Probability of Error Analysis under Arbitrary Fading and Power Allocation for Decode and Forward Cooperative Communication

CTW 2006
May 21-24

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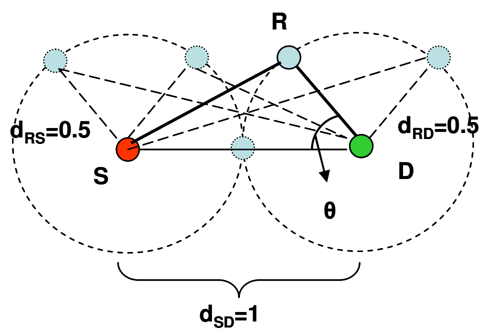

Classical Relay based cooperative communication with relay at different positions corresponding to $\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}$

Phase I: Source transmits; relay and destination listen
Phase II: Relay transmits decoded information to destination

$$\left.\begin{aligned} r_{SR} &= b_S \alpha_{SR} \sqrt{E_S} + n_{SR} \\ r_{SD} &= b_S \alpha_{SD} \sqrt{E_S} + n_{SD} \end{aligned}\right\} \text{phase I}$$

$$r_{RD} = \hat{b}_S \alpha_{RD} \sqrt{E_R} + n_{RD} \quad \text{phase II}$$

$$E_T = E_S + E_R$$

$$E_S = \delta E_T \text{ and } E_R = (1-\delta) E_T$$

$$\sigma_{ij}^2 = \left(d_{SD}/d_{ij}\right)^{\mu} \quad d_{SD} = 1$$

d_{ij} is the distance between i and j nodes

μ is the path loss coefficient ~ 3

Average received SNRs : $\bar{\gamma}_{SD} = \frac{\delta E_T \sigma_{SD}^2}{N_0}$ $\quad \bar{\gamma}_{SR} = \frac{\delta E_T \sigma_{SR}^2}{N_0}$ $\quad \bar{\gamma}_{RD} = \frac{(1-\delta) E_T \sigma_{RD}^2}{N_0}$

ASER for Decode and Forward

$$P_e = P_e^{SD} \cdot F_{\gamma_{SR}} + \left(1 - P_{e|\gamma_{SR} > \gamma^*}^{SR}\right) \cdot P_e^{(DIV)} + P_{e|\gamma_{SR} > \gamma^*}^{SR} \cdot P_e^{(X)}$$

$$F_{\gamma_{SR}} = \Pr\left(\gamma_{SR} < \gamma^*\right)$$

γ^* **: SNR threshold at relay**

P_e^{SD} **: Average symbol error probability (ASEP) source-to-destination**

$P_{e|\gamma_{SR} > \gamma^*}^{SR}$ **: ASEP source-to-relay link when received SNR is greater than threshold**

$P_e^{(DIV)}$ **: ASEP due to diversity combining at destination when relay decodes and retransmits correctly**

$P_e^{(X)}$ **: ASEP due to wrongful diversity combining at destination when relay decodes and retransmits incorrectly**

SEP for an arbitrary two dimensional signal constellation can be expressed in desirable exponential form.

Marginal MGF is known for Rayleigh, Rice and Nakagami-m, q channels

P_e^{SD} $\quad P_{e|\gamma_{SR} > \gamma^*}^{SR}$ **and** $P_e^{(DIV)}$ **can be readily evaluated using an MGF method**

For MPSK : $P_e(\gamma) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-\gamma \sin^2(\pi/M)}{\sin^2(\theta)}\right) d\theta$

$$P_{e|\gamma > \gamma^*}(\gamma) = \int_{\gamma^*}^{\infty} P_e(\gamma) f_\gamma(\gamma) d\gamma = \frac{1}{\pi} \int_{\gamma^*}^{(M-1)\pi/M} \phi_\gamma\left(\frac{\sin^2(\pi/M)}{\sin^2(\theta)}, \gamma^*\right) d\theta$$

$$\phi_\gamma(s,0) = \phi_\gamma(s) \equiv \text{MGF of } \gamma$$

Derivation of $P_e^{(X)}$ for BPSK

Decision statistic at destination after noise normalization and MRC combining: $\hat{Z} = \Re\left\{\alpha_{SD}^* r_{SD} + \alpha_{RD}^* r_{RD}\right\} = \delta \sqrt{E_T} \sigma_{SD}^2 \alpha_{SD}^2 + (1-\delta)\sqrt{E_T} \sigma_{RD}^2 \alpha_{RD}^2 + \Re\left\{\sqrt{\delta}\alpha_{SD}^* n_{SD} + \sqrt{1-\delta}\alpha_{RD}^* n_{RD}\right\}$

