1. Suppose $\mathbf{X}$ is a complex random vector. Let $\Sigma_{\mathbf{X}}$ and $\tilde{\Sigma}_{\mathbf{X}}$ be defined as in the notes. Show that:

(a) $\Sigma_{\mathbf{X}} = \frac{1}{2} (\Sigma_{\mathbf{X}} + \Sigma_{\mathbf{X}}^*) + j \frac{1}{2} (\Sigma_{\mathbf{X}}^* \mathbf{X}^T - \Sigma_{\mathbf{X}} \mathbf{X})$

(b) $\tilde{\Sigma}_{\mathbf{X}} = \frac{1}{2} (\Sigma_{\mathbf{X}}^T - \Sigma_{\mathbf{X}}^* \mathbf{X}) + j \frac{1}{2} (\Sigma_{\mathbf{X}}^T \mathbf{X} + \Sigma_{\mathbf{X}} \mathbf{X})$

Where $\mathbf{X} = \mathbf{X}_I + j \mathbf{X}_Q$.

2. Show that if $\mathbf{X}$ is a proper complex random vector so is $Y = AX + \mathbf{b}$ where $A$ and $\mathbf{b}$ are deterministic.

3. If orthogonal modulation is used, ie., $S_m(t) = \sqrt{2E} \rho_m(t)$ for $m = 1,2,\ldots,M$ for $t \in [0,T]$ show that if $r(t) = gae^{j\psi}S_m(t) + w(t)$:

(a) The sufficient statistic is $r_i = \int_0^T r(t) e^{-j\hat{\psi} \rho_i^*(t)} dt$ for $i = 1,2,\ldots,M$ when an estimate of $\psi$ is available (denoted as $\hat{\psi}$).

(b) The sufficient statistic is $r_i = \int_0^T r(t) \rho_i^*(t) dt$ for $i = 1,2,\ldots,M$ if an estimate of $\psi$ is not available.

4. Consider a linear modulation system where the signal is $S_m(t) = \sqrt{2E} \rho(t)$ for $m = 1,2,\ldots,M$ and $t \in [0,T]$. The received signal through a simple slow frequency non-selective fading channel with no inter-symbol interference is $r(t) = S_m(t)g(t)e^{j\psi(t)} + w(t)$ where $w(t)$ is a baseband white complex (proper) gaussian random variable.

(a) Define the term sufficient statistic.

(b) Show that $r = \int_0^T r(t) \rho^*(t) e^{-j\hat{\psi}} dt$ is the sufficient statistic in this problem where $\hat{\psi}$ is a good estimate of $\psi(t)$ for $t \in [0,T]$. 