ELEC 243
Problem Set 6
Homework Section
Due: February 27, 2015

H6.1 Work Problem 9.27 in K&I.

H6.2 Work Problem 9.46 in K&I.

H6.3 Work Problem 9.52 in K&I.

H6.4 You have a pair of sensors, A and B, which produce output voltages $v_A(t)$ and $v_B(t)$ respectively. Due to internal current limiting, each sensor can only drive a load resistance of 1kΩ or more. The desired output of your system,$v_o(t)$, is a linear combination of the two sensor values.

Using an ideal op amp, design a circuit to provide the output voltage $v_o(t) = -[2v_A(t) + 8v_B(t)]$. Provide values for all your components.

H6.5 A noninverting amplifier is designed using op amps to have a gain of +10, but is constructed with $\pm 10\%$ resistors. (The actual value of the resistance can depart from the nominal value by up to 10%.) What is the range of gain that can result?

H6.6 Operational amplifiers were originally developed to perform mathematical operations (addition, scaling, and integration) in analog computers. The primary function of analog computers was to solve complex differential equations (in the analysis and simulation of dynamic systems) in the days before digital computers were powerful enough to handle the task.

For example, suppose we wanted to solve the equation

$$a\frac{dx}{dt} + bx = f(t)$$

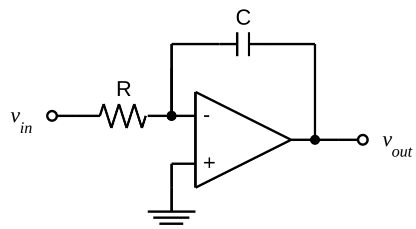
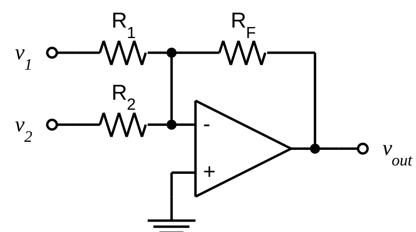
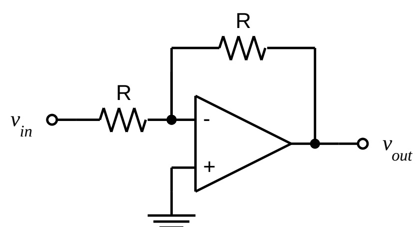
For compactness (and in keeping with tradition) we will use the "dot" notation where $\dot{x} \equiv \frac{dx}{dt}$, $\ddot{x} \equiv \frac{d^2x}{dt^2}$, etc., so our equation becomes

$$a\dot{x} + bx = f(t) \tag{1}$$

We proceed as follows:

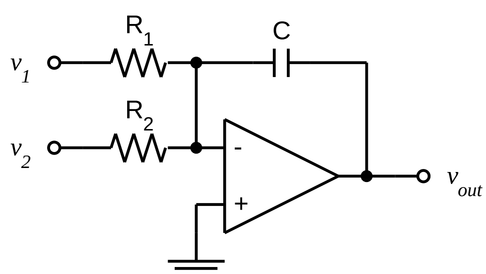
1. Assume we know $\dot{x}(t)$. Then we could rewrite (1) as an equation for $\dot{x}$: $\dot{x} = \frac{1}{a}f(t) - \frac{b}{a}x$.
2. $\dot{x}$ is in terms of f (which we know) and x (which is what we're looking for). But if we know $\dot{x}$ we could integrate it to find x: $x = \int \dot{x}dt$
3. Since we now know x and f we can scale them and add them together to get $\dot{x}$ that we claimed we knew in the first place (see step 1).
4. Since we now do in fact have $\dot{x}$, we also have x, which is what we were looking for: $x = \int \dot{x}dt = \int(\frac{1}{a}f(t) - \frac{b}{a}x)dt$.

The table below shows how to perform the necessary basic operations. Since the integrator and summer/scaler are inverting, we may need the inverter function to change sign if the minus signs don't cancel out.

Function	Circuit	Equation
Integrate		$v_{out} = -\frac{1}{RC}\int_0^t v_{in}(t)dt + V_0$
Sum, Scale		$v_{out} = -(\frac{R_F}{R_1}v_1 + \frac{R_F}{R_2}v_2)$
Invert		$v_{out} = -v_{in}$

The initial conditions (V_0) are established by connecting a voltage source of the desired value across the capacitor until $t = 0$, at which time it is disconected and the solution begins.

We can combine the functions of scaling, summation, and integration into a single circuit:

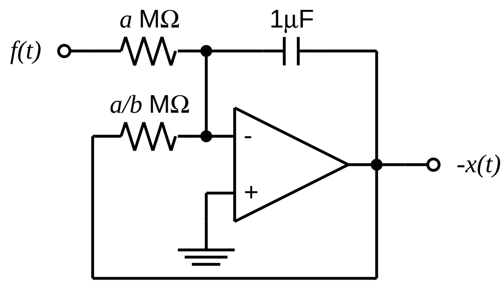


for which

$$v_{out} = -\int_0^t \left(\frac{v_1}{R_1C} + \frac{v_2}{R_2C}\right) dt + V_0$$

Values of R and C must be chosen to establish the desired time scale. For example, if we choose $R = 1\text{M}\Omega$ and $C = 1\mu$F we get $1/RC = 1/(10^6 \times 10^{-6}) = 1$ sec and the solution runs in “real time”.

Putting it all together, we can solve equation (1) with the following circuit:



Higher order derivatives can be handled by cascaded integrators.

Finally, we come to the problem:

Design an operational amplifier circuit to solve each of the following equations:

(a) $\dot{y} + 0.4y = 0$

(b) $5\ddot{y} + 4\dot{y} + 6y = f(t)$

H6.7 Find i in the circuit below. Assume an ideal op-amp.