Inter-Cell Scheduling in a Cellular MIMO System

Wan Choi and Jeffrey G. Andrews

Wireless Networking and Communications Group (WNCG)
Department of Electrical and Computer Engineering
The University of Texas at Austin
{wchoi, jgandrews}@mail.utexas.edu

The capacity and robustness of cellular MIMO systems is very sensitive to other-cell interference, which will in practice necessitate network level interference reduction strategies [1]–[3]. As an alternative to traditional static frequency reuse patterns, we investigate inter-cell scheduling among neighboring base stations, where adjacent base stations and an (arbitrarily chosen) home base station cooperatively schedule their transmissions to reduce other-cell interference. Just as frequency reuse achieves OCI reduction at the expense of decreasing spectral efficiency by the frequency reuse factor, cooperatively scheduled transmission reduces OCI at the expense of decreasing throughput by the transmit duty cycle. There are two important advantages of inter-cell scheduling relative to traditional frequency reuse. First, universal frequency reuse can still be adopted, achieving an interference averaging effect, simplifying frequency planning in deployment, and reducing the number of required frequency channels for the system. Second, inter-cell scheduling achieves an additional gain termed *expanded multiuser diversity* if straightforward opportunistic scheduling is employed among neighboring base stations.

In this paper, we study the potential extra gain of inter-cell scheduling over conventional frequency reuse in the context of cellular MIMO systems. A single antenna system is a special case of our study. Our contributions can be summarized as follows:

- We derive capacity bounds for inter-cell scheduling and show inter-cell scheduling achieves an *expanded multiuser diversity* gain in terms of ergodic capacity as well as almost the same amount of interference reduction as conventional frequency reuse.
- The analysis is based on a practical cellular environment including lognormal shadowing and user geometry, which have usually been neglected in previous MIMO studies despite its importance in cellular systems.
- The capacity analysis provides a lower bound on attainable capacity via joint multiple cell-site processing [4], [5] since inter-cell scheduling is the simplest joint multiple cell-site processing.
- We provide an altered view of multiuser diversity in the context of a multi-cell system. – The derived ergodic capacity bounds show that the multiuser diversity gain grows like $\sqrt{\log K}$ in a cellular system, whereas it has previously been known to grow only as $\log\log K$ when just short-term fading is considered [6], [7].

Even though TDMA is not the capacity achieving strategy for a multiple-antenna system, TDMA with optimal user selection is still a practical solution and widely used in commercial systems. In this context, we investigate the potential gains of inter-cell scheduling in MIMO TDMA systems with optimal user selection. We consider opportunistic scheduling where a transmission in each time slot is allocated only to the base station that is able to provide the highest throughput among the base stations consisting of one cluster (7 cells).

If opportunistic inter-cell scheduling is applied to TDMA systems with optimal user selection, the user with the highest instantaneous mutual information is selected for transmission among all the users in the cells involved. Therefore, the per-cell ergodic capacity of this system is given by

$$\mathbb{E}\left[\mathcal{C}_{coop}\right] = \frac{1}{N_s}\mathbb{E}\left[\max_{1\le k\le N_s K}\max_{\mathrm{Tr}(\mathbf{Q}_k)\le 1}\log\left|\mathbf{I}_{M_r}+\gamma_k\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^H\right|\right]$$

where γ_k is the large-scale interference plus noise power perceived at the user k at cell i, reflecting user geometry and lognormal shadowing. Because the sum of lognormal random variables is well approximated by a lognormal random variable [8] and the ratio of lognormal random variables is also a lognormal random variable, the distributions of γ_k is well modeled by lognormal distribution. The MIMO channels of all the users in the cell i, $\{\mathbf{H}_k^{(i)}\}$, are assumed to be static during a transmission slot and have an independently and identically distributed complex Gaussian distribution $\sim \mathcal{CN}(0,1)$. The matrix $\mathbf{Q}_k^{(i)}$ is the normalized covariance matrix of the transmitted signal with a transmit power constraint of $\mathrm{Tr}(\mathbf{Q}_k^{(i)}) \le 1$. The factor $1/N_s$ reflects the throughput loss by the transmission duty cycle, and N_s is the total number of cooperating cells. Compared to the system without inter-cell scheduling, the cardinality of the selection pool increases from K to N_sK at the expense of decreasing the throughput by the transmit duty cycle.

