7 Two Versions of Capacity

In digital transmission systems, bits are represented by the analog signals $s_0(t)$ and $s_1(t)$, both having duration T. When this signaling strategy is used over an additive white Gaussian noise channel, the probability of error in decoding the transmission of equally likely bits is given by

$$P_e = Q\left(\sqrt{\frac{\|s_1 - s_0\|^2}{2N_0}}\right) ,$$

where $\|s\|^2 = \int_0^T s^2(t)\,dt \equiv E$, the signal's energy, and $Q(x)$ is the tail-integral of the standardized Gaussian probability density function.

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\alpha^2/2}\,d\alpha$$

Imposing an energy constraint on the two signals, $E_{s_0}, E_{s_1} \leq E_0$, the smallest achievable probability of error results when $s_1(t) = -s_0(t)$ and each signal has the largest possible energy.

From a discrete-channel viewpoint, this encoding-analog transmission-decoding implementation can be accurately modeled as a binary symmetric channel (BSC) with a cross-over probability equal to P_e.

(a) Find an expression for and plot the capacity of this channel as a function of the signal-to-noise ratio.

We also have an expression for the capacity of an additive white Gaussian noise channel having bandwidth W.

$$C = W \log\left(1 + \frac{P}{N_0 W}\right) \text{ bits/s}$$

Here, P is the signal's power. The issue becomes reconciling the two ways of computing capacity.

(b) Before performing any calculations, which way of computing capacity yields the larger answer or are they the same?

(c) Calculate expressions for the two capacities so that they can be compared. Plot them to determine how they vary with signal-to-noise ratio.