

## 12 Rate-Distortion

We computed the rate distortion function for a discrete  $N$ -letter source that is uniformly distributed. Let's explore the limits of transmitting and receiving this source in more detail. This source is transmitted over a channel that has the input-output characteristic

$$\Pr[Y_j|W_i] = \begin{cases} 1 - \epsilon & j = i \\ \frac{\epsilon}{2} & j = (i - 1) \pmod{N} \\ \frac{\epsilon}{2} & j = (i + 1) \pmod{N} \end{cases} \quad i, j = 0, \dots, N - 1, \epsilon < \frac{1}{2}$$

(a) What receiver minimizes the Hamming (probability of error) distortion function?

$$d(W, \hat{W}) = \begin{cases} 0 & \hat{W} = W \\ 1 & \hat{W} \neq W \end{cases}$$

(b) Is this error criterion well-matched to the source and the channel?

(c) Find the rate-distortion function for this source under the Hamming distortion measure.