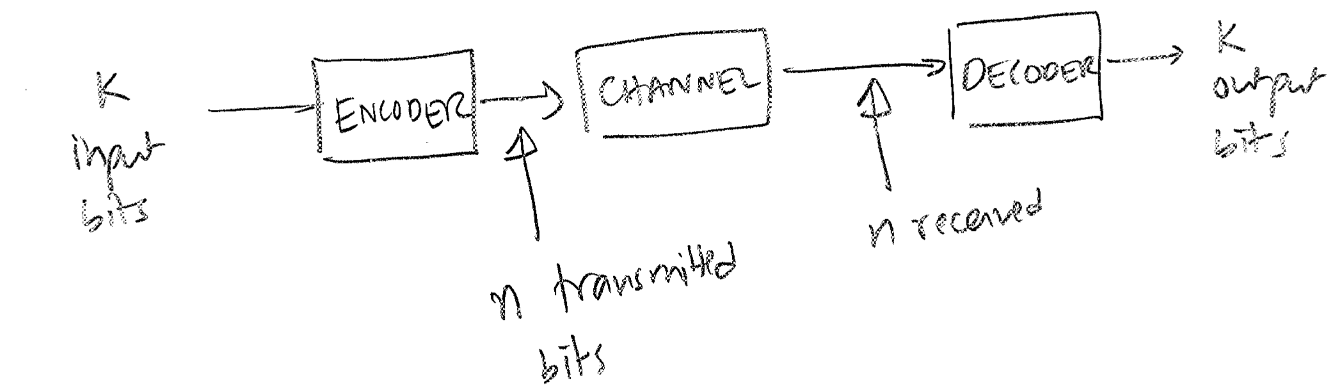
TRELLIS CODED MODULATION

In our treatment of block and convolutional codes, we reduced the probability of error by increasing the system bandwidth.

UNCODED

CODED



To maintain the same rate of information transfer, the system transmission rate had to be increased by a factor of $\frac{n}{K} > 1$.

BANDWIDTH EXPANSION

- In a sense, this gain in error probability performance would have been more exciting if no bandwidth expansion was necessary.

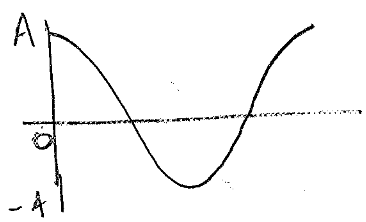
- To achieve just that, we need to closely look at the channel, which will be the Additive White Gaussian Noise (AWGN) channel.

BASICS OF MODULATION

- On a continuous input channel like a copper wire or wireless channel, actual transmission of information is done via waveforms

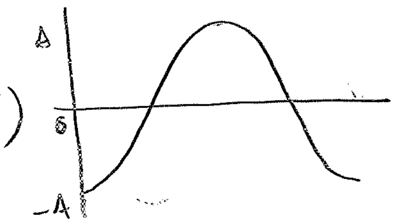
- For example,

$\cos(\omega t)$



can be used to represent 0

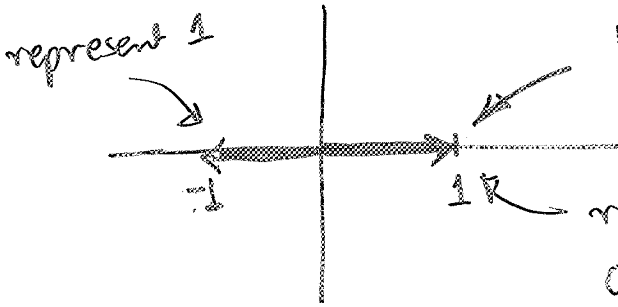
$\cos(\omega t + \pi)$



can represent 1

BINARY PHASE SHIFT KEYING
(BINARY ANTIPODAL MODULATION)

