Broadcast Capacity in Multihop Wireless Networks

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ABSTRACT

In this paper we study the *broadcast capacity* of multihop wireless networks which we define as the maximum rate at which broadcast packets can be generated in the network such that all nodes receive the packets successfully in a limited time. We employ the Protocol Model for successful packet reception usually adopted in network capacity studies and provide novel upper and lower bounds for the broadcast capacity for arbitrary connected networks. In a homogeneous dense network these bounds simplify to $\Theta(W/\max(1, \Delta^d))$ where W is the wireless channel capacity, Δ the interference parameter, and d the number of dimensions of space in which the network lies. Interestingly, we show that the broadcast capacity does not change by more than a constant factor when we vary the number of nodes, the radio range, the area of the network, and even the node mobility. To address the achievability of capacity, we demonstrate that any broadcast scheme based on a backbone of size proportional to the Minimum Connected Dominating Set guarantees a throughput within a constant factor of the broadcast capacity. Finally, we demonstrate that broadcast capacity, in stark contrast to unicast capacity, does not depend on the choice of source nodes or the dimension of the network.

Categories and Subject Descriptors

Computer Systems Organization [**Computer- Communication Network**]: Network Architecture and Design Wireless communication; Data [**Coding and Information Theory**]: Formal models of communication

General Terms

Performance, Theory

Keywords

Broadcast Capacity, Broadcast Scheme, Ad Hoc Networks, Multihop Wireless Networks, Unicast Capacity

1. INTRODUCTION

In wireless networks, broadcast plays a particularly important role, relaying a message generated by one node to all other nodes. Broadcast is an integral part of a variety of protocols that provide basic functionality and efficiency to higher-layer services. Coordinated and distributed computing, a prime task in sensor networks, provides but one example. Also, multicast protocols and a host of unicast ad hoc routing protocols rely on broadcast, such as Dynamic Source Routing (DSR), Ad Hoc On Demand Distance Vector (AODV), Zone Routing Protocol (ZRP), and Location Aided Routing (LAR) [1].

There has been a growing interest to understand the fundamental capacity limits of wireless networks [2–9]. Results on network capacity are not only important from a theoretical point of view but also provide guidelines for protocol design in wireless networks. Hitherto, most research on network capacity has focused on the capacity of unicast connections between random source and destination nodes.

In this paper we study the capacity of wireless networks for broadcasting. We define the *broadcast capacity* $\lambda_{\mathcal{B}}$ of a multihop wireless network as the maximum rate of generation of broadcast packets by a set of nodes $\mathcal{B}$ in the network such that all nodes receive the packets successfully. To the best of our knowledge, so far only one paper studies the capacity of wireless networks for broadcasting [10]. That paper models the locations of the nodes by a Poisson point process and the wireless channel as Gaussian and considers $\lambda_{\mathcal{B}}$ when $\mathcal{B}$ consists of a single generating node. It proves that the broadcast capacity is a constant factor of a computed upper bound when the Poisson intensity of the nodes is fixed and the number of nodes goes to infinity. The paper does not derive bounds for the broadcast capacity when the number of nodes is finite or when the nodes have a different distribution in the plane than Poisson.

As the first contribution of this paper, we develop novel bounds of the broadcast capacity $\lambda_{\mathcal{B}}$ for a general wireless network with a finite number of nodes, an arbitrary topology, and for an arbitrary set of generating nodes $\mathcal{B}$. We assume that the wireless channel capacity has a fixed rate W (bits per second) and model successful packet receptions with the Protocol Model, a popular model for wireless channels in the literature on network capacity [2, 11–13].

Surprisingly, we find that the broadcast capacity does not change by more than a constant factor when we vary the number of nodes, the radio range and the area of the network. For the special case of large homogeneous networks we find that the broadcast capacity is $\Theta(\frac{W}{\max(1,\Delta^d)})$ where Δ

is the interference parameter of the wireless channel and d is the number of dimensions of the space in which the network lies. We adopt standard notation from complexity theory; $O(.)$, $\Omega(.)$, and $\Theta(.)$ describe asymptotic upper, lower, and tight bounds respectively.

Interestingly, we prove that mobility cannot significantly increase the broadcast capacity of wireless networks. In contrast to unicast capacity which can increase by as much as $\Omega(\sqrt{n})$ (n is the number of network nodes in a fixed area) with the help of mobility [5], the broadcast capacity changes at most by a factor of $O(\max(1, \Delta^d))$ with mobility.

As our second contribution, we study the throughput of *broadcast schemes* of wireless multi-hop networks. The simplest approach to broadcasting is *blind flooding*, in which every node rebroadcasts the packet. Blind flooding, however, produces redundant broadcasts and wastes precious bandwidth and power [14]. Many broadcast schemes have been proposed for ad hoc networks that are far more efficient than flooding (see [15–17] and references therein). So far, no theoretical analysis of the achievable throughput of these schemes has been performed.

We define the *maximum throughput* $\lambda_{S,\mathcal{B}}$ of a given broadcast scheme, S, as the maximum aggregate rate of generation of broadcast packets by a set of nodes $\mathcal{B}$ in the network such that the scheme can disseminate the packets to all nodes successfully. We establish a close tie between $\lambda_{S,\mathcal{B}}$ and the backbone size of a broadcast scheme S. Indeed, for $\lambda_{S,\mathcal{B}}$ to be within a constant factor of the broadcast capacity it is necessary and sufficient that its backbone uses a bounded number of nodes per radio range area. Such backbones always have size within a constant factor of the size of the Minimum Connected Dominating Set (MCDS).

As our third contribution, we highlight the fundamental differences between unicast and broadcast capacity, in particular we compare and contrast how they are influenced by changes in the radio range (R) and the interference parameter (Δ). As mentioned earlier, for any network topology the broadcast capacity does not change more than by a constant when the radio range varies. However, varying the radio range strongly influences the unicast capacity for networks in two or three dimensional space. Curiously in one dimensional space, because unicast and broadcast are very similar, the radio range does not change the unicast capacity more than by a constant just like for the broadcast capacity.

We find that when the interference parameter (Δ) is large it has the same effect on both unicast and broadcast capacity; both vary according to $\Theta(\Delta^{-d})$. However, if Δ approaches zero then in some networks the unicast capacity can become as large as $\Theta(n)$, unlike the broadcast capacity which will vary at most by a constant factor.

The paper is organized as follows. In Section 2 we summarize existing work on the network capacity. We introduce a wireless network model and define relevant terms in Section 3. In Section 4 we compute upper and lower bounds for broadcast capacity for general wireless networks. We also derive specific results for the broadcast capacity for homogeneous dense networks. Section 5 studies the maximum throughput of broadcast schemes and compares them to the broadcast capacity. In Section 6 we compare the variation of broadcast and unicast capacities with different network parameters. Finally, we conclude the paper in Section 7. All proofs are placed in the Appendix.

2. RELATED WORK

Gupta and Kumar [2] study the network capacity for unicast connections between random sources and destinations in static wireless networks consisting of n nodes distributed in a circle of area A with wireless channel capacity W. They define the "transport capacity" of a wireless network with units of bit-meters per second as the maximum rate of the packets times the distance they travel between the source and the destination. Their main result is that the aggregate transport capacity of unicast connections is $\Theta(W\sqrt{An})$ in an arbitrary network with optimally placed nodes and $\Theta(W\sqrt{An/\log(n)})$ in a random network where the nodes are placed uniformly. As a result, the capacity of the network per node is $\Theta(W\sqrt{A/n})$ (in random networks $\Theta(W\sqrt{A/n\log(n)})$) when n grows in a fixed area. In order to achieve a throughput within a constant factor of the capacity, the radio ranges of the nodes must be set equal to $\Theta(\sqrt{A/n})$ (in random networks $\Theta(\sqrt{A\log(n)/n})$). The same authors also analyze three dimensional networks [11]. They prove that if the nodes are distributed in a sphere with volume V then the aggregate transport capacity is $\Theta(W\sqrt[3]{Vn^2})$.

Several other papers have enhanced the theory of network capacity. The results of Gupta and Kumar were generalized for a more accurate channel model [3]. Using percolation theory techniques [18] it was proved that $\Theta(W\sqrt{A/n})$ is achievable in random networks with high probability [4].

