HOMEWORK 7 — Receivers III

Exercise 1. Receiving with Estimators

Consider an On-Off Keying system where \( s_0(t) = A \cos(2\pi f_c t + \Theta) \) for \( 0 \leq t \leq T \) and \( s_1(t) = 0 \), for \( 0 \leq t \leq T \). The channel is the usual AWGN with zero mean and spectral height \( N_0/2 \).

(a) Assume \( \Theta \) is known at the receiver. What is the average probability of bit-error using an optimum receiver?

(b) Assume that we estimate the phase to be \( \hat{\Theta} \) and that \( \hat{\Theta} \neq \Theta \). Analyze the performance of the matched filter with an incorrect phase estimate, i.e. compute \( P_e \) as a function of the phase error.

(c) When does a noncoherent receiver become preferable? The expression for probability of error for non-coherent OOK receivers is given below. In other words, how large must the phase error be before switching to noncoherent reception?

\[
P_e \approx \frac{1}{2} e^{-\frac{A^2 T}{8 N_0}}
\]

Exercise 2. Constellations

Suppose that binary PSK is used to transmit information over an AWGN channel with the usual spectral height of \( \frac{N_0}{2} = 10^{-10} \) W/Hz. The transmitted signal energy is \( E_b = \frac{A^2 T}{2} \), where \( T \) is the bit interval and \( A \) is the signal amplitude. Determine the signal amplitude required to achieve an error probability of \( 10^{-6} \) if the data rate is

(a) 10 kilobits/sec
(b) 100 kilobits/sec
(c) 1 megabit/sec
Now consider the four and eight phase signal constellations shown. Determine the radii $r_4$ and $r_8$ of the circles such that the distance between two adjacent points in the two constellations is $d$. From the result, determine the additional energy transmitted in the 8-PSK signal to achieve the same error probability as the 4-PSK signal at high SNR, where the error probability is determined by errors in selecting adjacent points.

**Exercise 3. The Effect of Estimation**

A coherent phase-shift keyed system is operating over an AWGN channel with spectral height $\frac{N_0}{2}$. It uses the signals $s_0(t) = Ap_T \cos(2\pi f_c t + \theta_0)$ and $s_1(t) = Ap_T \cos(2\pi f_c t + \pi + \theta_1)$. Here, $|\theta_i| \leq \frac{\pi}{3}$ are constants and $f_cT$ is an integer value.

(a) Suppose $\theta_0$ and $\theta_1$ are known constants and that the receiver uses filters matched to $s_0(t)$ and $s_1(t)$. What are the values of $P_{e,0}$ and $P_{e,1}$?

(b) Now suppose $\theta_0$ and $\theta_1$ are unknown, and the receiver uses $\hat{s}_0(t) = Ap_T \cos(2\pi f_c t)$ and $\hat{s}_1(t) = Ap_T \cos(2\pi f_c t + \pi)$. What are $P_{e,0}$ and $P_{e,1}$ now? Suggestion: use a correlation receiver structure.

(c) What are the minimum values of the probabilities of error, as functions of $\theta_0$ and $\theta_1$?

All assignments are covered by the Honor Code. You may work together, but do not directly copy the solutions of other students.