conductance seen at port n+1 (input port). Y_s can be decomposed as,

$$Y_s = \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix} \tag{3}$$

where $W_{11} \in C^{z \times z}$, $W_{22} \in C^{(n-z) \times (n-z)}$, and $W_{33} \in C$. (C: complex numbers).
From (1), (2), (3),

$$V_1 = -V_2 W_{21} W_{11}^{-1} - v_{in} W_{31} W_{11}^{-1} \tag{4}$$

$$V_2 = v_{in} (W_{31} W_{11}^{-1} W_{12} - W_{32}) \widetilde{Y} \tag{5}$$

$$y_{in} = v_{in}^{-1} V_1 W_{13} + v_{in}^{-1} V_2 W_{23} + W_{33} \tag{6}$$

where, $\widetilde{Y} := (W_{22} - W_{21}W_{11}^{-1}W_{12} + Y_p)^{-1}$ (7)

As shown in equations (4) to (6), variables V_1, V_2, and y_{in} are all linear functions of the matrix $\widetilde{Y}$. The passivity condition on the termination matrix Y_p is equivalent to the following linear matrix inequitably,

$$\mathrm{Re}\{Y_p\} \succ 0 \qquad (8)$$

where the notation $\succ$ is used to show the inequality in the positive definite sense. The goal is to find a passive termination matrix Y_p such that some linear constraints on V_1 and y_{in} are satisfied, and a desired linear function of elements in V_1 is maximized. Because equations (4) to (6) are all linear, a simple linear programming method could be used to find the best $\widetilde{Y}$ matrix, but any arbitrary $\widetilde{Y}$ matrix does not necessarily satisfy equations (7) and (8). In a recent paper [3], it is proven that, for reciprocal antenna problems, the passivity condition (8) is equivalent to a linear matrix inequality shown in equation (9),

$$\begin{bmatrix} \mathrm{Re}\{W_{22} - W_{21}W_{11}^{-1}W_{12}\}^{-1} - \mathrm{Re}(\widetilde{Y}) & \mathrm{Im}(\widetilde{Y}) \\ \mathrm{Im}(\widetilde{Y}) & \mathrm{Re}(\widetilde{Y}) \end{bmatrix} \succ 0 \qquad (9)$$

Equations (4), (5), (6), and (9) convert the abovementioned optimization problem to a convex problem with a simple form. This means that, *the best solution (the globally optimum solution)* for this optimization problem can be found. This convex representation is equivalent to a linear matrix inequality (LMI) optimization problem, which can be handled efficiently using a proper software tool such as YALMIP [4] or SOSTOOLS [5].

The abovementioned convex method is used to find the globally optimum solution in the following optimization problem (Figure 3). In this example, a ground plane is located at Z=0µm, a 250µm 10Ω-cm silicon substrate is placed right above the ground layer, and a 20µm Silicon-dioxide (SiO_2) dielectric layer is mounted right on top of the Silicon substrate. These parameters are chosen to be similar to the ones that are used in today's standard Silicon process technologies. On-chip metal layers are usually implemented inside the SiO_2 dielectric layer. In this example, an on-chip 10×10 patch array is located at Z=250µm (1st metal layer) and an on-chip dipole antenna, with a width of 20µm and a length of 500µm, is placed at Z=270µm (top metal layer). The dimensions of each patch are 95µm by 95µm and the frequency of interest is 300GHz. The goal is to maximize the antenna gain at the bore-sight (θ=0º) and produce a null in one of these angles: θ= -75º, -60º, -45º, -30º, -15º, 15º, 30º, 45º, 60º, 75º. Ports 1 to 11 are the receiving ports located at a distance of 10mm (10λ_0) from the transmitting antenna in directions θ= -75º, -60º, -45º, -30º, -15º, 0º, 15º, 30º, 45º, 60º, 75º, respectively. The receiving ports are terminated by a fixed 50Ω resistance. Ports 12 to 101 connect every two adjacent patches (on the X-direction) and represent the passive termination matrix Y_p, and port 102 is the transmitting port which is connected to a fixed voltage of 1V. Table 1 summarizes the results. The results imply that there is always a non-zero correlation between the signal transmitted to the bore-sight (θ=0º) and directions θ= -75º, -60º, 60º, 75º. This means that no passive termination matrix connected to ports 12 to 101 can cause the antenna system to generate a null in one of the directions θ= -75º, -60º, 60º, 75º, and at the same time transmit a non-zero signal to the bore-sight (θ=0º). The convex optimization method also finds the best passive network that maximizes the received signal in the bore-sight while generating a null in one of these directions θ= -45º, -30º, -15º, 15º, 30º, 45º. The program is implemented in MATLAB and results are computed in less than ten minutes using a quad-core 2.3GHz computer.

