Homework 7

Due September 24, at class

Suppose that we are designing a fault-detection system for a nuclear power plant. A total of N sensors (Geiger Counters) are placed at various positions in the plant to monitor radioactivity levels. The sensors are checked once every ten minutes and nominally, when all is well, each sensor measures an average background radiation intensity $\lambda_0 = 5$ over each ten minute time period. Assume that all sensors make independent, but identical measurements.

We want to detect the very beginnings of a problem, so if the intensity increases to $\lambda_1 = 6$ in a measurement period, then we decide that a plant wide system check is in order. Design a binary hypothesis test for this detection problem following the guidelines below.

a. Give the LRT and design a minimum probability of error detector.

b. Simplify the LRT statistic to a test statistic only involving the sufficient statistic. Apply a monotonically increasing transformation to simplify further.

c. Determine the distribution of the sufficient (test) statistic under both hypotheses. (HINT: Use the characteristic function to show that a sum of Poisson variates is again Poisson distributed.)

d. Calculate the minimum probability of error achievable in this problem as a function of N. If an error rate less than 0.01 is required, then what is the minimum number of sensors needed?