

4 Data Processing Theorem

Let X be a Gaussian random variable having mean m and variance σ_X^2 . The random variable Y equals $X + N$, where N is independent of X and Gaussian with zero mean and variance σ_N^2 . In other words, X serves as the input to an additive Gaussian channel and Y serves as the output.

- (a) Find the mutual information between the input and the output. Show that this quantity depends on the signal-to-noise ratio σ_X^2/σ_N^2 and not on the mean m .
- (b) Assume the input mean changes from m_1 to m_2 . Find the Kullback-Leibler distance between the two distributions.
- (c) Compute γ , defined to be the ratio of the output to input Kullback-Leibler distances. Show that it too depends only on the signal-to-noise ratio.
- (d) How is $\mathcal{I}(X; Y)$ related to γ in this case? Is the relationship monotonic?
Note: This relationship does not hold for all possible probability distributions.