- On the phase plane

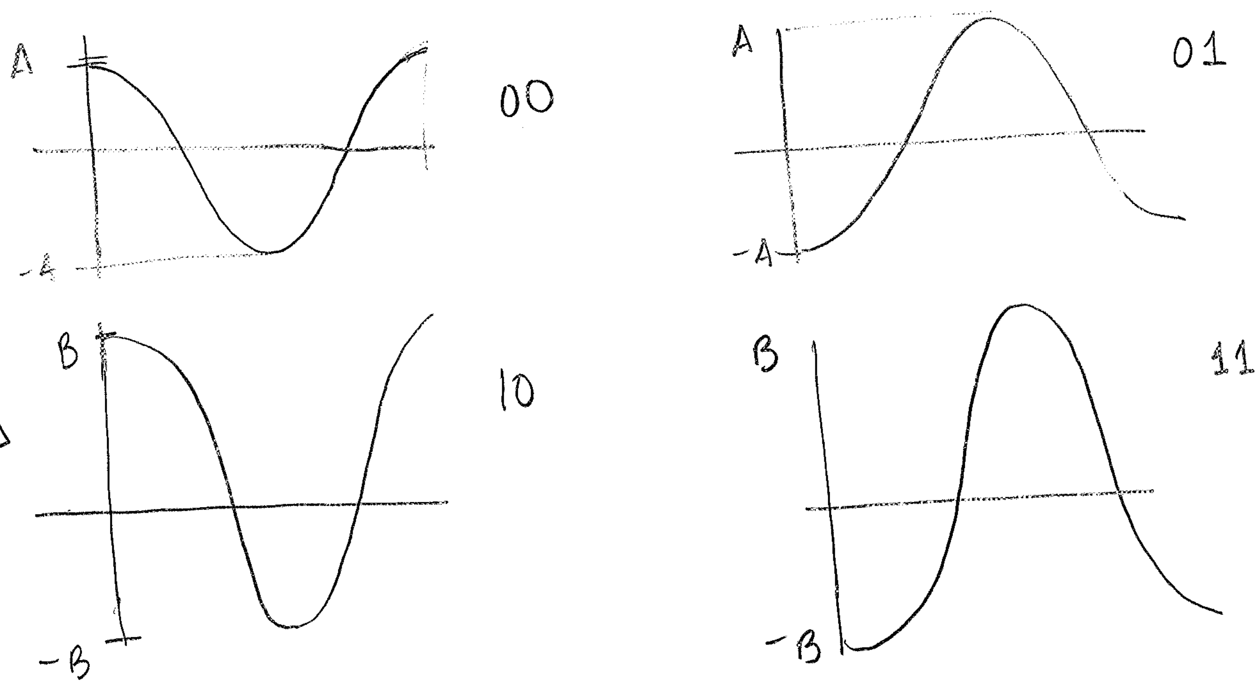


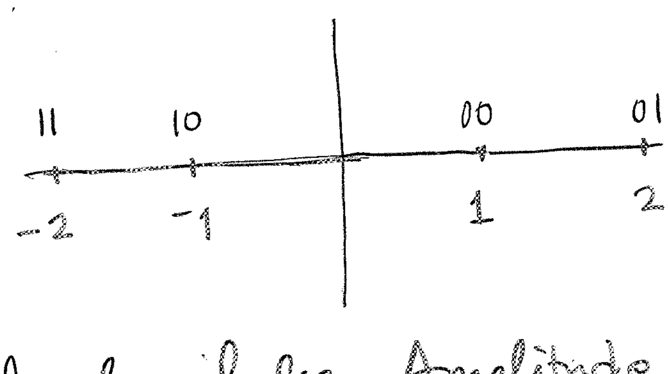
represents energy of the waveform

$$= \int_0^{2\pi} \cos^2(t)\,dt$$

- If I use 4 waveforms to represent 2 bits

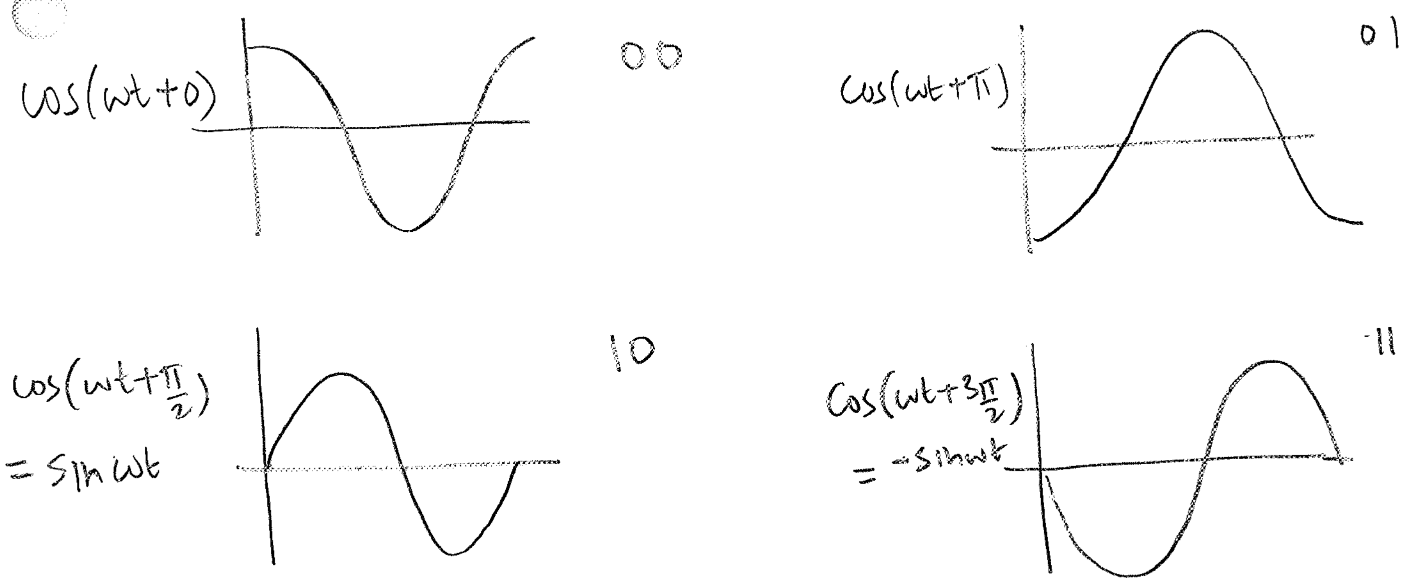
$\boxed{B > A}$

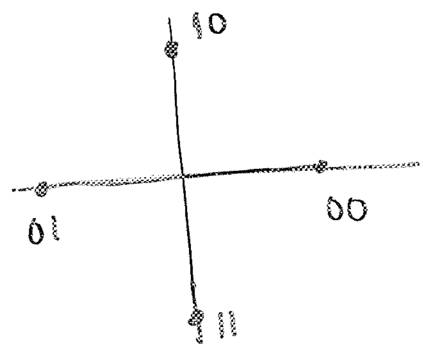




4 level Pulse Amplitude Modulation

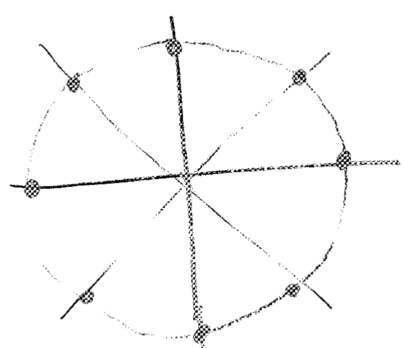
- Alternately, I would obtain 4 points as follows





4 - PHASE SHIFT KEYING (4-PSK)