For wireless mobile networks, Grossglauser and Tse [5] show that per node capacity can be increased to $\Theta(1)$ if packet delay is left unbounded. They propose a mobility-based routing method in which the number of retransmissions of the unicast packets between source and destination is reduced to 2. Many other efforts demonstrate that there is a trade-off between the capacity and the delay in wireless mobile networks, for different mobility patterns and constraints on delay [19–25].

Introducing a new direction in network capacity research, Zheng studies the "broadcast capacity" of static wireless networks [10]. The paper models the locations of the nodes through a Poisson point process and the channel as a Gaussian wireless channel whose capacity is given by the Shannon Signal to Interference plus Noise Ratio (SINR). It then computes the asymptotic bottleneck of the network by using the distance properties of a Poisson points process and the SINR of the Gaussian channel. The idea is as follows. There exists a node with high probability that is at a large distance from all other nodes. Since the capacity of the channel between any pair of nodes decreases with the increasing distance between them, the maximum receiving rate is low for that node. This receiving rate provides an upper bound for broadcast capacity.

Zheng proves that the broadcast capacity is a constant factor of the computed upper bound when the Poisson intensity of the nodes is fixed and number of nodes goes to infinity. However, the paper does not address the issue whether the upper bound $O(\log n)$ is achievable or not, in the case when the intensity tends to infinity as the number of nodes grows, and leaves the problem for the future study.

This work does not derive bounds for the broadcast capacity when the number of nodes is finite or when the nodes have a different distribution than Poisson in the plane.

Our work differs from Zheng's work in several ways. First, we compute bounds of the broadcast capacity for general wireless networks with a finite number of nodes and an arbitrary topologies. Second, we also take into account the interference faced by packet transmissions of one node from transmissions of its neighbors which was neglected by Zheng in her derivation of an upper bound for capacity [10]. Our results show that this factor has a strong effect on broadcast capacity.

Note that all the above mentioned papers as well as this paper assume only point-to-point coding at the receivers. If the nodes are allowed to cooperate and use sophisticated multi-user coding then a per-node capacity of a higher order than that described above can be achieved [26–28]. A full discussion of these results is beyond the scope of this paper

3. WIRELESS CHANNEL MODEL AND BASIC NOTIONS

In this section we describe the wireless network model and define several terms relevant to our analysis of broadcast capacity.

3.1 Network and Connectivity

We consider a wireless network consisting of n wireless nodes. Let X_i for $i = 1, 2, \ldots, n$ denote the location of the different nodes. For simplicity we also use X_i to refer to the i^{th} node itself. All nodes use the same bit-rate W to transmit data.

We denote by $G(R)$ the geometric graph formed by the nodes when each node has transmission radius R. The vertices of $G(R)$ are the nodes of the network. Two nodes X_i and X_j are adjacent, that is joined by an edge, in $G(R)$ if and only if $|X_i - X_j| \leq R$. Note that increasing R can only increase the number of edges in $G(R)$.

A *dominating set* of a network graph is defined as the set of nodes such that every node in the network is either in the set or has an adjacent node which is in the set. In other words, a dominating set is the set of nodes in a wireless network which cover all nodes. A *Connected Dominating Set* (CDS) is a dominating set such that the subgraph induced by its nodes is connected. A *Minimum Connected Dominating Set* (MCDS) is a CDS of the graph with the minimum number of nodes. If a broadcast packet is received by all nodes in the network then the set of nodes which transmit the packet build a CDS. Clearly, an MCDS uses the minimum number of transmissions to disseminate the broadcast packet to all nodes.

An *Independent Set* is defined as a set of nodes such that no two of them are adjacent in the graph. Nodes of an independent set are spaced far apart from each other, that is they are *sparse* in the network. A *Maximum Independent Set* (MIS) is an independent set of the graph with the maximum size.

Note that the different sets we have defined above will change with the radio range. In this paper, we thus include the radio range R in the notation for the defined sets. For example MCDS(R_1) is an MCDS of $G(R_1)$ and MIS(R_2) is an MIS of $G(R_2)$. We use the symbol $\#$ for representing the size of a set.

In this paper we assume that for the radio range R the network is connected. When we vary the radio range of the nodes we assume that the range does not get so small that the network becomes disconnected.

3.2 Channel Model

We employ the Protocol Model for modeling successful transmissions [2, 11–13]. The rules for successful reception of a packet are as follows.

Assume that node X_i transmits a packet to node X_j. Then the transmission is successfully received by X_j if and only if

1. the Euclidean distance between X_i and X_j is less than R
$$|X_i - X_j| \leq R, \tag{1}$$
2. and for every node X_k that transmits during transmission of X_i to X_j
$$|X_k - X_j| \geq (1 + \Delta)R. \tag{2}$$

We refer to Δ as the *interference parameter*.

The circular area with radius R and center of X_i is called *transmission area* of X_i. Only nodes located within this area can receive packets from X_i successfully. The larger circular area with radius $(1 + \Delta)R$ and centered at X_i is called the *interference area* of X_i. During a transmission from X_i, any node X_j within this area is blocked from receiving a packet from any node other than X_i. The annular part of the interference area that lies outside of transmission area is called the *shadow area*. It follows that the nodes in the shadow area of X_i do not receive any packet successfully while X_i is transmitting.

The Protocol Model allows us to analyze the broadcast capacity for general network topologies and apply graph theoretic methods for the analysis. Furthermore, this model is easier to understand than Physical Models such as the ones of [2, 3] and it allows to study the effects of interference in terms of the simple parameter Δ on the network capacity.

3.3 Broadcast Capacity and Maximum Throughput

We define broadcast capacity for a subset $\mathcal{B} := \{\mathcal{B}_1, \mathcal{B}_2, \ldots\}$ of nodes that generate broadcast packets. The reason we do so is that in some networks only a few nodes may be required to broadcast packets. In such cases we are interested to know the maximum rates at which this particular subset of nodes can successfully broadcast packets rather than the maximum broadcast rates when all nodes (or only a single node) generate broadcast packets.

Assume that $\mathcal{B}_i$ generates packets at rate $\lambda_{\mathcal{B}_i} \geq 0$. We say that the rate vector $[\lambda_{\mathcal{B}_i}]_i$ is *achievable* if all nodes of the network receive all generated broadcast packets successfully within some given time $T_{\max} < \infty$.

In this paper we study the maximum achievable broadcast rates for originating nodes $\mathcal{B}$ when the fraction of the aggregate rate that each node uses is prespecified. That is, given a vector of weights $\mathbf{g} = [g_i]_{i=1}^{\#\mathcal{B}}$, $g_i > 0$ such that $\sum_i g_i = 1$ we study the *broadcast capacity*

$$\lambda_{\mathcal{B}}(\mathbf{g}) := \sup\{a : \lambda_{\mathcal{B}_i} = g_i a, [\lambda_{\mathcal{B}_i}]_i \text{ is achievable}\} \tag{3}$$

and the *maximum throughput* of scheme S

$$\lambda_{S,\mathcal{B}}(\mathbf{g}) := \sup\{a : \lambda_{\mathcal{B}_i} = g_i a, [\lambda_{\mathcal{B}_i}]_i \text{ is achievable for } S\}. \tag{4}$$

The broadcast capacity, $\lambda_{\mathcal{B}}(\mathbf{g})$, and maximum throughput of scheme S, $\lambda_{S,\mathcal{B}}(\mathbf{g})$, are related:

$$\lambda_{S,\mathcal{B}}(\mathbf{g}) \leq \sup_S \lambda_{S,\mathcal{B}}(\mathbf{g}) = \lambda_{\mathcal{B}}(\mathbf{g}). \tag{5}$$

We later derive bounds for broadcast capacity $\lambda_{\mathcal{B}}(\mathbf{g})$, and maximum throughput of scheme S, $\lambda_{S,\mathcal{B}}(\mathbf{g})$. We find that the corresponding lower and upper bounds are always within a constant factor of each other and independent of $\mathbf{g}$. We thus subsequently drop the argument $\mathbf{g}$ and simply refer to broadcast capacity as $\lambda_{\mathcal{B}}$ and maximum throughput of S as $\lambda_{S,\mathcal{B}}$.

4. BROADCAST CAPACITY OF WIRELESS NETWORKS

In this section we compute bounds for the broadcast capacity of wireless networks which determine the capacity up to a small constant factor. We first prove bounds that apply to arbitrary connected networks. We then improve these bounds for homogeneous dense networks and finally discuss whether mobility can improve the broadcast capacity.

