11 Noisy Communication

We have a discrete channel with unusual error characteristics. The N input symbols $X_0, X_1, \ldots, X_{N-1}$ map to N output symbols $Y_0, Y_1, \ldots, Y_{N-1}$. Output symbol is related to input symbol by the probabilistic transfer function

$$\Pr[Y_j|X_i] = \begin{cases} \frac{1}{2} & j = i \\ \frac{1}{2} & j = (i+1) \mod N \end{cases}$$

(a) Find the capacity of this channel.

(b) As the block length increases indefinitely, what source rate can be transmitted reliably over this channel?

(c) If the block length is one, what source rate can be reliably transmitted over this channel?