Outage Capacity of Relay Channels in Low SNR: An Adaptive Strategy

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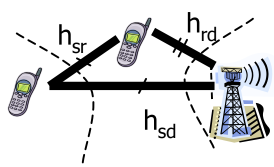
Problem setup

- We are interested in Outage analysis in relay channels at LSNR ($\rho \to 0$).
- Slowly Fading Rayleigh Channels.
- Half-duplex Constraint.



At LSNR: $R = \alpha \log(1+\rho) \simeq \alpha\rho + \mathcal{O}(\rho^2)$

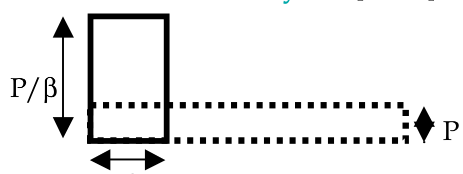
From Max. Flow Min. Cut

Theorem1 For transmission rate $R = \alpha\rho$ a lower bound on the outage probability is given by:

$$P_{out}(\alpha) \geq \alpha^2 - \alpha^3 + O(\alpha^4)$$

• How close can we get to the lower bound??

• Bursty Amplify and Forward (BAF) achieves the first order only α^2 [AT05]



$$I_{BAF} = \rho\left[|h_{sd}|^2 + \frac{|h_{rd}|^2|h_{sr}|^2}{|h_{rd}|^2 + |h_{sr}|^2 + \frac{\beta}{\rho}}\right]$$

Theorem2 The outage probability achieved by BAF at LSNR, with $\frac{\beta}{\rho} = o(1)$, is given by the following expression:

$$P_{o,BAF} = \alpha^2 - \frac{2}{3}\alpha^3 \ln\alpha + 5.139\alpha^3 + \mathcal{O}(\alpha^4 \ln\alpha)$$

• How about Decode and Forward?

Consider a version of DF in which relay is used if it decodes after a fraction of time, f=1/2, Then:

$$I_{DF} = \begin{cases} \rho|h_{sd}|^2 + \rho|h_{rd}|^2 & |h_{sr}|^2 \geq 2\alpha; \\ \rho|h_{sd}|^2 & \text{Otherwise.} \end{cases}$$

Theorem3 The outage of DF at LSNR is given by:

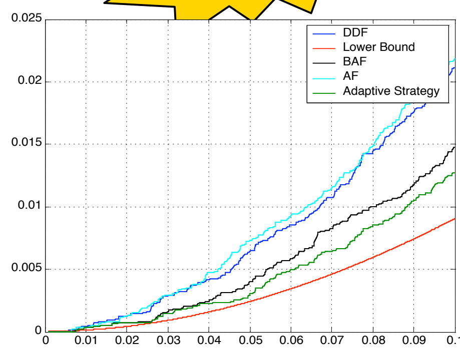
$$P_{o,DF} = \frac{5}{2}\alpha^2 - \frac{25}{3}\alpha^3 + \mathcal{O}(\alpha^4)$$

• *Note that the result can be generalized for any fraction of time* $f = 1 - \frac{1}{n}$

An Adaptive Strategy can improve outage!!

Partial CSI

(one extra bit)



Gain (G) in using the adaptive strategy w.r.t. BAF:

$$G = \frac{P_{o,BAF} - P_{low}}{P_{o,Ada} - P_{low}} = -\log\alpha \to \infty \text{ as } \alpha \to 0$$

• Is there a scheme that can improve the outage up to the second order??

• **Yes**!! Use an adaptive scheme that chooses between (BAF/DDF) based on the quality of the S-R channel and the rate R, hence :

We consider an adaptive scheduling scheme which chooses DF if $|h_{sr}|^2 \geq 2\alpha$ and otherwise uses BAF

• Only need partial CSI at the sender about the quality of the source-relay link (Just 1 bit)

$$P_{o,Ada} = Pr[I_{DDF} \leq \alpha\rho]\ Pr[|h_{sr}|^2 \geq 2\alpha] + Pr[I_{BAF} \leq \alpha\rho/|h_{sr}|^2 < 2\alpha]\ Pr[|h_{sr}|^2 < 2\alpha]$$

Theorem4 The adaptive scheme which switches between BAF and DF meets the lower bound of the outage expansion up to the second order. The achieved outage probability is:

$$P_{o,Ada} = \alpha^2 + 2\alpha^3 + O(\alpha^4 \ln\alpha).$$

References:
[SSA06] Masoud Sharif, Venkatesh Saligrama, George Atia, "Outage Capacity of Relay Channels in Low SNR: An Adaptive Strategy," submitted to CTW 2006.
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[AGS05] K. Azarian, H. El Gamal and P. Schniter, "On the achievable Diversity Multiplexing Tradeoff in Half-Duplex Cooperative Channels," IEEE Trans. Info Theory, 2005.
[LTW04] J.N. Laneman, D.N. Tse, G. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," IEEE Trans. Info. Theory, Dec. 2004.