We assume throughout that the transmission radio range R of nodes is large enough to ensure that static networks are connected and that mobile networks stay connected with probability one as the nodes move.

4.1 Broadcast Capacity for General Wireless Networks

We now determine different upper and lower bounds for the broadcast capacity that apply to any arbitrary connected wireless network. The accuracy of any one of these bounds varies with the network scenario. While one bound may be more accurate than another bound in one particular network, in a different network the opposite may be true.

Ideally we would like to compute the broadcast capacity up to a constant factor. One way to do this is to determine upper and lower bounds that are *tight*, that is if they differ by at most a constant factor.

Our first bounds are summarized in Theorem 1. Since in a successful broadcast every node must receive the data, the broadcast capacity cannot be higher than the maximum data rate at which a node can receive data. We thus obtain the upper bound, W, which is a hard upper bound for any network (small, large, static or mobile).

We obtain the lower bound with the help of the MCDS of a network. Note that given an MCDS, we can broadcast a message throughout the network by making each MCDS node retransmit it once. As we demonstrate in the proof, this can always be performed with less than $\#\text{MCDS}(R)+1$ transmissions which gives the lower bound.

THEOREM 1. *For an arbitrary connected wireless network*

$$\frac{W}{\#\text{MCDS}(R)+1} \leq \lambda_{\mathcal{B}} \leq W. \tag{6}$$

The lower bound in Theorem 1 can be difficult to evaluate in practice for any arbitrary network. We present bounds that are easier to evaluate in the next theorem. Theorem 2 shows that if the nodes can be covered by a dominating set of size M then the broadcast capacity is larger than W times a factor that depends only on M. To use this lower bound all we have to do is find a dominating set and then compute its size M. Clearly the bound approaches W when M becomes smaller. One practical way to reduce M is to increase the radio range of nodes, R.

THEOREM 2. *If in a connected wireless network all nodes are covered by M transmissions, then*

$$\frac{W}{3M-1} \leq \lambda_{\mathcal{B}} \leq W. \tag{7}$$

Following a different line of reasoning, Theorem 3 gives another lower bound for the broadcast capacity which only depends on the interference parameter Δ. This bound outperforms the lower bound of Theorem 1 for very large networks. We derive the result using a TDMA scheduling method that reduces the interference between simultaneous transmissions. As a consequence this bound improves on the one in Theorem 1 which was derived by allowing only one transmission at any given time.

THEOREM 3. *For an arbitrary connected wireless network in the plane*

$$\frac{W}{21\lceil 1+\sqrt{2}(2+\Delta)\rceil^2} \leq \lambda_{\mathcal{B}}. \tag{8}$$

Note that the interference parameter Δ is usually a small number in wireless ad hoc networks. Thus the denominator of the lower bound in Theorem 3 will in practice not get very large.

The bounds we find for the network capacity in Theorems 1 and 3 are enough to determine the network capacity up to a constant. We next go further and study more carefully the effect of interference and topology of the network on the broadcast capacity. We assume that the network is large enough to contain at least two nodes which are outside each other's interference area.

Theorem 4 computes an upper bound in terms of the ratio of the size of MCDS(R) to the size of MIS(ΔR). Essentially, #MCDS(R) equals the minimum number of retransmission required to broadcast a packet and #MIS(ΔR) the maximum possible number of successful simultaneous transmissions in the network possible. We call an individual transmission of the broadcast packet *successful* if at least one node receives the transmitted packet successfully. Intuitively, the ratio #MCDS(R)/#MIS(ΔR) thus approximately represents the number of packet transmission time units required to broadcast a message. Consequently, the broadcast capacity is inversely proportional to this quantity.

THEOREM 4. *For an arbitrary connected wireless network*

$$\lambda_{\mathcal{B}} \leq W \cdot \frac{\#\text{MIS}(\Delta R)}{\#\text{MCDS}(R)}. \tag{9}$$

Corollary 5 shows that if $\Delta > 2$ and the network is large enough that there are at least two nodes which do not interfere with each other then the broadcast capacity has a smaller upper bound than that in Theorem 1. In addition, if the network size is so small as compared to the interference range that only one successful transmission per time can occur, then the broadcast capacity upper bound becomes $W/\#\text{MCDS}(R)$.

COROLLARY 5. *In an connected wireless network, if* $\#\text{MIS}(\Delta R) > 1$ *and* $\Delta > 2$ *then*

$$\lambda_{\mathcal{B}} \leq \frac{W}{\lceil \frac{\Delta-2}{2} \rceil}$$

and if $\#\mathrm{MIS}(\Delta R) = 1$ *then*

$$\lambda_{\mathcal{B}} \leq \frac{W}{\#\mathrm{MCDS}(R)}$$

.

The results of the Theorems 1, 3 and 4 can be summarized as follows:

Case (i): If $\#\mathrm{MIS}(\Delta R) = 1$

$$\frac{W}{1 + \#\mathrm{MCDS}(R)} \leq \lambda_{\mathcal{B}} \leq \frac{W}{\#\mathrm{MCDS}(R)}. \qquad (10)$$

Case (ii): If $\#\mathrm{MIS}(\Delta R) > 1$ and $\Delta \leq 2$

$$\max(\frac{W}{1 + \#\mathrm{MCDS}(R)}, \frac{W}{21\lceil 1 + \sqrt{2}(2 + \Delta)\rceil^2}) \leq \lambda_{\mathcal{B}} \leq W. \qquad (11)$$

Case (iii): If $\#\mathrm{MIS}(\Delta R) > 1$ and $\Delta > 2$

$$\max(\frac{W}{1 + \#\mathrm{MCDS}(R)}, \frac{W}{21\lceil 1 + \sqrt{2}(2 + \Delta)\rceil^2}) \leq \lambda_{\mathcal{B}} \frac{W}{\lceil \frac{\Delta - 2}{2} \rceil}. \qquad (12)$$

We have already provided some intuition for cases (i) and (ii) listed above. We now discuss the third case which makes more explicit the effect of the interference parameter on the broadcast capacity.

Case (iii) considers a network that is large compared to the interference area of a single node. In addition it assumes that the interference parameter is very large. We observe from (12) that the upper bound decreases at a markedly slower rate with Δ than the lower bound. While the upper bound has order $\Theta(\frac{W}{\Delta})$, the lower bound has order $\Theta(\frac{W}{\Delta^2})$.

Interestingly, there exist two very different classes of network topologies such that for one the broadcast capacity tracks the upper bound of (12) and for the other the broadcast capacity tracks the lower bound of (12). Consider the topology in which the nodes are distributed on a line. For this topology the capacity is a constant factor of the upper bound. Now consider the topology in which the nodes are homogeneously distributed and dense in a plane. Here the capacity is a constant factor of the lower bound. The broadcast capacity for these two cases will be studied in greater detail in Theorem 8.

This observation motivates us to seek a more accurate relationship between topology and broadcast capacity. Let us suppose that the topology is such that during a broadcast we are forced to retransmit the same packet K_i times within the interference range of a particular node X_i. Then X_i can receive broadcast data at a maximum rate of W/K_i. We conclude that $W/\max_i(K_i)$ is an upper bound of broadcast capacity, which is caused by interference.

We now approximate the quantity $\max_i(K_i)$ using an $\mathrm{MIS}(R)$. Because an $\mathrm{MIS}(R)$ is an independent dominating set, the number of its elements that lie in the interference range of X_i is approximately equal to the number of times we retransmit the same packet within its interference range, K_i. Consequently, we approximate $\max_i(K_i)$ by the *bottleneck factor*, $K(R)$, which we define for a particular $\mathrm{MIS}(R)$ as the maximum number of its elements that lie in the interference range of any single node. Note that $K(R)$ is a function of radio range R.

Theorem 6 provides tight bounds on the capacity in terms of the bottleneck factor.

THEOREM 6. *Assume that in a connected wireless network* $\#\mathrm{MIS}(\Delta R) > 1$*. Then, there are positive constant numbers* c_1 *and* c_2 *independent of the network parameters such that*

$$c_1 \frac{W}{K(R)} \leq \lambda_{\mathcal{B}} \leq c_2 \frac{W}{K(R)}. \qquad (13)$$

Moreover, we can show that $\lceil \frac{\Delta+1}{3} \rceil \leq K(R) \leq \lfloor 4(\Delta+1.5)^2 \rfloor$.