Let $Y = \delta \sigma_{SD}^2 \alpha_{SD}^2 + (1-\delta) \sigma_{RD}^2 \alpha_{RD}^2$ **and** $E\left\{n^2\right\} = \frac{\delta \alpha_{SD}^2 + (1-\delta)\alpha_{RD}^2}{2} N_0$

$$P_e^{(X)} = \Pr\left(\hat{Z} > 0 \middle| b_s = -1\right) = \Pr\left(n > Y\sqrt{E_T}\right) = P_1 + P_2$$

$$P_1 = \int_0^{\infty} \int_{\alpha_{RD}\sqrt{\frac{1-\delta}{\delta}}}^{\infty} Q\left(\left(\delta \alpha_{SD}^2 - (1-\delta)\alpha_{RD}^2\right)\sqrt{\frac{2E_T}{N_0} \frac{1}{\delta \alpha_{SD}^2 + (1-\delta)\alpha_{RD}^2}}\right) f_{\alpha_{SD}}(\alpha_{SD}) f_{\alpha_{RD}}(\alpha_{RD}) d\alpha_{SD} d\alpha_{RD}$$

$$P_2 = \int_0^{\infty} \int_0^{\alpha_{RD}\sqrt{\frac{1-\delta}{\delta}}} \left[1 - Q\left(-\left(\delta \alpha_{SD}^2 - (1-\delta)\alpha_{RD}^2\right)\sqrt{\frac{2E_T}{N_0} \frac{1}{\delta \alpha_{SD}^2 + (1-\delta)\alpha_{RD}^2}}\right)\right] f_{\alpha_{SD}}(\alpha_{SD}) f_{\alpha_{RD}}(\alpha_{RD}) d\alpha_{SD} d\alpha_{RD}$$

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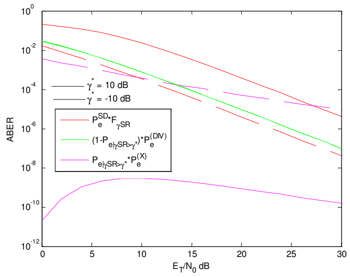
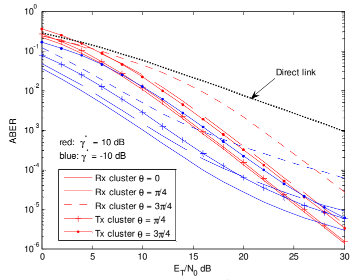
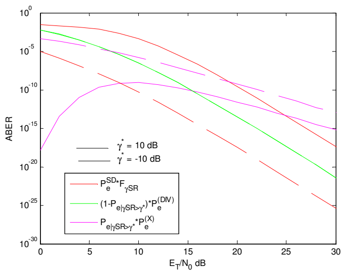
Equal Power Allocation



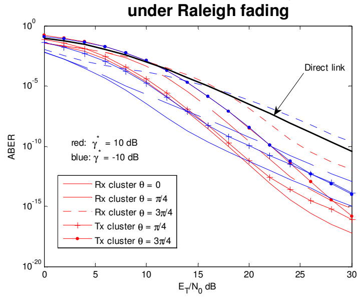
Contribution of individual terms to ABER under Rayleigh fading Rx Cluster $\theta = \pi/4$



Performance of BPSK DF system under Raleigh fading



Contribution of individual terms to ABER under Nakagami-m (m=4) fading Rx Cluster $\theta = \pi/4$

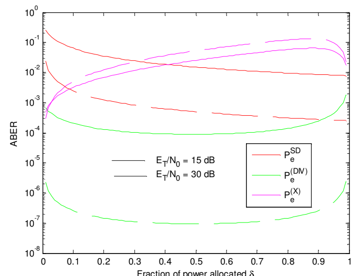


Performance of BPSK DF system under Nakagami-m (m= 4) fading

1. Decode and forward system performs better than direct link under Rayleigh fading
2. DF system with lower threshold level performs better at lower E_T/N_o values
3. Higher threshold level performs better at higher E_T/N_o values
4. Effect of wrongful combining dominates at high SNR
5. Best performance when the relay is situated in the middle
6. At SNR threshold of 10 dB relay in Tx cluster performs better than relay in Rx Cluster
7. At SNR threshold of -10 dB relay in Tx cluster performs better than relay in Rx Cluster only at high E_T/N_o
8. In Nakagami-m (m=4) channels due to better links performance is better than Rayleigh
9. Performance trends similar to Rayleigh

