ELEC 243
Problem Set 2
Homework Section
Due: January 30, 2015

Homework Problems.

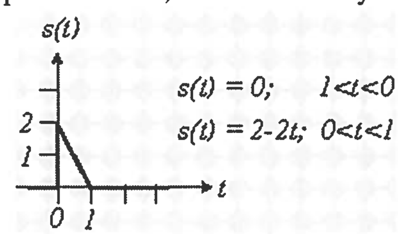
H2.1 Consider the triangle function $\Lambda(t)$:

$$\Lambda(t) = \begin{cases} 0 & |t| > \frac{1}{2} \\ 2 - 4|t| & |t| < \frac{1}{2} \end{cases}$$

Decompose the function $\Lambda(t)$ into a linear combination of scaled and shifted versions of the unit ramp function $r(t)$.

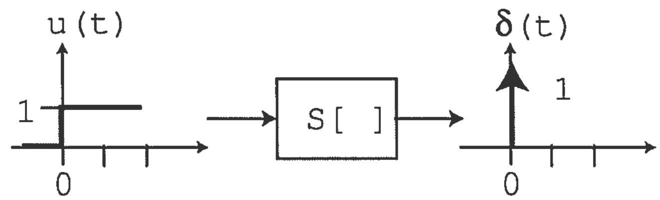
H2.2 A system S with an input signal $x(t)$ produces an output signal $S[x(t)] = k\frac{d}{dt}x(t)$. Prove that the system is linear.

H2.3 The signal $s(t)$ shown is the input to a linear, time invarient system.



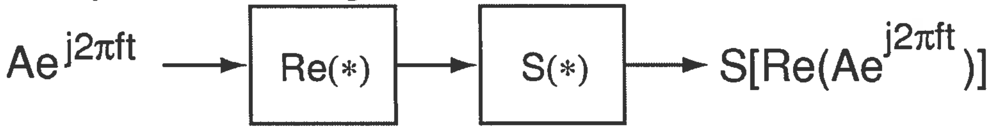
Express the signal as a linear combination of delayed and weighted step functions and ramps (the integral of the step function). Sketch your decomposition.

The response of the system to a step function is an impulse of area 1 at time $t = 0$:



Find the output of the system, $w(t)$, when the the input is $s(t)$. Give both an analytical expression and a sketch of the output.

H2.4 Consider a system S that is linear, and has real-valued outputs for real-valued inputs. Show that if the input is the real part of a complex exponential, the output is the real part of the system's output to the complex exponential: $S[Re(Ae^{j2\pi ft})] = Re[S(Ae^{j2\pi ft})]$. In other words, in the diagram below, show that the two systems can be exchanged.



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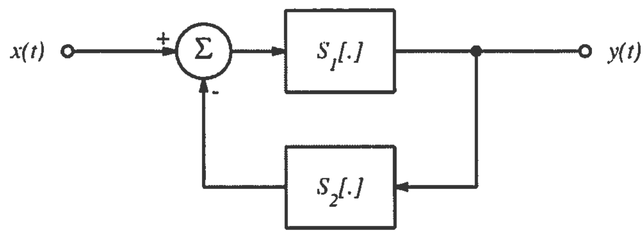
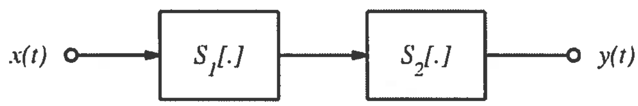
H2.5 Two systems are connected in the feedback configuration as shown below. The feed-forward system is a simple amplifier: $S_1[x(t)] = G \cdot x(t)$, where G is a constant greater than 1, and the output of the feedback system $S_2[\cdot]$ is simply 1/10 of its input (an attenuator). Find the overall system response $\frac{y(t)}{x(t)}$. Approximate the system response if the gain G is very much greater than 1, say 10,000.



H2.6 Consider a cascade connection of two systems.



For the particular system functions given below, determine whether the interchanging the order of the systems changes the overall system function or output $y(t)$.

Case	$S_1[\cdot]$	$S_2[\cdot]$
A	$\|x(t)\|$	$G \cdot x(t)$
B	$\frac{d}{dt}x(t)$	$x(t-\tau), \tau > 0$
C	$G \cdot x(t)$	$x(t) + b$

H2.7 As promised, here is a problem involving the turkey specific heat capacity measurement system we discussed in class. Recall that the behavior of the system is described by the following equation:

$$mc\frac{d}{dt}\Delta T + \frac{1}{\theta}\Delta T = P_{in}$$

where

$$\begin{aligned}
\Delta T &= T - T_0 \\
T &= \text{temperature of the turkey} \\
T_0 &= \text{ambient temperature} \\
m &= \text{mass of the turkey} \\
c &= \text{specific heat capacity of the turkey} \\
\theta &= \text{oven to ambient thermal resistance} \\
P_{in} &= \text{input power to the oven}
\end{aligned}$$

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To perform the measurement, 500 W of power was applied to the oven (i.e. $P_{in} = 500u(t)$) and the temperature of the turkey was measured every 20 min. The results are shown in the table on the right. The ambient temperature was 25°C and the mass of the turkey was 8 kg.

t (min)	T (°C)
0	25.0
20	45.1
40	63.5
60	80.3
80	95.7
100	109.7
120	122.6

Based on these measurements, find the specific heat capacity of the turkey (c) and the thermal resistance of the oven (θ).