4.2 Broadcast Capacity for Homogeneous Wireless Networks

We now study the broadcast capacity for a wireless network model which is widely employed in studying unicast network capacity. In this model, the nodes are distributed uniformly in a circle or square of area A, the number of nodes grows to infinity.

The broadcast capacity bounds have a simple form in homogeneous dense networks because the maximum number of simultaneous transmissions and the MCDS size can be estimated in terms of area and radio range for these networks. Moreover, the bounds we compute here are closer than the bounds which we computed before for arbitrary wireless networks in Section 4.1.

Theorem 7 gives tight bounds for the broadcast capacity in a homogeneous dense network where the nodes are distributed within a square area.

THEOREM 7. *If the nodes are uniformly distributed in a square with area* $A \geq \frac{1}{4}\Delta^2 R^2$*, and* $R \geq \sqrt{\frac{15A\log(n)}{n}}$ *then,*

$$\frac{W}{\lceil 2 + (1 + \Delta)\sqrt{5}\rceil^2} \leq \lambda_{\mathcal{B}} \leq W \min(1, \frac{96}{\pi\Delta^2}) \qquad (14)$$

almost surely for large n*.*

Theorem 8 generalizes this result for the homogeneous networks where the nodes are distributed in a d-dimensional cube.

THEOREM 8. *If the nodes are uniformly distributed in a* d*-dimensional cube with volume* $V \geq c\Delta^d R^d$ *(*$c > 0$ *a constant number) and* $R \geq \sqrt{d+3}\sqrt[d]{\frac{3V\log(n)}{n}}$*, then there exist positive numbers* c_1 *and* c_2 *independent of network parameters, such that*

$$c_1 \frac{W}{\max(1, \Delta^d)} \leq \lambda_{\mathcal{B}} \leq c_2 \frac{W}{\max(1, \Delta^d)} \qquad (15)$$

almost surely for large n*.*

It is well known that the radio range R must be larger than $R_c := \sqrt[d]{\frac{V\log(n)}{n}}$ in order to have connectivity in the network with high probability [29]. We see that $R \geq \sqrt{d+3}\sqrt[d]{3}R_c$ satisfies the condition of Theorem 8. One might wonder if (15) holds when $R_c < R \leq \sqrt{d+3}\sqrt[d]{3}R_c$. Indeed, it can be shown that this is the case. The proof of this result is however quite involved and is hence not included in this paper.

4.3 Broadcast Capacity for Mobile Networks

Previous work by Grossglauser and Tse [5] showed that mobility can considerably increase the aggregate unicast capacity of homogeneous dense networks to $\Theta(n)$. The question is: Can mobility similarly increase the broadcast capacity of homogeneous dense networks?

Our results in previous sections reveal that mobility can increase capacity by at most a factor that depends on Δ. From Theorem 1 we see that W is a hard upper bound for the broadcast capacity of any wireless network including mobile ones. Using this result and the lower bound of Theorem 8 we can bound the maximum increase in broadcast capacity that mobility can deliver. When $\Delta < 1$, mobility can increase the broadcast capacity by at most a constant factor of $1/c_1$ and when $\Delta \geq 1$ by at at most a factor Δ^d/c_1. Intuitively, a mobile network can have a higher capacity than a static network if mobility moves nodes away from each other's shadow areas.

In mobile networks, if we can engineer (control) the mobility of the nodes then the broadcast capacity can always be increased to W. For example, one way of doing this is to move all nodes in the network into the transmission area of one particular node of the network. Then this node can broadcast at rate W successfully. However, again the mobility does not increase the broadcast capacity more than a factor which only depends on Δ and d, in contrast to unicast capacity which can be increased by factor of $\Theta(n^{\frac{1}{d}})$ using the mobility.

5. MAXIMUM THROUGHPUT OF BROADCAST SCHEMES

In this section we study the maximum throughput of different broadcast schemes. Several schemes have been proposed in the literature that efficiently broadcast a packet (see [15–17] and references therein). Although the efficiency of these schemes has been analyzed extensively, no work has been performed to analyze their maximum throughput and compare it with the broadcast capacity.

We show that the maximum throughput of a broadcast scheme depends on properties such as the size and topology of the broadcast backbones it uses. The size of the backbone has a strong influence on the throughput because it represents the number of retransmissions for every broadcast packet. Intuitively, the broadcast schemes with smaller backbone sizes are more efficient in utilizing the capacity of the network and so can have higher throughput. Theorem 9 provides a lower bound in terms of backbone size for the maximum throughput of the broadcast scheme.

THEOREM 9. *Let* Backbone(R) *be the broadcast backbone of a given broadcast scheme with the radio range R in a connected wireless network. Then,*

$$\frac{W}{\#\text{Backbone}(R)+1} \leq \lambda_{S,\mathcal{B}} \leq \lambda_{\mathcal{B}} \leq W.$$

Theorem 10 computes an upper bound for the maximum throughput of a given scheme in terms of the backbone size.

THEOREM 10. *Let* Backbone(R) *be the broadcast backbone of a connected wireless network. Then,*

$$\lambda_{S,\mathcal{B}} \leq W \frac{\#\text{MIS}(\Delta R)}{\#\text{Backbone}(R)}. \quad (16)$$

From Theorem 10 we see that if $\#\text{MIS}(\Delta R)$ is bounded above by a constant number as the number of nodes grows then the upper bound of capacity in (16) and the lower of Theorem 9 are tight. In practice this condition will hold if the area of the network remains constant as the number of nodes increases.

For the rest of the discussion we require notions of efficient and inefficient broadcast schemes. An *efficient broadcast scheme* is a scheme that bounds the number of broadcast nodes per radio range. As a result it always builds a backbone of size within a constant factor of the size of the MCDS. Some schemes which build broadcast backbones using an MIS have been proved to be efficient [30–37]. We refer to all other broadcast schemes as *inefficient broadcast schemes*.

We prove in Theorem 11 that for efficient broadcast schemes the maximum throughput is always within a constant factor of the broadcast capacity. Note that in our analysis we only take into account the effect of backbone size on capacity and not other factors such as the scheduling of the packet transmissions from different nodes which is a MAC layer operation.

THEOREM 11. *For an efficient broadcast scheme there is a positive number c which depends only on the scheme (and not the network parameters) such that for an arbitrary connected wireless network*

$$c\lambda_{\mathcal{B}} \leq \lambda_{S,\mathcal{B}} \leq \lambda_{\mathcal{B}}.$$

We show in Theorem 12 that for inefficient broadcast schemes, there exists a sequence of networks of increasing size for which the maximum broadcast throughput tends to zero. A simple example of this situation is when number of nodes in a bounded area goes to infinity and we use blind flooding for broadcast. However, recall from Theorem 3 that the broadcast capacity $\lambda_{\mathcal{B}}$ has a positive lower bound. We conclude that $\lambda_{S,\mathcal{B}}/\lambda_{\mathcal{B}}$ cannot be lower bounded by a positive constant for inefficient broadcast schemes. Thus inefficient schemes must not be employed for broadcast in dense networks.

THEOREM 12. *If in a connected wireless network broadcast scheme $\Delta > 0$ and $\frac{\#\text{MCDS}(R)}{\#\text{Backbone}(R)} \to 0$ as number of nodes $n \to \infty$ then*

$$\lambda_{S,\mathcal{B}} \to 0.$$

Theorem 12 not only emphasizes the *need for efficient broadcast* algorithms in dense networks, it also clearly indicates the danger and potential pitfalls for *neglecting interference* when analyzing of broadcast protocols. Indeed, We see that if $\Delta > 0$ then efficiency is a necessary and sufficient condition for a scheme to have maximum throughput within a constant factor of the broadcast capacity. However, note that if $\Delta = 0$ then efficiency is not a necessary condition any more. Figure 1 shows a network with interference parameter $\Delta = 0$. We see that the backbone size is $\frac{n-1}{2}$ and all backbone nodes on the small circle can send simultaneously to the nodes on the large circle. Hence when the node in the center is the originating node of broadcast the throughput equals $\frac{W}{2}$; half the time the center node transmits and during the other half all nodes on the small circle transmit

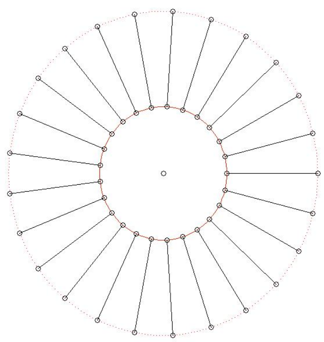

Figure 1: The radii of the small and large circles are aR and $(1+a)R$ respectively, where $0 < a < 1$. Since the interference parameter is zero, if all nodes on the small circle transmit simultaneously then the nodes on the large circle will receive successfully.

simultaneously. However, the MCDS size for this network is less than 13 irrespective of n. Thus we see that although $\frac{\#\text{MCDS}(R)}{\#\text{Backbone}(R)} \to 0$, the throughput is positive.

