18 Communicating Data

We have an analog source that has been sampled and the samples quantized to three bits. The source can be chararacterized as producing three statistically independent, binary-valued, random variables W_1, W_2, W_3. Each source symbol is represented by a Gaussian random variable X_k, $k = 1, 2, 3$, and communicated over an additive Gaussian noise channel. The three channels are used simultaneously and have noise variances 1, 2, and 3. The *total* communication power $\sum_k \mathsf{E}[X_k^2]$ is constrained to be no greater than 6. The fidelity of this communication system is judged by the mean-squared error between the quantized amplitude and the reconstructed quantized amplitude.

$$d(\mathbf{W}, \widehat{\mathbf{W}}) = \sum_{k=1}^{3} \left(W_k - \widehat{W}_k\right)^2 2^{-2k}$$

(a) What is the channel capacity?

(b) Find an expression for the rate-distortion function. You may only be able to write it by including a difficult-to-find parameter; leave it in these terms.

(c) The rate-distortion function and the capacity calculation each implicitly specify how the power should be allocated among the three channels. In general, which bit should be sent over which channel? Do their suggested power allocations agree in detail?