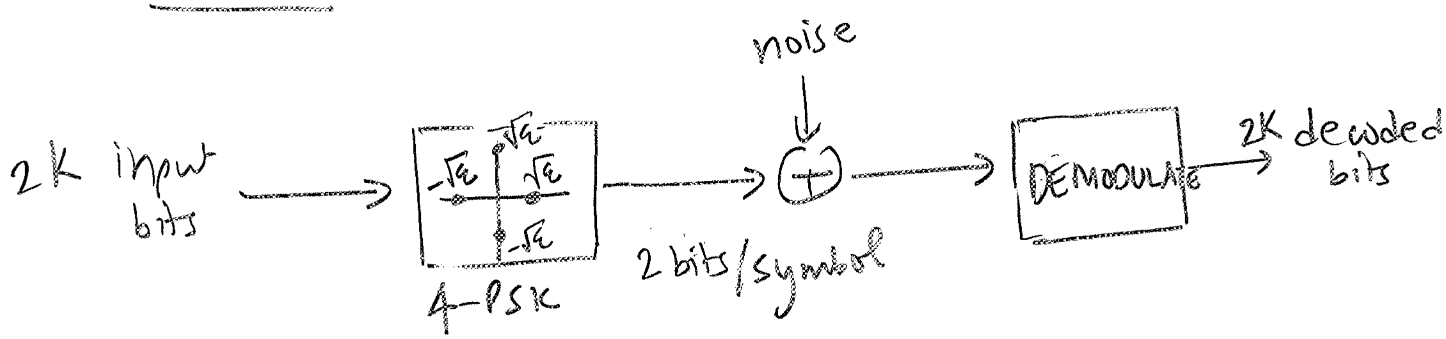
- 8-PSK



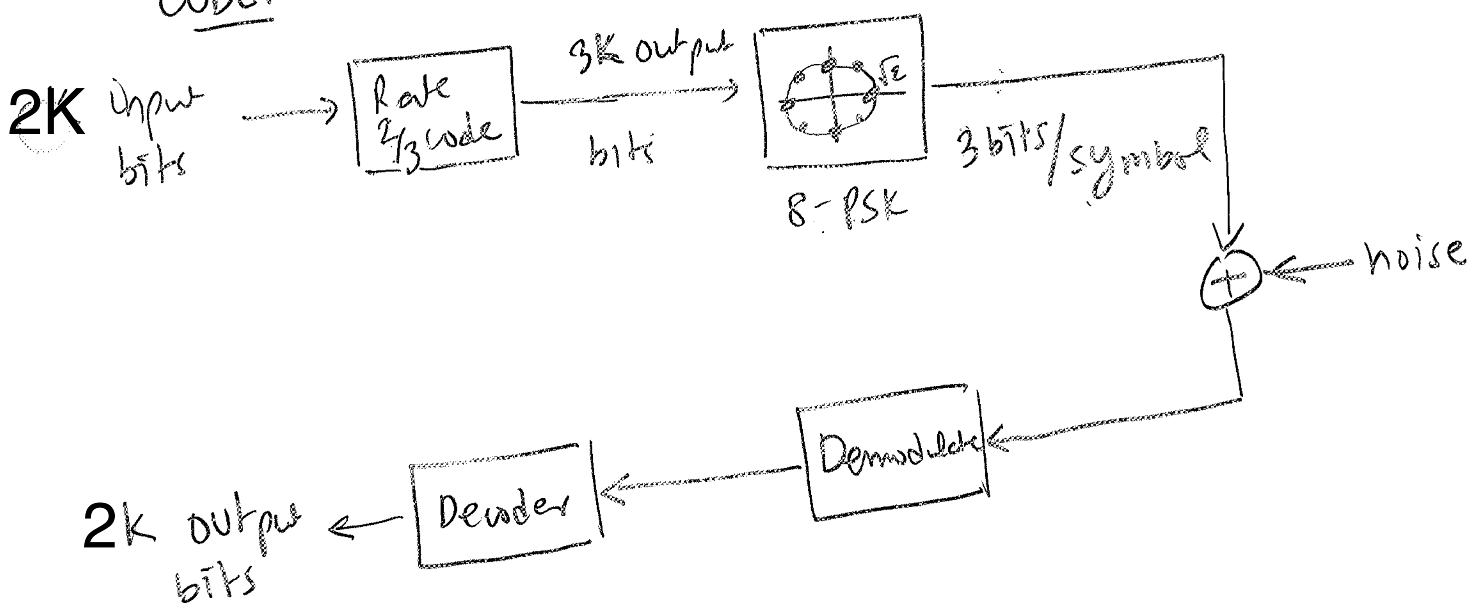
NOW, if we wanted a coded signal with no bandwidth expansion over uncoded signal, we could do the following.

EXAMPLE

<u>UNCODED</u>



<u>CODED</u>

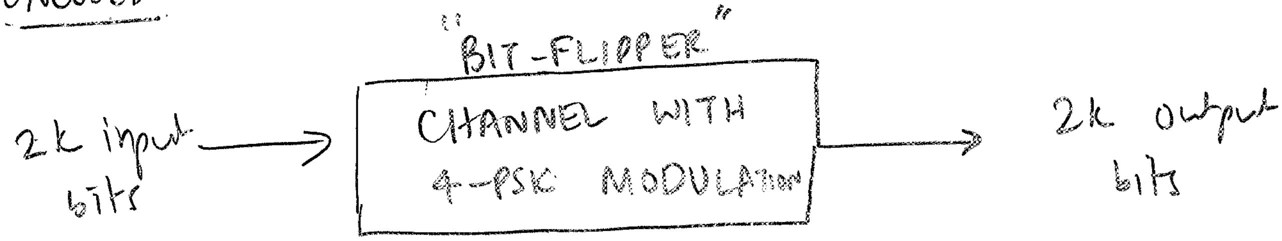


- SO TO KEEP THE SYSTEM BANDWIDTH CONSTANT, WE INCREASED THE SIZE OF MODULATION BY $\frac{1}{R}$, R = coding rate.

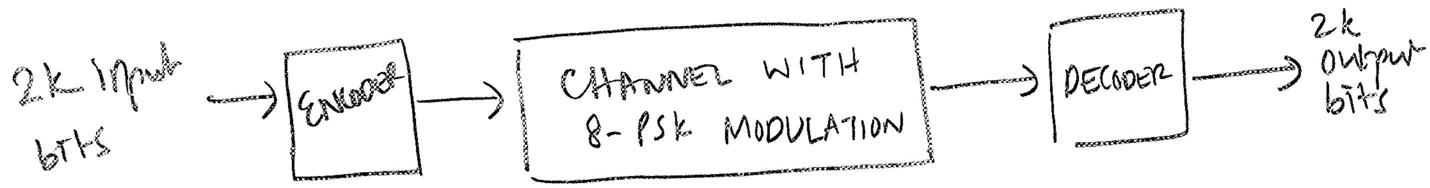
GREAT! SO EASY (YOU WISH!)

LET'S take a closer look at the two systems

UNCODED



CODED



THIS CHANNEL MAKES MORE ERRORS SINCE CONSTELLATION POINTS ARE BUNCHED CLOSER THAN 4-PSK

- ON AWGN, 8-PSK requires extra 4-dB of signal power to match the 4-PSK channel
- so IF coding has to help, it has to have a coding gain of more than 4 dB.

(CODING GAIN = SAVINGS IN POWER OVER UNCODED SYSTEM (WITH SAME MODULATION) TO ACHIEVE IDENTICAL BIT ERROR PROBABILITY)

- THOUGH POSSIBLE TO GET MORE THAN 4 dB CODING GAIN, THE CODES REQUIRED ARE VERY COMPLEX.

- <u>KEY TRICK</u>: DON'T TREAT MODULATION AS A SEPARATE OPERATION FROM CODING

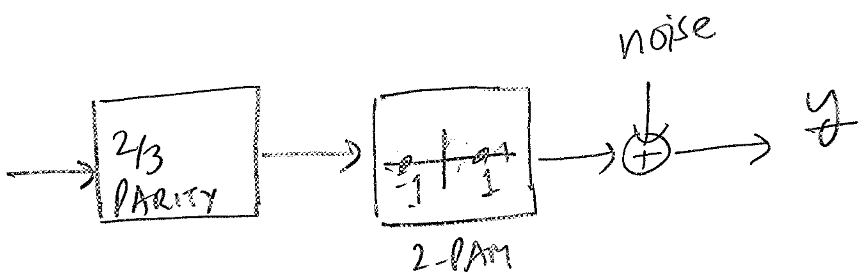
(UNGERBOECK' <u>1982</u>)