6. COMPARISON OF THE BROADCAST AND UNICAST CAPACITIES

In this section we highlight some fundamental differences between broadcast capacity and unicast capacity. We do so by comparing how the broadcast and unicast capacity of wireless networks vary with radio range R and interference parameter Δ. Finally, we study the capacity per node for both unicast and broadcast. All the analysis in this section pertains to homogeneous dense networks.

6.1 Effect of Radio Range

As discussed in Section 4, changing the radio range does not change the broadcast capacity by more than a constant factor.

Unicast capacity, in stark contrast, has been shown to depend strongly on R [2,11]. It has been shown that for nodes distributed uniformly in a circle with area A, the maximum number of simultaneous transmissions is $\Theta(\frac{A}{\Delta^2 R^2})$ and unicast packets need to be retransmitted on average $\Theta(\frac{\sqrt{A}}{R})$ times. Therefore the aggregate capacity of the network is limited to $\Theta(W\frac{A}{\Delta^2 R^2}/\frac{\sqrt{A}}{R}) = \Theta(W\frac{\sqrt{A}}{\Delta^2 R})$. The same technique can be applied when the nodes are distributed in a sphere with volume V to prove that network capacity has order $\Theta(W\frac{V}{\Delta^3 R^3})/\frac{\sqrt[3]{V}}{R}) = \Theta(W\frac{\sqrt[3]{V^2}}{\Delta^3 R^2})$. Both cases show that the radio range must be minimized in order to maximize throughput.

We now consider a one dimensional space and assume that the nodes are distributed uniformly on a line segment of length D. The maximum number of simultaneous transmissions is $\Theta(\frac{D}{\Delta R})$ and the average number of retransmissions for unicast packets is $\Theta(\frac{D}{R})$. Consequently, the network capacity becomes $\Theta(W\frac{D}{\Delta R}/\frac{D}{R}) = \Theta(\frac{W}{\Delta})$. Just like for broadcast capacity we see here that unicast capacity becomes independent of the radio range and the size of the area.

In order to better understand this similarity between broadcast and unicast capacity in one dimensional networks, observe that when the maximum number of simultaneous transmissions is proportional to the average number of unicast (resp. broadcast) retransmissions then the unicast (broadcast) capacity becomes proportional to W. Since for a broadcast packet the average number of retransmissions is proportional to the maximum number of simultaneous transmissions, the capacity of the network for broadcasting packets will remain constant and not depend on the radio range. The same thing happens to unicast packets when the nodes are distributed on a line.

6.2 Effect of Interference Parameter

The broadcast capacity of large homogeneous networks is $\Theta(\frac{W}{\max(1,\Delta^d)})$ which does not depend on Δ when $\Delta < 1$. When $\Delta > 1$ the broadcast capacity decreases by the factor Δ^d.

Earlier studies of capacity set the inference parameter (Δ) to a constant value [2,11]. The impact of this parameter on capacity, in particular if it is very small or very large, has not been studied. Addressing this issue is important because Δ appears in unicast capacity bounds. In fact, a very small or large value of Δ affects the unicast capacity significantly. When the inference parameter is large ($\Delta >> 1$), the unicast capacity will decrease by a factor of $\Theta(\frac{1}{\Delta^d})$.

In some cases, when the interference parameter is small ($\Delta << 1$) the unicast capacity increases largely. Earlier work presents the upper bound $\frac{\sqrt{8An}}{\pi}\frac{W}{\Delta}$ and $\sqrt[3]{\frac{6V}{\pi}}\frac{W\sqrt[3]{n^2}}{\Delta}$ for the aggregate unicast capacity in 2 and 3 dimensional space [2,11]. Figure 2 shows a wireless network in the 2 and 3 dimensional space and a particular traffic pattern. If the horizontal and vertical spacing between nodes in Fig. 2 are related through $\varepsilon \geq \sqrt{3\Delta}$, we can easily show that the aggregate transport capacities are $W\sqrt{\frac{n}{2\varepsilon(1+\Delta)}}$ and $W\sqrt[3]{\frac{n^2}{4\varepsilon^2(1+\Delta)}}$ for sufficiently large n. If we set $\varepsilon = \sqrt{3\Delta}$ then we see that the aggregate unicast capacities of these networks are proportional to $\frac{1}{\sqrt[4]{\Delta}}$ and $\frac{1}{\sqrt[3]{\Delta}}$. In the special case of $\Delta \to 0$, we can choose $\varepsilon = O(1/n) \geq \sqrt{3\Delta}$ and the aggregate unicast capacity becomes $\Theta(n)$.

6.3 Capacity per node

We here consider the broadcast capacity *per node*, that is $\lambda_{\mathcal{B}}$ divided by the number of nodes n. If $\mathcal{B}$ consists of all nodes in the network and all nodes get an equal share of the broadcast capacity, then the broadcast capacity per node is $\Theta(\frac{W}{n})$ which is less than the unicast capacity per node by a factor of $\Theta(\sqrt{n})$ in static networks and by a factor of $\Theta(n)$ in mobile networks.

However, $\mathcal{B}$ and $\mathbf{g}$ (see Section 3) can be chosen arbitrarily, that is only a few nodes can generate broadcast packets and these can also share the bandwidth unequally between them. For arbitrary $\mathcal{B}$ and $\mathbf{g}$ we proved that we can broadcast at aggregate rate $\Theta(W)$, that is within a constant factor of $\sup_{\mathcal{B}} \lambda_{\mathcal{B}}$. That means the capacity for broadcasting is flexible for any choice of $\mathcal{B}$ and $\mathbf{g}$ in the network. In the case of unicast, however, we do not enjoy such flexibility. In order to achieve the unicast capacity, the source and destination nodes must have special locations in the network and send data at appropriate rates.

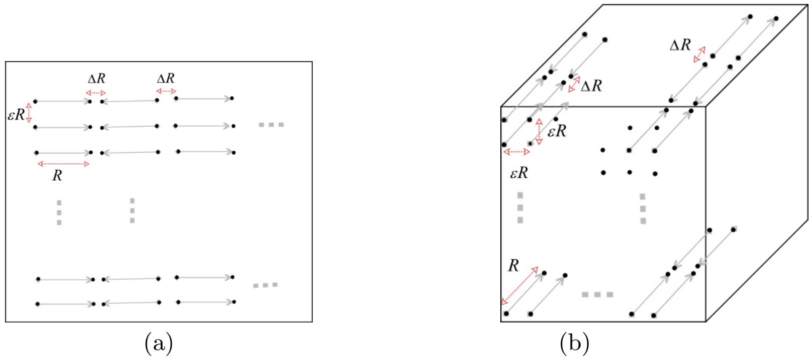

Figure 2: Wireless networks in 2 and 3 dimensional space. The flows do not interfere with each other if $\varepsilon \geq \sqrt{3}\Delta$. The aggregate transport capacities of these networks are $W\sqrt{\frac{n}{2\varepsilon(1+\Delta)}}$ and $W\sqrt[3]{\frac{n^2}{4\varepsilon^2(1+\Delta)}}$ for large n.

7. CONCLUSION AND FUTURE WORK

We have proved using the Protocol model that the broadcast capacity of wireless networks is proportional to the wireless channel capacity (W). Furthermore, it does not change by more than a constant factor when the radio range, the area of the network and the number of nodes in the network vary. However, we explicitly computed the effect of the interference parameter Δ on the capacity which depends on the topology of general networks. In the particular case of homogeneous dense networks the broadcast capacity decreases by a factor of $O(\frac{1}{\max(1,\Delta^d)})$. Interestingly, we showed that in contrast to the impact of mobility on unicast capacity, mobility cannot change the broadcast capacity by more than a factor which only depends on the interference parameter Δ.

