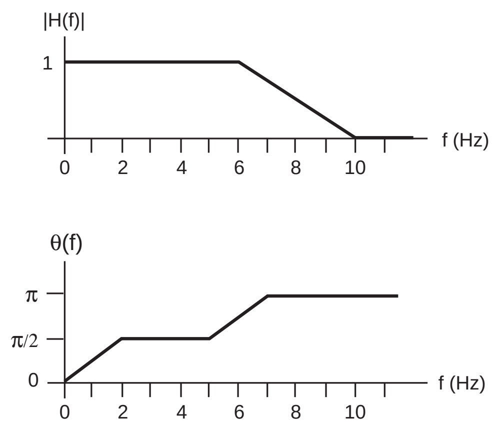
ELEC 243
Problem Set 4
Homework Section
Due: February 13, 2015

H4.1 The input signal to a system has only 4 nonzero frequency components in its spectrum, with the magnitudes and phases given below. (I have just given the positive f values; you now know what the corresponding negative frequency values are.

f(Hz)	Magnitude	Phase
1	2	0
3	8	$\frac{\pi}{4}$
8	3	$\frac{\pi}{2}$
10	2	$\frac{\pi}{2}$

(a) What is the minimum sampling frequency you can use to sample the **input signal** without losing information?

The signal is processed by a system that has a frequency transfer function with magnitude and phase as given in the graphs below:



(b) Sketch the spectrum of the output signal, magnitude and phase (just for positive f).

(c) What is the minimum sampling frequency you can use to sample the **output signal** without losing information?

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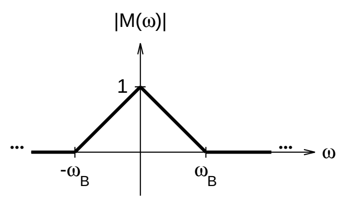
H4.2 The frequency shifting trick we used to prove the Sampling Theorem has a number of other applications. One of these is *modulation*, the process used in communication systems to move an information containing signal from baseband to a more appropriate location in the frequency domain. In radio communication systems, this is done so that (a) the signal can propagate as an electromagnetic wave with reasonable sized antennas and (b) so that many signals can share the same airwaves by moving each to a different frequency domain *channel*.

There are a variety of modulation schemes, but the simplest is *amplitude modulation*, or *AM* for short. This is achieved by multiplying the *message signal* $m(t)$ by a sinusoidal *carrier signal* $\cos(\omega_c t)$ where ω_c is the *carrier frequency* of the trasmitted signal $x(t)$. In equations: $x(t) = m(t)cos(\omega_c t)$.

(a) Using the properties of the Fourier Transform, sinusoids, etc., show that this process achieves the desired result. I.e, it moves the spectrum of $m(t)$ from baseband to ω_c.

(b) If the message signal has a spectrum of the shape shown below, sketch the spectrum $X(\omega)$ of the transmitted signal. (Assume that $\omega_B < \omega_c$.) Be sure to include both positive and negative frequencies.

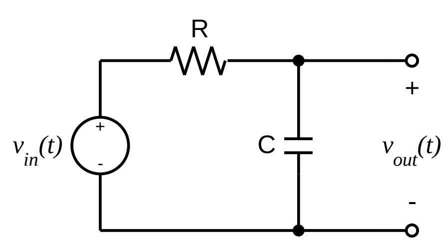


(c) Suggest a process for *demodulating* the received signal, i.e. for recovering the original signal $m(t)$ from a received version of $x(t)$.

H4.3 for the following circuit, show that the relationship between v_{in} and v_{out} is given by:

$$RC\frac{dv_{out}}{dt} + v_{out} = v_{in}$$

Hint: the i-v relationship for a capacitor is $i = C\frac{dv}{dt}$. This is the equivalent of Ohm's Law for capacitors.



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H4.4 We know from the sampling theorem that it is necessary to remove high frequency components from a signal before sampling it in order to avoid aliasing. A circuit which passes low frequencies and blocks high frequencies is called a *low pass filter*. The circuit in the previous problem is the simplest possible low pass filter.

- **(a)** Using your results from that problem, and from problem H3.7 on last week's assignment, find the transfer function $H(\omega)$ for this circuit.
- **(b)** Show that the circuit is in fact a low pass filter by calculating and plotting $|H(\omega)|$ for an appropriate range of ω.
- **(c)** The frequency at which the magnitude of a filter's transfer function is equal to $1/\sqrt{2}$ of its maximum value is called the *cutoff frequency*. What is the cutoff frequency of the above filter?

H4.5 Work Problem 8.32 in K&I.

H4.6 Work Problem 8.35 in K&I.