THE KEY INGREDIENTS OF TCM ARE

- SET PARTITIONING
- TRELLIS ENCODING
- SOFT VITERBI DECODING

SOFT DECODING

LET'S use a very simple error control scheme. And use 2-PAM (or BPSK)



$$\left.\begin{array}{l} 0 \longrightarrow 1 \\ 1 \longrightarrow -1 \end{array}\right\} \text{ MODULATION}$$

$$\begin{array}{ccccc} 00 & \longrightarrow & 000 & \longrightarrow & (1, 1, 1) \\ 01 & \longrightarrow & 011 & \longrightarrow & (1, -1, -1) \\ 10 & \longrightarrow & 101 & \longrightarrow & (-1, 0, -1) \\ 11 & \longrightarrow & 110 & \longrightarrow & (-1, -1, 1) \end{array}$$

ENCODING — TRANSMITTED SIGNAL

- MINIMUM Hamming distance between codewords is 2 $\Rightarrow$ can detect only one error
- HARD decoding: ① Convert each received symbol into 0 or 1

  ② Pass it to decoder

  ③ If Hamming distance between demodulated signal & codewords > 0
  $\rightarrow$ SIGNAL AN ERROR

- NOW Hard decoding is not the best idea

Suppose we transmitted $(+1, +1, +1)$

and received $(+1.2, -0.15, +0.9)$

then demodulator output $(+1, -1, +1)$ ← not a valid codeword

<u>BUT</u> compute EUCLIDEAN DISTANCES from each of the codewords

$d_1 = \sqrt{1.3725}$ ←

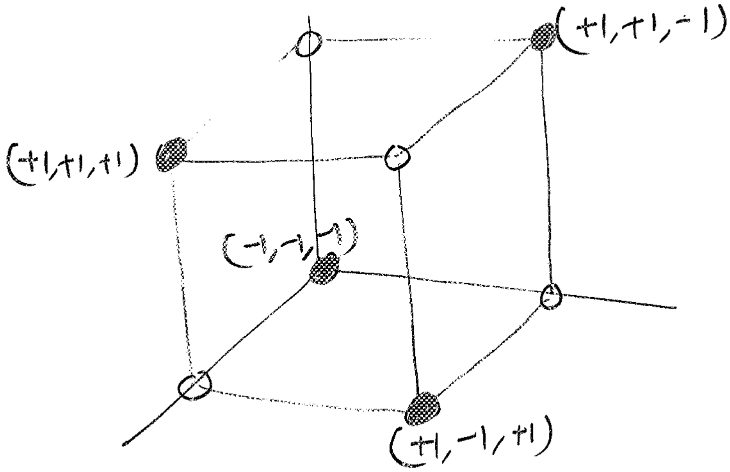
$d_2 = \sqrt{4.3725}$

$d_3 = \sqrt{9.7725}$

$d_4 = \sqrt{5.5725}$

- Codeword with minimum distance is the right codeword
- SO ALWAYS SELECT MINIMUM DISTANCE CODEWORD
- THIS IS CALLED <u>SOFT DECODING</u>

- SOFT DECODING has a <u>3dB</u> gain over HARD decoding (NOTE : This gain is not over uncoded system)

- 

- SIGNALING IS DONE IN EUCLIDEAN SPACE
- DECODING IN THAT SPACE OPTIMAL

LESSONS

1) SOFT decoding did better because we exploited the knowledge of channel by performing optimal decoding

2) MORE IMPORTANTLY, it gives us a geometrical interpretation of what codes are trying to do — place points in a multidimensional space which have good distance properties.
(COMMONLY REFERRED AS PACKING PROBLEM IN GEOMETRY).

BACK TO TCM

- TCM is a joint coding and modulation
- Objective is to increase distance between constellation points without increasing bandwidth
- Done by carefully mapping input data to points in a multidimensional space (space spanned by consecutive symbols).

SET PARTITIONING

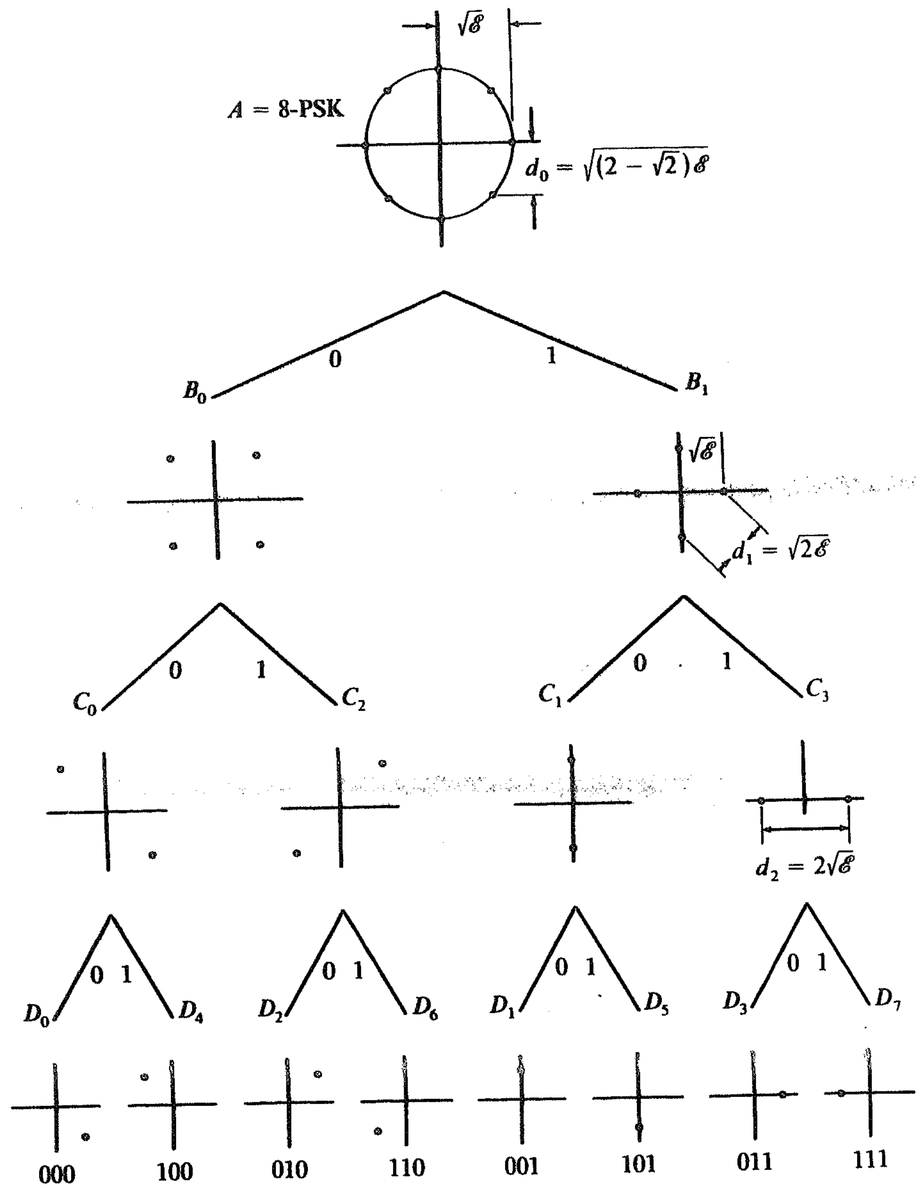


FIGURE 5.4.1
Set partitioning of an 8-PSK signal set.