In addition, we studied the maximum throughput of broadcast schemes for wireless multihop networks. We proved that a necessary and sufficient condition for a broadcast scheme to achieve a maximum throughput within a constant factor of the broadcast capacity is that it relay on a backbone which has a bounded number of nodes per radio range area. Finally, we studied the fundamental differences between unicast and broadcast capacity and the effects of network parameters on them both. We found that broadcast capacity does not depend on the choice of source nodes or the dimension of the network unlike unicast capacity which does.

For future work, we will study the broadcast capacity in mobile wireless networks under different mobility models, for example random uniform mobility and engineered (controlled) models. In addition, we will investigate how different channel models like the physical model [2] and Shannon channel model [3, 10] influence the broadcast capacity.

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Appendix

Proof of Theorem 1: Consider an arbitrary node X_i in the network. The maximum rate of transmission or reception of data by X_i is W.

Since X_i must either receive or generate all broadcast packets, the broadcast capacity has a hard upper bound of W, irrespective of the mobility of the nodes.

To prove the lower bound, we design a TDMA scheme that provides a broadcast rate equal to $\frac{W}{\#\mathrm{MCDS}(R)+1}$. Call the different nodes of $\mathcal{B}$, $\mathcal{B}_i$ ($i = 1, 2, \ldots, \#\mathcal{B}$). Set $m = \#\mathrm{MCDS}(R)$. First $\mathcal{B}_i$ ($i = 1$) transmits w_i bits at rate W to its MCDS neighbor with the lowest index. Then in each of the next m time slots of length w_i/W the MCDS node with the lowest index that has not yet transmitted the received w_i bits rebroadcasts them to all nodes in its radio range. We thus broadcast w_i bits to the entire network in time $(m+1)w_i/W$, that is at rate $W/(m+1)$. We repeat the same procedure for $i = 2, 3, \ldots, \#\mathcal{B}, 1, \ldots$ of set $\mathcal{B}$. Node $\mathcal{B}_i$ thus generates broadcast data at rate

$$\lambda_{\mathcal{B}_i} = \frac{w_i}{\frac{m+1}{W}\sum_i w_i} = \frac{W}{m+1}\frac{w_i}{\sum_i w_i} \tag{17}$$

and $\lambda_{\mathcal{B}} = W/(m+1)$. From (17), we see that by choosing the w_i's appropriately we can support any rates $\lambda_{\mathcal{B}_i}$ that sum to $W/(m+1)$. $\square$

Proof of Theorem 2: Consider M nodes which cover all nodes of the network. These nodes build a dominating set. We can build a CDS by connecting 2-hop and 3-hop away nodes of the dominating set on a spanning tree [32]. Therefore at most $2(M-1)$ additional nodes are needed in order to build a CDS. Thus $\#\mathrm{MCDS}(R) \leq 3M - 2$. Theorem 1 then gives the result.

Proof of Theorem 3: We design a TDMA scheme for any arbitrary wireless network which has a broadcast throughput equal to the lower bound. We do so in three steps.

Step 1: Divide the network into cells and build a cell graph by connecting occupied cells.

Divide the plane into square cells with diagonal R such that the coordinates of their centers are $(i\frac{R}{\sqrt{2}}, j\frac{R}{\sqrt{2}})$ for $i, j \in \mathbb{Z}$ (see Fig. 3). Note that by design every pair of nodes in the same cell are within each other's radio range. Next, we build a "cell graph" over the occupied cells (the cells which contain at least one node); these cells are colored grey in Fig. 3. The vertices of the cell graph are the occupied cells and two cells are connected (adjacent) if there exists a pair of nodes, one in each cell, that are less than R distance apart. Because the network is connected it follows that the

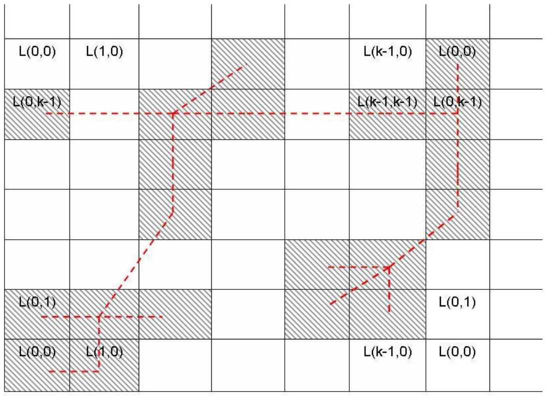

Figure 3: TDMA scheme collision free for broadcasting. It uses k^2 colors to schedule the cells transmissions

cell graph is connected. We then build a spanning tree over the cell graph which we use to route broadcast packets.

Step 2: Color the cells with k^2 colors $L(0,0), L(0,1), \dots, L(k-1,k-1)$ such that cells with the same color are far apart.

We assign color $L(r_i, r_j)$ to the cell with center $(i\frac{R}{\sqrt{2}}, j\frac{R}{\sqrt{2}})$, where $r_i = i(mod)k$. The value of k is chosen large enough such that when two nodes in different cells with the same color transmit simultaneously, all of the nodes in their transmission range can receive successfully. It is easy to show by geometry that $k = \lceil 1+\frac{R+(1+\Delta)R}{R/\sqrt{2}}\rceil = \lceil 1+\sqrt{2}(2+\Delta)\rceil$ is large enough to have this property. We then divide time into k^2 time slots which correspond to k^2 different colors. In each time slot only nodes in the cells with the corresponding color are allowed to transmit.

Step 3: Schedule packet transmissions among the nodes.

For every pair of adjacent cells on the spanning tree of the cell graph we choose two nodes which connect the cells to be relays. When a packet needs to be forwarded from cell $S1$ to adjacent cell $S2$, the relay in cell $S1$ forwards the packet to the relay in cell $S2$. The relay in $S2$ rebroadcast the packet to all nodes in $S2$. If $S2$ is not a leaf vertex on the spanning tree of the cell graph then its relays which connect it to other cells will forward the packet and the process continues this way till the broadcast packet disseminates to all nodes.

By geometry, it is easy to show that every cell has at most 20 adjacent cells. There are thus at most 20 relays in each cell. We divide the time slot corresponding to each color into 21 equal time slots of length T. In the first time slot corresponding to every color, each node $\mathcal{B}_i \in \mathcal{B}$ generates $WT\lambda_{\mathcal{B}_i}/(21k^2\lambda_{\mathcal{B}})$ bits for broadcast, if more than one $\mathcal{B}_i$'s are located in the same they broadcast the packets in some order after each other. In the remaining 20 time slots of any particular color, the relays of cells with that color transmit (to other nodes the cell or to the corresponding relay of adjacent cell) any broadcast data that they have received but not yet forwarded.

Note that every relay can forward packets at rate $\frac{W}{21k^2}$ with this setup. Thus when $\lambda_{\mathcal{B}} < \frac{W}{21k^2}$, the broadcast backlog at every relay will always be less than WT bits.

Proof of Theorem 4: First, we prove that the number of simultaneous successful transmissions in the network is bounded by $\#\text{MIS}(\Delta R)$. We call a broadcast transmission successful if at least one node receives the transmitted packet successfully. Consider two simultaneous transmitters X_i, X_j and assume that X_i is transmitting a bit b to a node X_r. For a successful transmission, it is necessary that $|X_i - X_r| \leq R$ and $|X_j - X_r| \geq (1+\Delta)R$. Then $|X_j - X_i| \geq |X_j - X_r| - |X_r - X_i| \geq \Delta R$. This means that simultaneous transmitters need to be at least distance ΔR apart from each other. Since $\text{MIS}(\Delta R)$ is the largest set of nodes with the property that every pair of its elements are at least ΔR apart from each other, $\#\text{MIS}(\Delta R)$ is an upper bound on the number of simultaneous transmissions. If the number of simultaneous transmission becomes larger than this, then some of transmissions become redundant because no node receives their packets successfully.

