Optimal number of PET detectors per ring while employing compressive sensing techniques

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Objective: The aim of this work is to theoretically investigate the optimal number of PET detectors per ring while recovering similar image quality to a scanner with a full detector ring using compressive sensing (CS).

Methods: The data model that relates the sinogram (measurement) and image is described as a linear transform \( Y = Gm \) where \( m \) is the image, \( G \) is the system matrix and \( Y \) is the measurement. To recover the image, the problem must be well-posed which requires that: 1) A solution to the problem exits, 2) The solution is unique, and 3) The solution is totally dependent on the data. We accomplish this by interpreting the “condition number” (CN) of a matrix as a criterion of stability. The CN is defined as the ratio of the largest to smallest singular value of a matrix, and shows how a perturbation of a matrix input (image) affects the output (measurement). If the matrix is ill-conditioned (large CN) a small perturbation of the data may cause large output differences that are propagated during image reconstruction. We simulated a 32×32 image with 184×158 sinogram and thereby a system matrix size of 28336×1024. An increasing number of detectors were then removed from the scanner which is equivalent to deleting a row in the system matrix. The CN was then calculated and plotted versus number of removed lines of response. The locations of rows removed were determined with masks that simulate the response of detector removal in a PET scanner in an equidistant manner. Simulation of a 128×128 image and sinogram of 367×315 (system matrix of 115605×16384) was also performed.

Results: CN increased with increasing removal of detectors as expected. Further analysis of the plot shows that a 60% decrease in detectors results in 11.0% and 8.9% increase in log CN for 32×32 and 128×128 images respectively.

Conclusion: The number of measurement required for CS techniques that ensures stable and well-posed recovery can be found through the plot of CN as a function of samples.