Estimation Diversity in Distributed Sensing

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Joint Estimation With Slow Fading Channels (Orthogonal MAC)

\[ E(\hat{\theta} - \theta)^2 \]

Different fading channel gain

Different observation quality

\[ \theta(t) \]

\[ x_{\alpha_1} \quad x_{\alpha_2} \quad x_{\alpha_K} \]

Sensor 1

Sensor 2

Sensor K

Fusion Center

\[ P_1 \quad P_2 \quad P_K \]

\[ +\sigma^2_1 \quad +\sigma^2_2 \quad +\sigma^2_K \]

\[ \xi^2_1 \quad \xi^2_2 \quad \xi^2_K \]
Best Linear Unbiased Estimator (BLUE)

- Why BLUE?
  - Universal (for any joint distribution)
  - Simple processing
  - Optimal in Gaussian case

- With ideal channels:
  - Infinite bandwidth (no need for lossy quantization)
  - Infinite power (or no channel distortion)
  - BLUE gives:

\[
\bar{\theta}_K = \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{x_k}{\sigma_k^2}
\]

\[
E( |\bar{\theta}_K - \theta|^2 ) = \left( \sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1}
\]
Joint Estimation in Fading Environment

- Channel gain with exponential distribution: \( g_k = \frac{C_0}{d_k^2} |r_k|^2 \)

- Slow fading: random but static within each observation period

- Multiple nodes (say, K nodes):
  - Reduce observation noise by linear combination
  - Diversity over independent fading channels

- To compare the performance for different K’s
  - Let us fix the total power budget
  - Each node also has an individual power limit (later)
Average vs. Outage Performance

Average distortion in fast fading channels:

$$E\{\text{Var}[\hat{\theta}]\} = \int \text{Var}[\hat{\theta}] f_g(g) \, dg$$

Distortion outage probability in slow fading channels:

$$P_{D_0} = \text{Prob}\{\text{Var}[\hat{\theta}] > D_0\}$$
Equal Power Allocation

- We may not know the channel at Tx side
- We may just be lazy
- We may just want to investigate the performance lower bound

Observation SNR: \( \gamma_k = \frac{W^2}{\sigma_k^2} \)

Channel SNR: \( s_k = \frac{g_k}{\xi_k^2} \)

\[
\text{Var}[\hat{\theta}] = W^2 \left( \sum_{k=1}^{K} \frac{P s_k}{\gamma_k^{-1} P s_k + K} \right)^{-1}
\]
Outage Performance

With a distortion target $D_0$

**Theorem:**

\[ P_{D_0} \sim \exp\left(-KI_s(a)\right) \]

where $I_s(a)$ is the rate function of $s_k$

Proof based on Large Deviation Theorem: for i.i.d. $\beta_k$

\[
\text{Prob} \left\{ \frac{1}{K} \sum_{k=1}^{K} \beta_k < a \right\} \leq \exp\left(-KI_{\beta}(a)\right)
\]

Where

\[
I_{\beta}(a) = \sup_{\theta \in \mathbb{R}} (\theta a - \log M_{\beta}(\theta))
\]
Estimation Diversity With Rayleigh Fading

pdf of channel SNR:

\[ f_s(x) = \frac{1}{2\eta^2} \exp\left\{ -\frac{x}{2\eta^2} \right\} \]

\[ I(a) \sim \log P \quad \text{When } P \text{ is large} \]

Outage Probability:

\[ \log \mathcal{P}_{D_0} \sim -K \log P \]

K is the slope of the probability curve at high P: Diversity order
Average Performance

![Graph showing expected distortion versus total power for different numbers of nodes. The graph plots expected distortion on a logarithmic scale against total power on a logarithmic scale. The x-axis represents total power $P_{\text{total}}$, ranging from $10^{-2}$ to $10^{0}$. The y-axis represents expected distortion, also on a logarithmic scale, ranging from $10^{-3}$ to $10^{-1}$. There are three curves: 1 node (dashed line), 2 nodes (dotted line), and 3 nodes (solid line). As the total power increases, the expected distortion decreases for all cases.](Image)
Outage Performance

Outage Probability (Equal Power)

Total Power $P_{\text{total}}$

- 1 node
- 2 nodes
- 3 nodes
Minimizing Distortion with Sum Power Constraint

\[
\begin{align*}
\min_{\alpha_k} & \quad \left( \sum_{k=1}^{K} \frac{\alpha_k s_k}{\sigma_k^2 \alpha_k s_k + 1} \right)^{-1} \\
\text{s. t.} & \quad \sum_{k=1}^{K} 4W^2 \alpha_k \leq P_{\text{total}} \\
& \quad \alpha_k \geq 0, \quad k = 1, \ldots, K.
\end{align*}
\]

Equivalent:

\[
\min_{\alpha_k} \quad - \sum_{k=1}^{K} \frac{\alpha_k s_k}{\sigma_k^2 \alpha_k s_k + 1}
\]
Closed-form Solution

Assume: \[ s_1 \geq s_2 \geq \ldots \geq s_K \]

\[
\alpha_{k}^{opt} = \begin{cases} 
0, & k > K_1 \\
\frac{1}{s_k \sigma_k^2} (\sqrt{s_k \eta_0} - 1), & k \leq K_1 
\end{cases}
\]

\[
\text{Var}[\hat{\theta}] = \left( \sum_{k=1}^{K_1} \frac{1}{\sigma_k^2} \left( 1 - \frac{1}{\eta_0} \sqrt{\frac{1}{s_k}} \right) \right)^{-1}
\]
Bad Sensors Won’t Help
Outage Performance with Adaptive Power Gain
Performance Loss with Additional Individual Power Constraint

![Graph showing outage probability vs. total power for different power constraint scenarios.]
Conclusions

- Joint design of the application layer and the link layer is beneficial.
- Achievable estimation diversity gain + power gain in slow fading environment.
- Bad sensors won’t help.