- EACH PARTITION INCREASES THE MINIMUM DISTANCE BETWEEN POINTS IN THAT SUBSET — IMPORTANT FOR THE SUCCESS OF TCM.

(DIGITAL COMMUNICATIONS, PROAKIS '89)

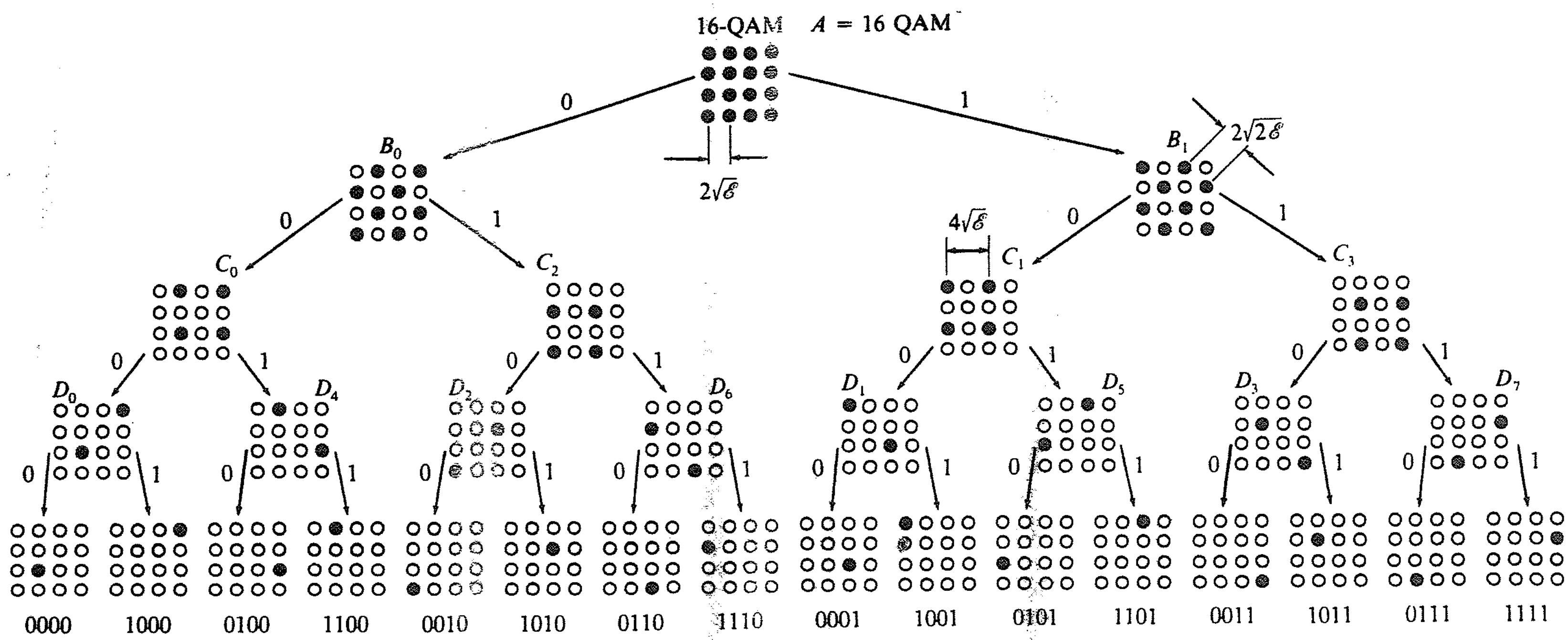
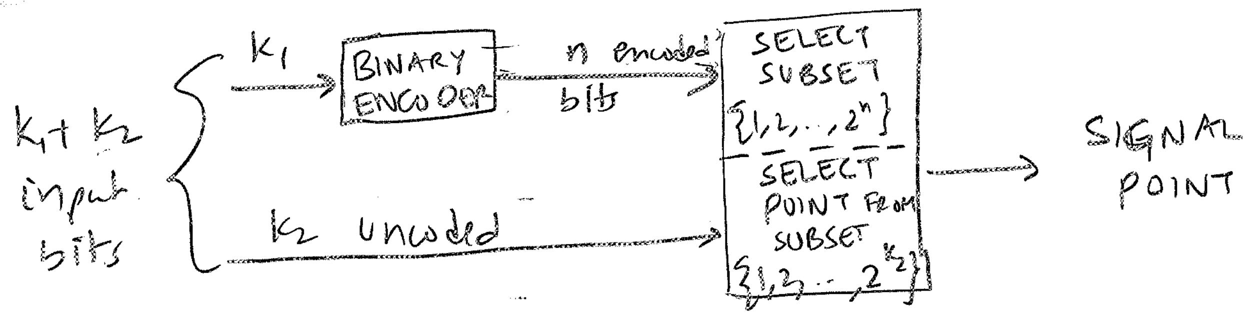


FIGURE 5.4.2
Set partitioning of 16-QAM signal.

FOR M-QAM, each signal partition increases the minimum Euclidean distance by $\sqrt{2}$, i.e., $\frac{d_{i+1}}{d_i} = \sqrt{2}$ for all i

(DIGITAL COMMUNICATIONS, PROAKIS '89)

General structure of combined encoder/modulator

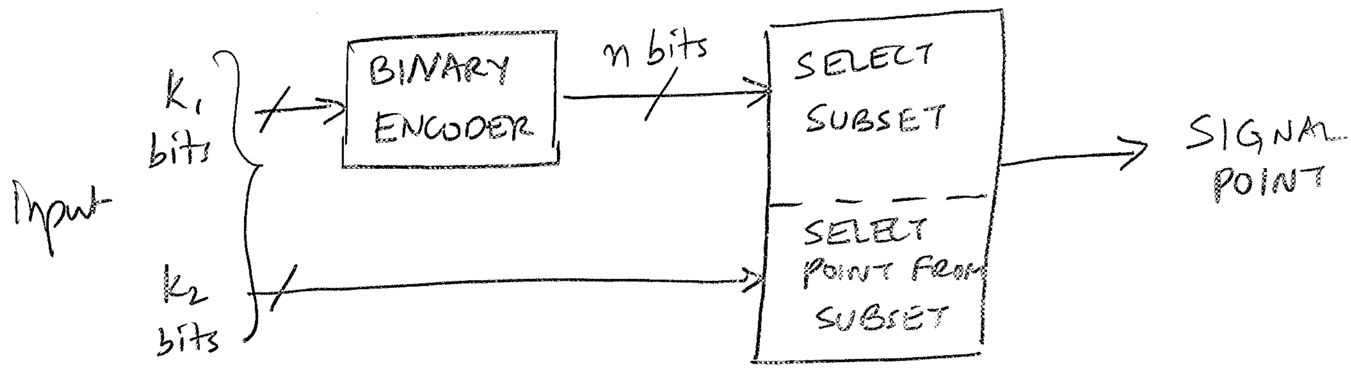


- Use part of input, encode it to select one of the partitions
- Use rest of the input to select one of the points in the partition
- The binary encoder can be
  - a block code, or
  - a convolutional code
- We will use a convolutional code because
  - soft decoding is easy (Viterbi)
  - flexibility in design for long blocks.