As the cardinality of the selection pool increases, the multiuser diversity gain correspondingly increases. This effect can be interpreted as *expanded multiuser diversity*. This gain can be measured by comparing the cell throughput with that of TDMA systems with conventional frequency reuse, with reuse factor $1/N_s$. In order to investigate the exact gain of expanded multiuser diversity, we could rely on computer simulations but analysis for large K can provide insights into the nature of the expanded multiuser diversity gain. In the analysis for large K, we rely on some known theorems and lemmas on the asymptotic behavior of the maximum of n i.i.d. random variables when n is sufficiently large [7], [9], [10]. Then, we can derive a theorem that gives bounds on the achievable per-cell ergodic capacity for large K.

Theorem 1: When M_t, M_r, and P are fixed, the capacity of optimal TDMA with *cooperatively scheduled transmission* for large K is bounded by

$$\begin{aligned}&\frac{\min(M_r,M_t)}{N_s}\log\left(1+\frac{b-a\log\log N_sK}{\min(M_r,M_t)}\log N_sK\right)\\&\cdot\left(1-\mathcal{O}\left(\frac{1}{\log N_sK}\right)\right)\left(1-\frac{1}{\log N_sK}\right)\leq\mathbb{E}\left[C_{coop}\right]\leq\\&\frac{\min(M_r,M_t)}{N_s}\log\left(1+\frac{b+\gamma_e a}{\min(M_r,M_t)}\big(\log N_sK\right.\\&\left.+M_rM_t\log\log N_sK\big)\right)+\mathcal{O}\left(\frac{1}{\sqrt{\log N_sK}}\right)+\mathcal{O}(1)\end{aligned}$$

where

$$\begin{aligned}b&=\exp\left\{(2\log N_sK)^{1/2}\sigma+\mu\right\},\\a&=b\sigma/(2\log N_sK)^{1/2}.\end{aligned}$$

Therefore, we have

$$\lim_{K\to\infty}\frac{\mathbb{E}\left[C_{coop}\right]}{\frac{1}{N_s}\min(M_r,M_t)\sigma_{coop}\sqrt{2\log N_sK}}=1.$$

The insights from this theorem can be summarized in 2 key results as follows:

- *Key Result 1:* Theorem 1 indicates that the capacity of TDMA systems with cooperatively scheduled transmission scales like $\frac{1}{N_s}\min(M_r,M_t)\sigma\sqrt{2\log N_sK}$ whereas the capacity of TDMA systems with frequency reuse scales like $\frac{1}{N_s}\min(M_r,M_t)\sigma'\sqrt{2\log K}$.
- *Key Result 2:* Theorem 1 also shows that the multiuser diversity gain in MIMO TDMA system grows like $\min(M_r,M_t)\sqrt{\log K}$ when the geometry of mobile stations and lognormal shadowing are considered, while it has been previously known to grow like $\min(M_r,M_t)\log\log K$ when only the short-term fading is considered [6], [7].

