

**5 Letter Merging**

Let  $X$  be drawn from a finite discrete alphabet  $\{a_1, a_2, \dots, a_n\}$ .  $Y$  is the result of merging two of the letters: its alphabet consists of  $\{b, a_3, \dots, a_n\}$  where  $b$  is the union of  $a_1$  and  $a_2$ .

- (a) Show that letter merging decreases entropy.
- (b) What is the mutual information between  $X$  and  $Y$ ?