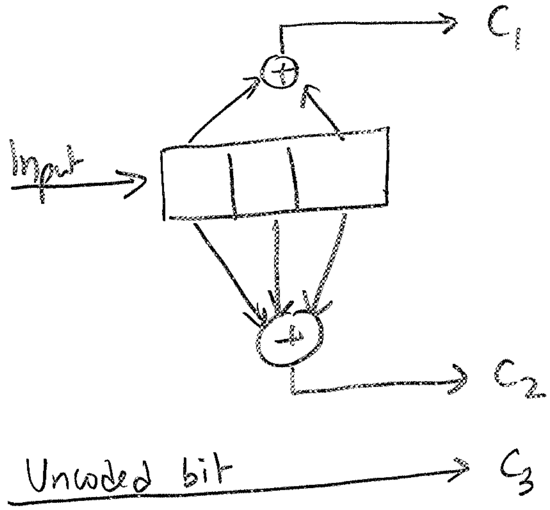
Nov-14

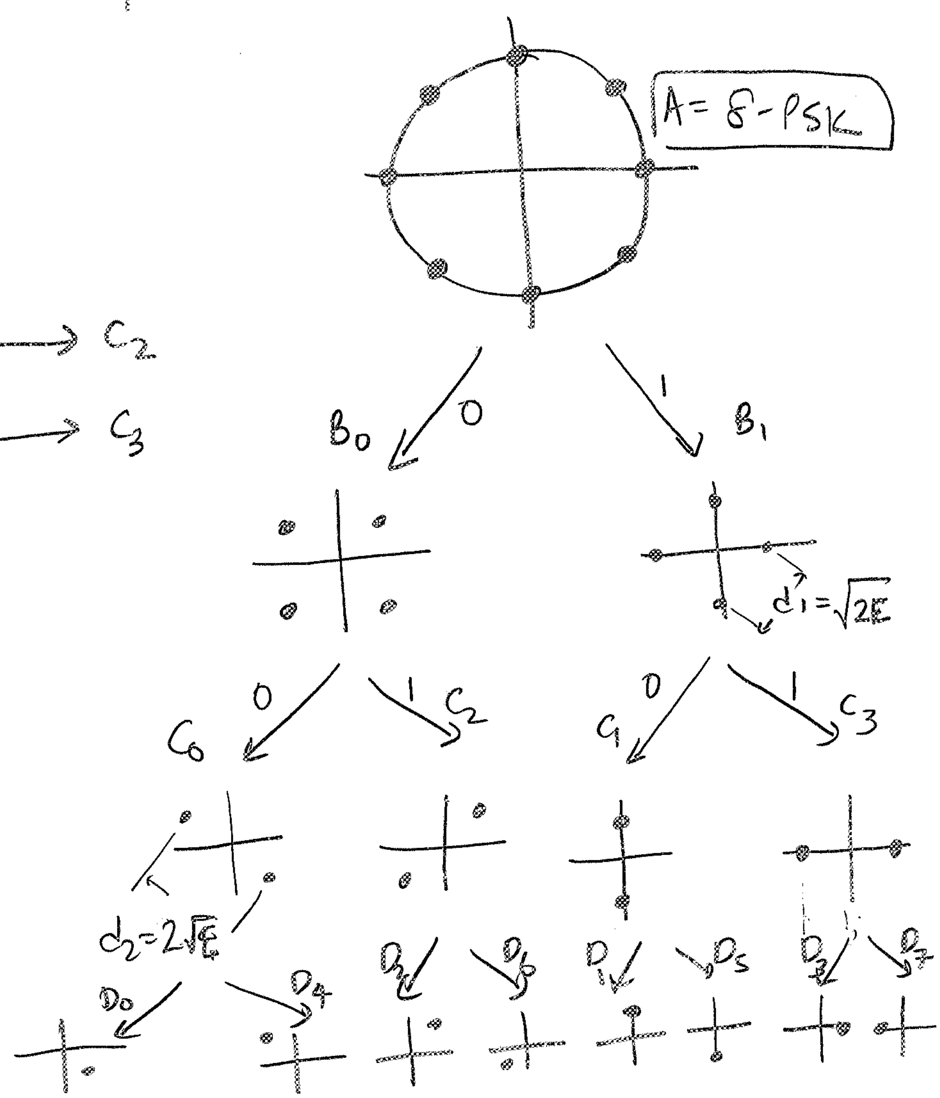
TRELLIS CODED MODULATION

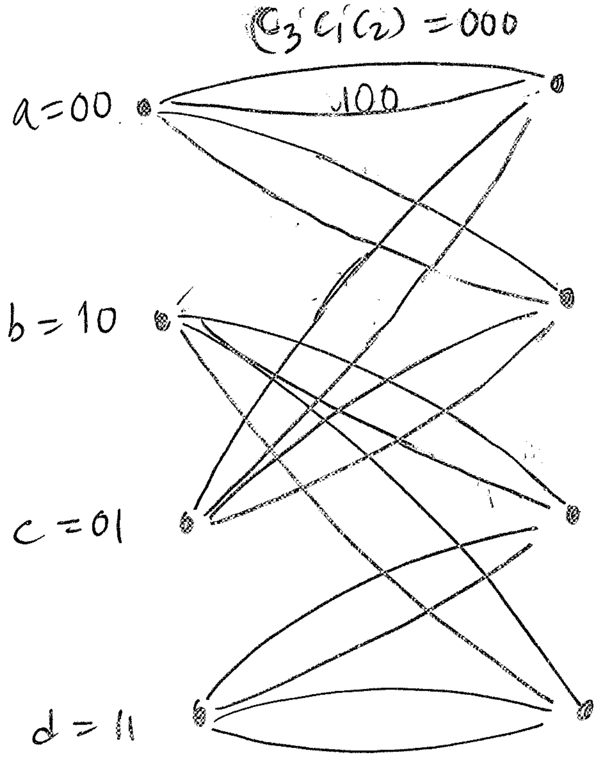
GENERIC COMBINED ENCODER / MODULATOR

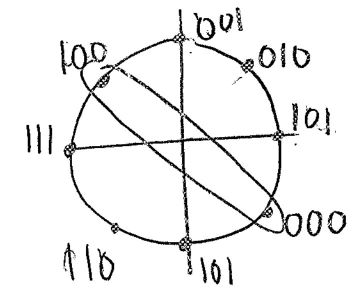


EXAMPLE: BINARY ENCODER = RATE $\frac{1}{2}$ CONVOLUTIONAL CODE







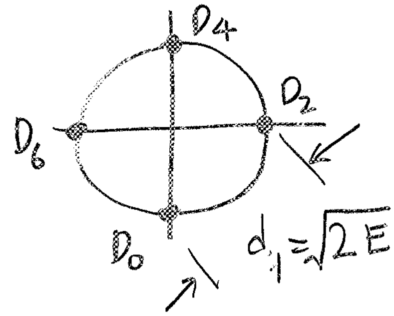


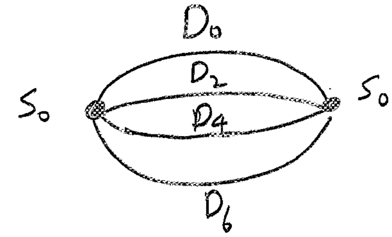
Four state trellis

- The basic 4-state trellis is same as that of the rate ½ convolutional code.
- The uncoded bit c_3 induces parallel paths
- The distance between the parallel paths is $d_2 = 2\sqrt{E}$

To see how we gain over an uncoded system, let's look at 4-PSK

UNCODED

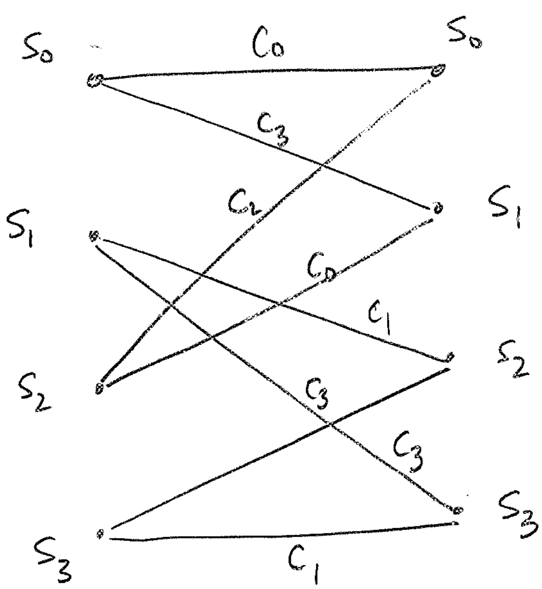




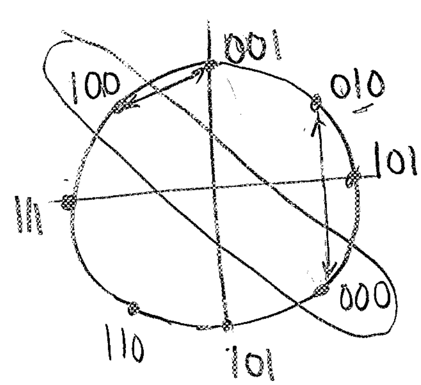
ONE STATE TRELLIS

MINIMUM DISTANCE = $\sqrt{2E}$

CODED



- Each branch corresponds to one of the 4 subsets C_0, C_1, C_2, C_3
- For 8-PSK, each of C_i contains 2 signal points
- Hence, each transition contains 2 points