Summary

A convex optimization method is introduced that is capable of finding globally optimum solutions for a broad range of antenna optimization problems. This technique can be employed in designing smart antennas where reconfigurability is achieved by varying termination impedances on an antenna structure. The capability of finding globally optimum solutions helps us to study the fundamental limits of complex antenna structures where thousands of optimization variables exist.

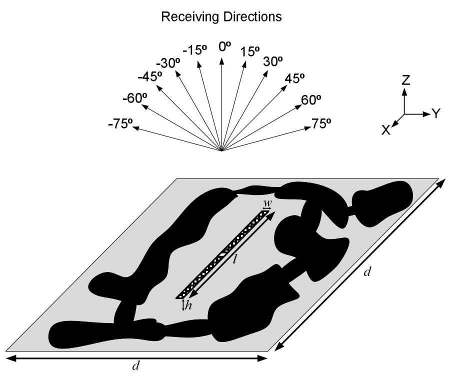

Figure 1) A dipole on a ground plane

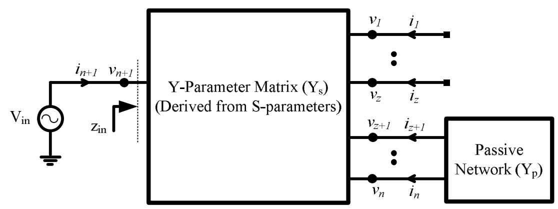

Figure 2) Circuit model of the problem

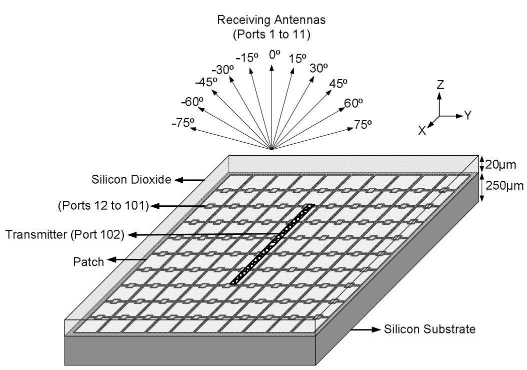

Figure 3) A dipole on a loaded metal array

Objective	Constraint	Results
Maximize $\lvert V_6\rvert$ (0º)	V_1=0 (θ=-75º)	Infeasible
Maximize $\lvert V_6\rvert$ (0º)	V_2=0 (θ=-60º)	Infeasible
Maximize $\lvert V_6\rvert$ (0º)	V_3=0 (θ=-45º)	V_6= (-3.6+3.1i)×1e-3
Maximize $\lvert V_6\rvert$ (0º)	V_4=0 (θ=-30º)	V_6= (-4.3+3.4)×1e-3
Maximize $\lvert V_6\rvert$ (0º)	V_5=0 (θ=-15º)	V_6= (-2.2+2.7i)×1e-3
Maximize $\lvert V_6\rvert$ (0º)	V_7=0 (θ=15º)	V_6= (-2.2+2.7i)×1e-3
Maximize $\lvert V_6\rvert$ (0º)	V_8=0 (θ=30º)	V_6= (-4.3+3.4)×1e-3
Maximize $\lvert V_6\rvert$ (0º)	V_9=0 (θ=45º)	V_6= (-3.6+3.1i)×1e-3
Maximize $\lvert V_6\rvert$ (0º)	V_{10}=0 (θ=60º)	Infeasible
Maximize $\lvert V_6\rvert$ (0º)	V_{11}=0 (θ=75º)	Infeasible

Table 1) Results of the optimization method

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