Assume that the broadcast packets are generated with the rate of $\lambda_{\mathcal{B}}$ bits per second. Denote by N_T the number of generated broadcast bits in $[0,T]$. By definition $\lambda_{\mathcal{B}} = \lim_{T\to\infty}\frac{N_T}{T}$. Also, denote by $N_B(b)$ the number of times any bit b is transmitted in order to be received by all nodes. The total number of bits that must be transmitted for the broadcast bits generated in $[0,T]$ is thus $\sum_{b=1}^{N_T}\sum_{i=1}^{N_B(b)} 1$. Since all broadcast packets are received in a limited time ($T_{\max}$), at time $T + T_{\max}$ all transmissions of N_T bits are finished. Therefore,

$$\sum_{b=1}^{N_T}\sum_{i=1}^{N_B(b)} 1 \leq \#\text{MIS}(\Delta R)(T+T_{\max})W \qquad (18)$$

Since $N_B(b) \geq \#\text{MCDS}(R)$

$$N_T\#\text{MCDS}(R) \leq \sum_{b=1}^{N_T}\sum_{i=1}^{N_B(b)} 1 \qquad (19)$$

By combining the two pervious inequalities we have

$$\lambda_{\mathcal{B}} = \lim_{T\to\infty}\frac{N_T}{T} \leq W\frac{\#\text{MIS}(\Delta R)}{\#\text{MCDS}(R)}\lim_{T\to\infty}\frac{T+T_{\max}}{T}. \qquad (20)$$

Proof of Corollary 5: If $\#\text{MIS}(\Delta R) > 1$ consider the circles with radius $\frac{\Delta R}{2}$ centered at nodes of a fixed $\text{MIS}(\Delta R)$. Since every two nodes of $\text{MIS}(\Delta R)$ are at distance larger than ΔR from each other, the circles are disjoint. Now consider a fixed $\text{MCDS}(R)$. Its nodes build some paths between the nodes at the centers of the circles. Since the MCDS nodes connect the center to some nodes outside the circle, we can show by geometry that the circle contains at least $\lceil\frac{\Delta}{2}-1\rceil$ MCDS nodes when $\Delta > 2$. Thus

$$\#\text{MIS}(\Delta R)\lceil\frac{\Delta}{2}-1\rceil \leq \#\text{MCDS}(R). \qquad (21)$$

By combing (21) with Theorem 4, we have

$$\lambda_{\mathcal{B}} \leq \frac{W\#\text{MIS}(\Delta R)}{\#\text{MCDS}(R)} \leq \frac{W}{\lceil\frac{\Delta}{2}-1\rceil}. \qquad (22)$$

The case $\#\text{MIS}(\Delta R) = 1$ is straightforward.

Proof of Theorem 6: First, we compute the upper and lower bounds for $K(R)$. Consider a node $X_u \in \text{MIS}(\Delta R)$. The circle $C(X_u, \Delta R)$ (X_u is the center and ΔR is the radius) does not contain all nodes in the network because $\#\text{MIS}(\Delta R) > 1$. Now consider the CDS built by connecting 2-hop and 3-hop away nodes in $\text{MIS}(R)$ [31, 32]. We can show by geometry that the CDS which connects X_u to the nodes outside of the circle contains at least $\lceil\frac{\Delta+1}{3}\rceil$

nodes of MIS(R). From the definition of $K(R)$ we thus have $K(R) \geq \lceil \frac{\Delta+1}{3} \rceil$.

To prove the upper bound, consider all nodes of MIS(R) in the interference range of a fixed node X_i. Since the nodes are more than R apart from each other, the circles with radius $\frac{R}{2}$ around the MIS(R) nodes are disjoint. The number of these circles whose centers are in the interference range of X_i is less than $\pi(\Delta+1+.5)^2R^2/\pi(.5R)^2 = 4(\Delta+1+.5)^2$ (the circles are disjoint inside $C(X_i,(1+\Delta)R+.5R)$). Hence $K(R) \leq 4(\Delta+1+.5)^2$.

We next compute c_1 by designing a TDMA scheme with a broadcast throughput larger than $\frac{W}{21*13(K(R)+32\Delta+80)}$. It follows that

$$\lambda_{\mathcal{B}} \frac{K(R)}{W} \geq \frac{K(R)}{21*13(K(R)+32\Delta+80)}. \tag{23}$$

Since we proved that $K(R) \geq \lceil \frac{\Delta+1}{3} \rceil$, it follows that

$$\frac{K(R)}{21*13(K(R)+32\Delta+80)} > 4*10^{-5} =: c_1. \tag{24}$$

Divide the plane into square cells and build the cell graph similar to the proof of Theorem 3. We color the cell graph vertices as follows. We consider a node (X_i) in an occupied uncolored cell, we check all colored cells which have at least one node inside of the circle $C(X_i,(\Delta+3)R)$ and then color the cell of X_i with the minimum color which has not been used for those cells. It is easy to show that with such a coloring scheme we can successfully broadcast to all nodes similar to the proof of Theorem 3.

Now we only need to compute a bound for the number of colors that are used for the cells. For every cell which has at least one node inside circle $C(X_i,(\Delta+3)R)$ fix one of its nodes in the circle. Consider the fixed nodes. Since MIS(R) is a dominating set, its nodes cover all fixed nodes. Clearly, the MIS(R) nodes which cover the fixed nodes are inside circle $C(X_i,(\Delta+4)R)$. By assumption, the number of MIS(R) nodes inside $C(X_i,(\Delta+1)R)$ is limited by $K(R)$. Also we show that the number of MIS(R) nodes in the area of $C(X_i,(\Delta+4)R)\backslash C(X_i,(\Delta+1)R)$ is less than

$$\frac{\pi(\Delta+4.5)^2R^2 - \pi(\Delta+1-.5)^2R^2}{\pi(.5R)^2} = 32\Delta + 80.$$

By a simple geometric argument we can show that every MIS(R) node covers at most 13 of the fixed nodes. Thus the number of cells represented by the fixed nodes is less than $13(K(R)+32\Delta+80)$. By applying the same broadcast scheme used in the proof of Theorem 3, we obtain that the throughput will be less than $\frac{W}{21(13(K(R)+32\Delta+80))}$ bits per second.

For computing c_2, we consider two cases and prove that $c_2 = 225$ satisfies both. First, if $K(R) \leq 25\Delta$ it is clear that $\lambda_{\mathcal{B}} \frac{K(R)}{W} \leq \frac{W}{\lceil \frac{\Delta-2}{2} \rceil} \frac{25\Delta}{W} \leq 225$. Second, if $K(R) \geq 25\Delta$ we consider the node X which has the maximum number of nodes of a MIS(R) in its interference range. We can bound the number of MIS(R) nodes inside the circle $C(X,(\Delta-1)R)$. Since the number of MIS(R) nodes in $C(X,(\Delta+1)R)\backslash C(X,(\Delta-1))$ is less than

$$\frac{(\Delta+1+.5)^2R^2 - (\Delta-1-.5)^2R^2}{(.5R)^2} = 24\Delta$$

the number of MIS(R) nodes inside $C(X,(\Delta-1)R)$ is at least $K(R) - 24\Delta R$.

In order to be able to transmit to all MIS(R) nodes in $C(X,(\Delta-1)R)$, a node must be located in circle $C(X,\Delta R)$. By geometry we can show that each transmission can cover at most five MIS(R) nodes. As a result every broadcast bit b must be transmitted at least $(K(R)-24\Delta)/5$ times in order to cover all MIS(R) nodes inside the circle $C(X,(\Delta-1)R)$. Clearly, the transmitters of the MIS(R) nodes are inside circle $C(X,\Delta R)$, and simultaneous transmitters must be ΔR from each other. Hence at most five successful simultaneous transmissions can occur at any time inside of $C(X,\Delta R)$. Similar to the proof of Theorem 4 we have

$$N_T(K(R)-24\Delta)/5 \leq \sum_{i=1}^{N_T}\sum_{b=1}^{M(b)} 1 \leq 5W(T+T_{\max}), \tag{25}$$

where $M(b)$ is the number of transmissions of bit b for it to reach all MIS(R) nodes in circle $C(X,(\Delta-1)R$. As $T \to \infty$ we have

$$\lambda_{\mathcal{B}} \frac{K(R)}{W} \leq \frac{25W}{K(R)-24\Delta} \frac{K(R)}{W} \leq 225. \tag{26}$$

Proof of Theorem 7: Divide the square area into square cells with side of length $\frac{R}{\sqrt{5}}$. It has been proved that if the side of the square cells is larger than $\sqrt{\frac{3A\log(n)}{n}}$ then there is at least one node in every cell almost surely for large enough n [12]. Since from our assumption $\frac{R}{\sqrt{5}} > \sqrt{\frac{3A\log(n)}{n}}$ we conclude that R is large enough to ensure that every cell contains at least one node. Consequently every cell is adjacent to its bordering cells in all four directions.