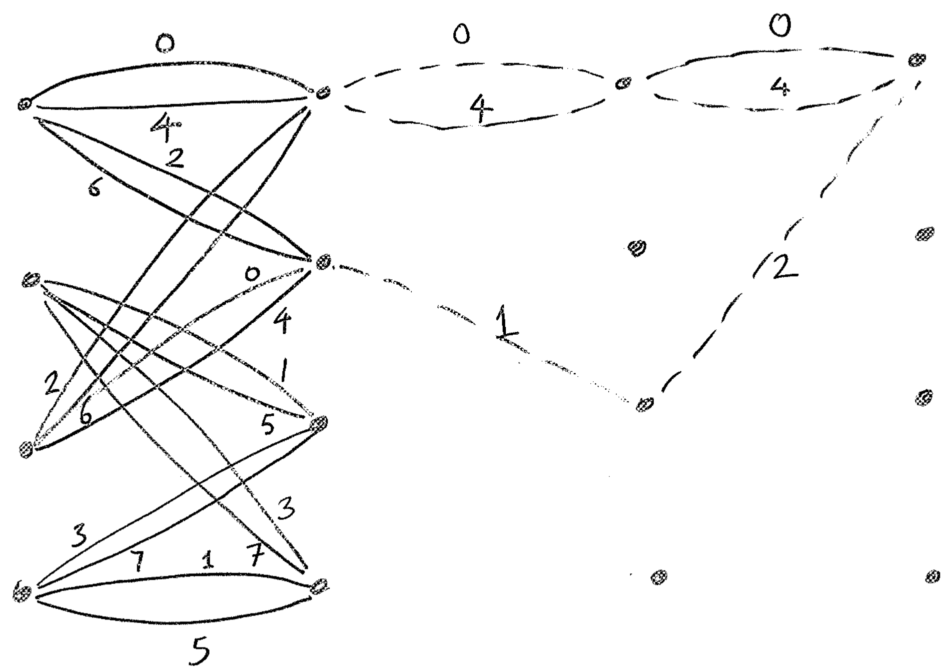


C_0 contains $(000, 100) \equiv (0,4)$ octal

C_1 " $(001, 101) = (1,5)$

C_2 " $(010; 110) = (2,6)$

C_3 " $(011, 111) = (3,7)$



- For this trellis, any two paths which diverge from one state and remerge at the same state after more than one transition have a squared Euclidean distance of

$$d(040, 212) = d_0^2 + 2d_1^2 = d_0^2 + d_2^2 = 4.585E$$

- Distance between parallel paths $= 4E = d_2^2$

$\Rightarrow$ Minimum distance $= d_2 = 2\sqrt{E}$

- Minimum distance is called <u>free Euclidean distance</u> and denoted by D_{fed}.

- In the coded 4-state trellis, $D_{fed}^{(4)} = 2\sqrt{E}$

  In the uncoded 1-state trellis, $D_{fed}^{(1)} = \sqrt{2E}$

(ASYMPTOTIC) CODING GAIN $= 10 \log_{10} \left(\dfrac{D_{fed}^{(4)}}{D_{fed}^{(1)}} \right)^2$

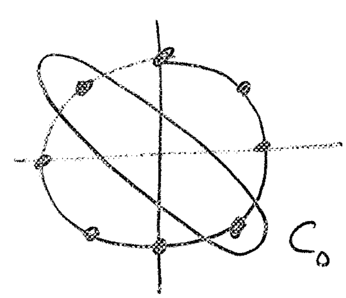
$= 10 \log_{10} 2 = 3\,dB$

- THIS PARTICULAR 4-STATE TRELLIS IS OPTIMAL! NO OTHER 4-STATE HAS HIGHER D_{fed}.