We numerically show the capacity gains of inter-cell scheduling over conventional frequency reuse through computer simulations, and compare with the derived asymptotic results. We consider various Rayleigh MIMO channels and propagation pathloss given by $L_k=(d_k/d_0)^{-l}$ where d is the distance from a base station to mobile station k and the pathloss exponent $l=3.5$. The total number of cells involved in cooperative scheduling is $N_s=7$ with the 48 nearest cells treated as OCI sources. Correspondingly, for comparison with traditional frequency reuse systems, $f=7$. The K users in each cell are randomly placed according to a uniform distribution. Figure 1 shows how the ergodic capacity is affected by the number of antennas (M_t,M_r), and how much inter-cell opportunistic scheduling helps relative to traditional frequency reuse. It also shows both the bounds and simulation results for the capacities of 2×2 and 4×4 MIMO TDMA systems. As expected, inter-cell opportunistic scheduling achieves higher capacity than frequency reuse, with a large user gain of about 1 bps/Hz for 2×2 MIMO and about 2 bps/Hz for a 4×4 MIMO system. The upper and lower bounds are accurate for large K, but optimistic for smaller K. The bounds on capacity attained by inter-cell scheduling converge faster because the effective number of users is N_sK, compared to just K in traditional frequency reuse.

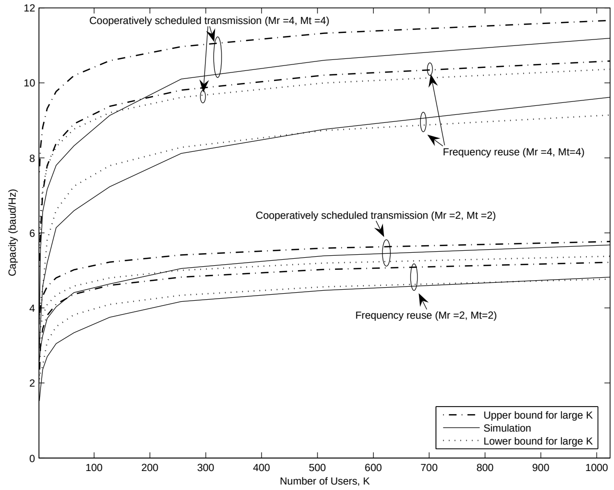

Fig. 1. Ergodic capacity of 2×2 and 4×4 MIMO TDMA systems with cooperatively scheduled transmission and with frequency reuse. The upper and lower bounds agree with simulations for large K.

REFERENCES

[1] H. Dai, A. Molisch, and H. Poor, “Downlink capacity of interference-limited MIMO systems with joint detection,” *IEEE Trans. on Wireless Communications*, vol. 3, no. 2, pp. 442–453, Mar. 2004.

[2] R. Blum, J. Winters, and N. Sollenberger, “On the capacity of cellular systems with MIMO,” *IEEE Communications Letters*, vol. 6, no. 6, pp. 242–244, June 2002.

[3] S. Catreux, P. F. Driessen, and L. J. Greenstein, “Attainable throughput of an interference-limited multiple-input multiple-output (MIMO) cellular system,” *IEEE Trans. on Communications*, vol. 49, no. 8, pp. 1307–1311, Aug. 2001.

[4] S. Jafar, G. Foschini, and A. Goldsmith, “Phantomnet: Exploring optimal multicellular multiple antenna systems,” in *Proc., IEEE Veh. Technology Conf.*, Sept. 2002, pp. 24–28.

[5] S. Shamai and B. Zaidel, “Enhancing the cellular downlink capacity via co-processing at the transmitting end,” in *Proc., IEEE Veh. Technology Conf.*, May 2001, pp. 1745–1749.

[6] P. Viswanath, D. N. C. Tse, and R. Laroia, “Opportunistic beamforming using dumb antennas,” *IEEE Trans. on Info. Theory*, vol. 48, no. 6, pp. 1277–1294, June 2002.

[7] M. Sharif and B. Hassibi, “A comparison of time-sharing, DPC, and beamforming for MIMO broadcast channels with many users,” *submitted to IEEE Trans. on Comm.*, 2004.

[8] S. Schwartz and Y. Yeh, “On the distribution function and moments of power sums with lognormal components,” *Bell Syst. Tech. Jour.*, vol. 61, no. 7, pp. 1441–1462, Sept. 1982.

[9] J. Hefferna and J. Tawn, *Extreme Value Theory*, RSS/EPSRC Graduate Training Programme Coursenotes, Lancaster University, 2003.

[10] S. G. Coles, *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag, 2001.