

8 Calculating a Rate-Distortion Function

The Blahut algorithm for numerically calculating the rate-distortion function is easy to program and runs quickly. This problem asks you to program this algorithm, to verify the results and to explore what the results mean. The algorithm for computing the conditional probability $p(\hat{w}|w)$ required to find the rate-distortion function is

$$p_n(\hat{w}|w) = \frac{p_n(\hat{w}) \exp\{-\lambda d(w, \hat{w})\}}{\sum_{\hat{w}} p_n(\hat{w}) \exp\{-\lambda d(w, \hat{w})\}}$$
$$p_{n+1}(\hat{w}) = \sum_w p_n(\hat{w}|w)p(w)$$

where $p(w)$ is the probability distribution of the input alphabet and $d(w, \hat{w})$ is the distortion measure. The parameter λ corresponds to some value of D , so varying λ finds $\mathcal{R}(D)$. The algorithm starts with an initial choice for $p(\hat{w})$, typically chosen to be a uniform distribution.

- (a) Assuming a uniformly distributed source alphabet, find the rate distortion function for four and five-letter, integer-valued, alphabets for each of two distortion measures.

$$d(w, \hat{w}) = \begin{cases} 0 & \hat{w} = w \\ 1 & \hat{w} \neq w \end{cases} \quad \text{(probability-of-error distortion)} \quad (1)$$

$$d(w, \hat{w}) = (\hat{w} - w)^2 \quad \text{(mean-squared error)} \quad (2)$$

- (b) Verify your answers. What should $\mathcal{R}(0)$ be in each case? Do your answers agree?
- (c) What should D_{\max} be in each case? Do your answers agree?
- (d) Does the curve have the correct shape?
- (e) Is either distortion measure well-matched at the rate-distortion limit? Examine the five-letter examples.