<u>HEURISTIC CONSTRUCTION RULES</u>

(a) Parallel transitions (if they occur) are assigned to signal points separated by the maximum Euclidean distance,

here it is $d_2 = 2\sqrt{E}$



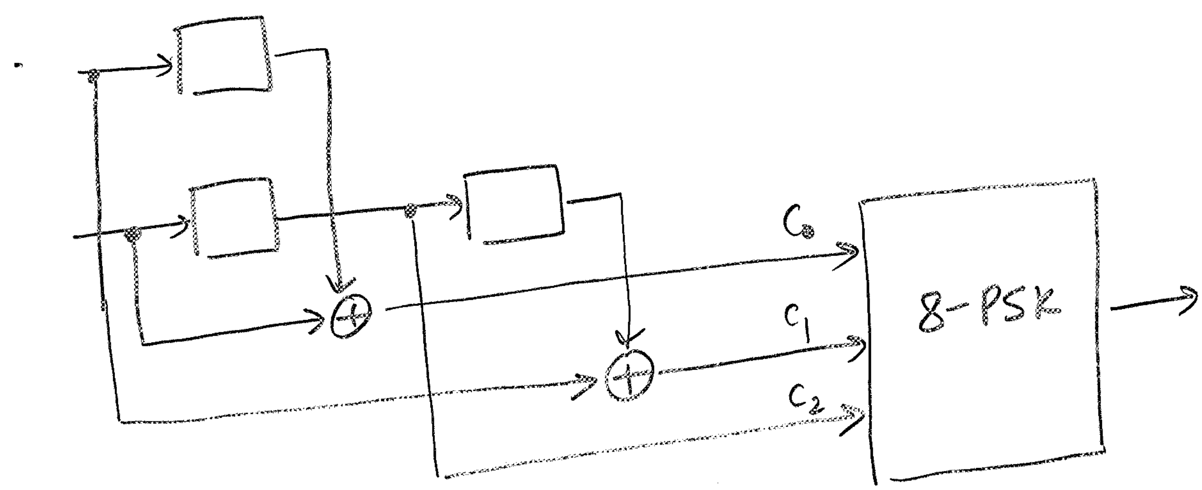
(b) The transition originating from & merging into any state is assigned the subsets (C_0, C_2) or (C_1, C_3) which have a maximum distance $d_1 = \sqrt{2E}$

(c) The signal points should occur with equal frequency.

Rules (a) and (b) guarantee that the Euclidean distance associated with single & multiple paths $>$ Euclidean distance of uncoded 4-PSK.

Rule (c) guarantees that the trellis code will have a regular structure

- FOR THE 4-STATE TRELLIS, the coding gain is limited by the parallel transitions.
- Eliminate parallel transitions by using a rate 2/3 code with more memory.



- $D_{fed} = \sqrt{d_0^2 + 2d_1^2} = \sqrt{4.585\,E}$

$$\text{Coding gain} = 10 \log_{10} \frac{4.585E}{2E}$$
$$= 3.6 \text{ dB}$$

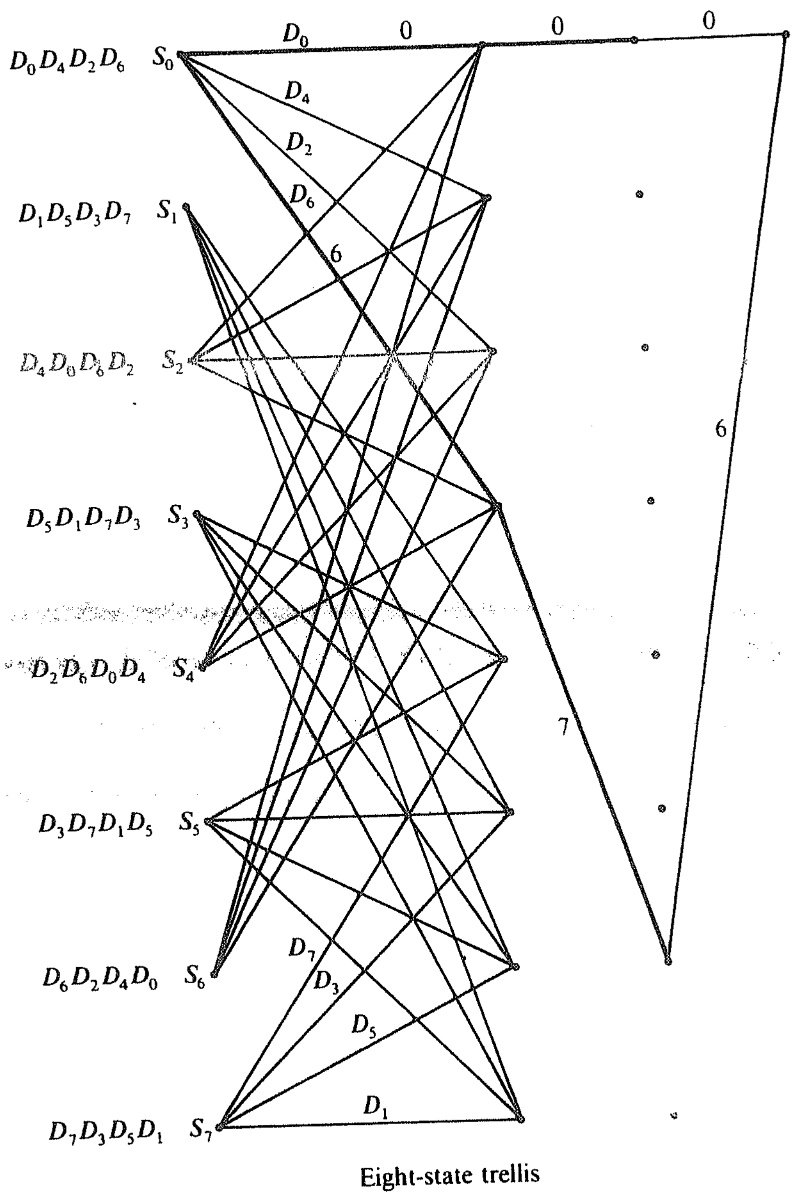


FIGURE 5.4.8
Eight-state trellis code for coded 8-PSK modulation.

DECODING is straight forward

(1) Each branch corresponds to a sub constellation of signals.

So compute the signal in the subconstellation which is closest to the received signal. Label the branch with the metric corresponding to the closest point

This step is called subset decoding.

(2) Now your trellis is completely labeled.

Perform usual Viterbi.

PERFORMANCE : Similar analysis applies

At high SNR, $P_e \approx N_{fed} \frac{1}{2} Q\left(\sqrt{\frac{D_{fed}^2}{4N_0}}\right)$

# of signal sequence with distance D_{fed}

this factor is typically ignored while computing coding gain BUT for large trellises, it cannot be ignored.