

# 1 Relation between Fisher Information and Kullback-Leibler Distance

The Fisher information of the continuous random variable  $X$  is defined to be

$$F(X) = \int p(x) \left[ \frac{\partial \ln p(x)}{\partial x} \right]^2 dx .$$

(a) Let  $\mathcal{D}(p(x)||q(x))$  be the Kullback-Leibler distance with  $\log = \ln$ . Show that

$$\left. \frac{\partial^2 \mathcal{D}(p(x+a)||p(x))}{\partial a^2} \right|_{a=0} = F(X)$$

(b) Based on this result, find the Taylor series approximation for  $\mathcal{D}(p(x+a)||p(x))$  as a function of  $a$  around  $a = 0$ .

(c) What interpretation(s) can you give to this result?