For broadcasting, every cell which receives the broadcast packet from a bordering cell forwards it to the other bordering cells by a single rebroadcast. This is possible because the radio range is large enough to cover all bordering cells simultaneously. We perform a TDMA scheduling similar to that in the proof of Theorem 3 and set $k = \lceil 2+(1+\Delta)\sqrt{5} \rceil$. The throughput becomes $\frac{W}{k^2}$ which gives the lower bound.

To prove the upper bound, we find a lower bound for #MCDS(R) using the cell structure explained above. We can show by geometry that every transmission covers less than 30 nodes in different cells. Since there are $A/(\frac{R}{\sqrt{5}})^2$ cells in the entire network,

$$\#\text{MCDS}(R) \geq \frac{5A}{R^2}/30. \tag{27}$$

We know that circles with radius $\Delta R/2$ centered at the MIS(ΔR) nodes are disjoint. Since the area A is a square with edges larger than $\Delta R/2$, at least $\pi(\Delta R/2)^2/4$ of these circles are inside it. Therefore

$$\#\text{MIS}(\Delta R) \leq \frac{A}{\pi\Delta^2R^2/16}. \tag{28}$$

By combining the inequalities (27), (28) and Theorem 4 we have

$$\lambda_{\mathcal{B}} \leq W\frac{A}{\pi\Delta^2R^2/16}/\frac{A}{6R^2} = \frac{96W}{\pi\Delta^2}. \tag{29}$$

Proof of Theorem 8: The technique of the proof is similar to that of Theorem 7. For the lower bound, we divide the space into d-dimensional cubic cells with edge length $\frac{R}{\sqrt{d+3}}$. Since, $\frac{R}{\sqrt{d+3}} > \sqrt{\frac{3A\log(n)}{n}}$, similar to Kulkarni's proofs we can show there is at least one node in each cell almost surely for large enough n [12]. In addition every cell is adjacent to the bordering cells in all $2d$ directions. For broadcasting, every cell which receives the broadcast packet from a bordering cell forwards it to the other bordering cells by a single rebroadcast. We color them with k^d colors $\{L(r_1, \ldots, r_d) : 0 \le r_i < k\}$. The parameter k is large enough to ensure that no two cells with the same color interfere with each other. For example, it can be shown that $k = \lceil 2 + (1+\Delta)R/\frac{R}{\sqrt{d+3}} \rceil = \lceil 2 + (1+\Delta)\sqrt{d+3} \rceil$ is large enough to satisfy this property. The throughput of this scheme is

$$\frac{W}{k^d} \ge \frac{1}{\lceil 2 + 2\sqrt{d+3} \rceil^d} \frac{W}{\max(1, \Delta^d)}. \quad (30)$$

Therefore, there exists $c_1 > 0$ which only depends on d such that $\lambda_{\mathcal{B}} > c_1 \frac{W}{\max(1,\Delta^d)}$.

We use Theorem 4 to prove the upper bound. Every transmission covers less than $\lceil 1 + 2\sqrt{d+3} \rceil^d$ nodes in different cells. Therefore the number of transmissions needed to cover all nodes is larger than $\frac{V}{(R/\sqrt{d+3})^d} / \lceil 1 + 2\sqrt{d+3} \rceil^d$. This give a lower bound for #MCDS(R). We next consider the d-dimensional spheres with radius $\Delta R/2$ centered at the MIS(ΔR) nodes. The fraction $c/2^d$ of the volume of each sphere is inside the cube which contains all nodes. Hence #MIS(ΔR) is bounded by $(2^d/c)(V/(\Delta R/2)^d)$. From Theorem 4 we find that

$$\begin{aligned} \lambda_{\mathcal{B}} &\le W \frac{\#\text{MIS}(\Delta R)}{\#\text{MCDS}(R)} \le W \frac{4^d V/(\Delta R)^d}{c\sqrt{d+3}^d V / \lceil 1 + 2\sqrt{d+3} \rceil^d R^d} \\ &= W \frac{4^d \lceil 1 + 2\sqrt{d+3} \rceil^d}{c\sqrt{d+3}^d \Delta^d}. \end{aligned}$$

This bound along with Theorem 1 prove that there exists $c_2 > 0$ such that $\lambda_{\mathcal{B}} < c_2 \frac{W}{\max(1,\Delta^d)}$.

Proof of Theorem 9: The proof is similar to the proof of Theorem 1. Here we consider the Backbone(R) instead of MCDS(R).

Proof of Theorem 10: The proof is similar the proof of Theorem 4. Here the number of transmissions for every broadcast bit b is at least #Backbone(R).

Proof of Theorem 11: If #MIS(ΔR) = 1, from 9 we have

$$\frac{W}{1 + \#\text{Backbone}(R)} \le \lambda_{S,\mathcal{B}}. \quad (31)$$

Since the scheme S is efficient, there is a positive number M_1 such that #Backbone(R) $\le M_1$#MCDS(R). Therefore

$$\frac{W}{1 + M_1\#\text{MCDS}(R)} \le \lambda_{S,\mathcal{B}} \quad (32)$$

since we know from Theorems 4 that $\lambda_{\mathcal{B}} \le \frac{W}{\#\text{MCDS}(R)}$, therefore we can show

$$\begin{aligned} \lambda_{S,\mathcal{B}} &\ge \frac{W}{M_1\#\text{MCDS}(R) + 1} \\ &\ge \frac{\#\text{MCDS}(R)}{M_1\#\text{MCDS}(R) + 1} \lambda_{\mathcal{B}} \\ &\ge \frac{1}{M_1 + 1} \lambda_{\mathcal{B}} \end{aligned}$$

If #MIS(ΔR) > 1, we divide the plane into square cells and color the occupied squares similar to proof of the Theorem 6. The number of backbone nodes in each square cell is less than some number M_2. Hence by applying the same TDMA method as proof of Theorem 6 we obtain throughput larger than $\frac{W}{M_2 13(K(R)+32\Delta+80)}$. Therefore,

$$\frac{W}{M_2 13(K(R) + 32\Delta + 80)} \le \lambda_{S,\mathcal{B}} \quad (33)$$

and since from the proof of Theorem 6 we have $\lambda_{\mathcal{B}} \le 225 \frac{W}{K(R)}$ we obtain that

$$\begin{aligned} \lambda_{S,\mathcal{B}} &\ge \frac{W}{M_2 13(K(R) + 32\Delta + 80)} \\ &\ge \frac{K(R)}{M_2 13(K(R) + 32\Delta + 80)225} \lambda_{\mathcal{B}} \\ &\ge \frac{\lceil \frac{\Delta+1}{3} \rceil}{M_2 13(\lceil \frac{\Delta+1}{3} \rceil + 32\Delta + 80)225} \lambda_{\mathcal{B}} \\ &\ge \frac{1}{M_2 13 * 145 * 225} \lambda_{\mathcal{B}} \end{aligned}$$

Proof of Theorem 12: We use the upper bound of Theorem 10 to prove that throughput tend to zero. The upper bound can be written as

$$W \cdot \frac{\#\text{MIS}(\Delta R)}{\#\text{MCDS}(R)} \cdot \frac{\#\text{MCDS}(R)}{\#\text{Backbone}(R)}.$$

For $\Delta > 0$, we can bound #MIS(ΔR)/#MCDS(R) as follows. The circles with radius $\Delta R/2$ centered at the MIS(ΔR) nodes are disjoint. We can show by geometry that every MCDS(R) node covers less than

$$\frac{\pi(R + .5\Delta R)^2}{\pi(.5\Delta R)^2} = (\frac{2}{\Delta} + 1)^2$$

nodes in the MIS(ΔR). This shows that

$$\frac{\#\text{MIS}(\Delta R)}{\#\text{MCDS}(R)} < (\frac{2}{\Delta} + 1)^2. \quad (34)$$

Hence if #MCDS(R)/#Backbone(R) $\to 0$ then the upper bound tends to 0 and the maximum throughput goes to 0.