9 Calculating a Constrained Capacity

As noted in Cover & Thomas, the Blahut algorithm can be used to calculate channel capacity as well as the rate-distortion function. The algorithm given there does not, however, incorporate a power-constraint. It computes what amounts to the unconstrained capacity C_0.

(a) What $p(x)$ corresponds to the constrained channel capacity? In other words, what is

$$\arg\max_{p(x):\mathsf{E}[\rho(x)]\leq P} I(X;Y)$$

(b) What value of λ, the Lagrange multiplier, yields C_0, the unconstrained capacity?

We consider here a four-letter-in, four-letter-out symmetric channel in which the transition probabilities are $(0.7,.15,.1,.05)$. Modify the Blahut algorithm found in the text and program it to evaluate the constrained capacity of a discrete channel in which $p(y|x)$, the channel transfer function, is specified by a matrix. Use a uniform distribution as your initial choice for the channel-input probability distribution.

(c) What is the theoretical value of C_0 for this channel?

(d) Assuming a uniform power-constraint function $[\rho(x) = 1]$, what is the capacity versus-power-plot for this channel?

(e) Now assume $\rho(x) = [1\ 2\ 4\ 9]$. Again, find the capacity versus power plot $C(P)$.

(f) From this result, what properties would you infer $C(P)$ has?