Wrongful Combining

Optimum SNR Threshold



Variation of ASEP terms under Raleigh fading

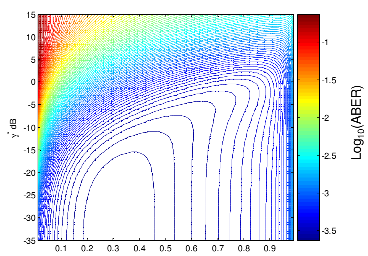
$$\frac{dP_e}{d\gamma^*} = f_{\gamma_{SR}}\left(\gamma^*\right)\left[P_e^{SD} + P_e\left(\gamma^*\right)\left(P_e^{(DIV)} - P_e^{(X)}\right)\right] = 0 \quad ➤ \quad P_e\left(\gamma^*\right) = \frac{P_e^{SD}}{P_e^{(X)} - P_e^{(DIV)}}$$

For BPSK : $P_e(\gamma) = \frac{1}{2} erfc\left(\sqrt{\gamma}\right)$ ➤ $\gamma_{opt}^{\ *} = \left[erfc^{-1}\left(\frac{2P_e^{SD}}{P_e^{(X)} - P_e^{(DIV)}}\right)\right]^2$

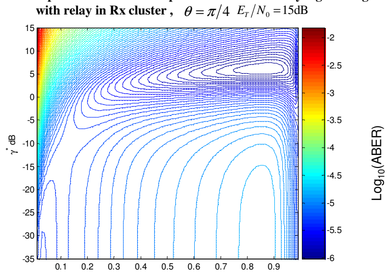
In practice however $P_e^{(X)}$ cannot be determined.

In practical operating environment $P_e^{(X)} \ll P_e^{(DIV)}$ **also** $P_e^{(X)} < 0.5$ ➤ $\gamma_{opt}^{\ *} = \left[erfc^{-1}\left(4P_e^{SD}\right)\right]^2$

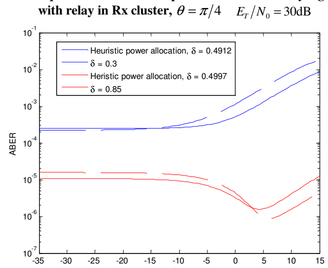
SNR Threshold and Power Allocation



Contour plot for BPSK ABER performance under Rayleigh fading with relay in Rx cluster , $\theta = \pi/4$ $E_T/N_0 = 15$dB



Contour plot for BPSK ABER performance under Rayleigh fading with relay in Rx cluster, $\theta = \pi/4$ $E_T/N_0 = 30$dB



BPSK ABER performance with Heuristic Power allocation

The contour plots show regions with minimum ABER
- ➤ A set of values of SNR threshold satisfy this condition
- ➤ A set of values of fractional power allocated satisfy this condition

Choosing higher SNR threshold will not only minimize wrongful combining but also improve rate (symbols per channel used)

$$R_{eff} = \Pr(\gamma_{SR} > \gamma^*)\frac{R}{2} + \Pr(\gamma_{SR} < \gamma^*)R \qquad R/2 < R_{eff} < R$$

Heuristic power allocation:
- ➤ Closed form solutions exist for optimum power allocation for transmit diversity systems in Rayleigh [Cavers 99] and Nakagami-m [Alouini 03] and approximate solution for Rice [Annamalai 04]
- ➤ Depends on fading parameter "m" estimation for Nakagami-m channels

$$P_l^* = m_l \max\left(\frac{E_T/N_0}{\sum_{j=1}^{L} m_j} + \frac{\sum_{j=1}^{L}\frac{m_j}{\sigma_j^2}}{b\sum_{j=1}^{L} m_j} - \frac{1}{b\sigma_l^2}, 0\right)$$ **Water filling method**

1. Use the above method to set the fractional power δ
2. Now plot ABER performance by varying SNR threshold

- ➤ Using fractional power allocation by Heuristic method provides performance comparable to the best possible power allocation seen in the contour plots
- ➤ However it is more important to find optimum SNR threshold that minimizes ABER
- ➤ Future effort will investigate methods to find optimum SNR threshold for practical use and upper bounds for performance

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