Estimation Diversity in Distributed Sensing

Shuguang Cui, University of Arizona
Andrea Goldsmith, Stanford University
Jinjun Xiao, Zhi-quan Luo, University of Minnesota
H. Vincent Poor, Princeton University

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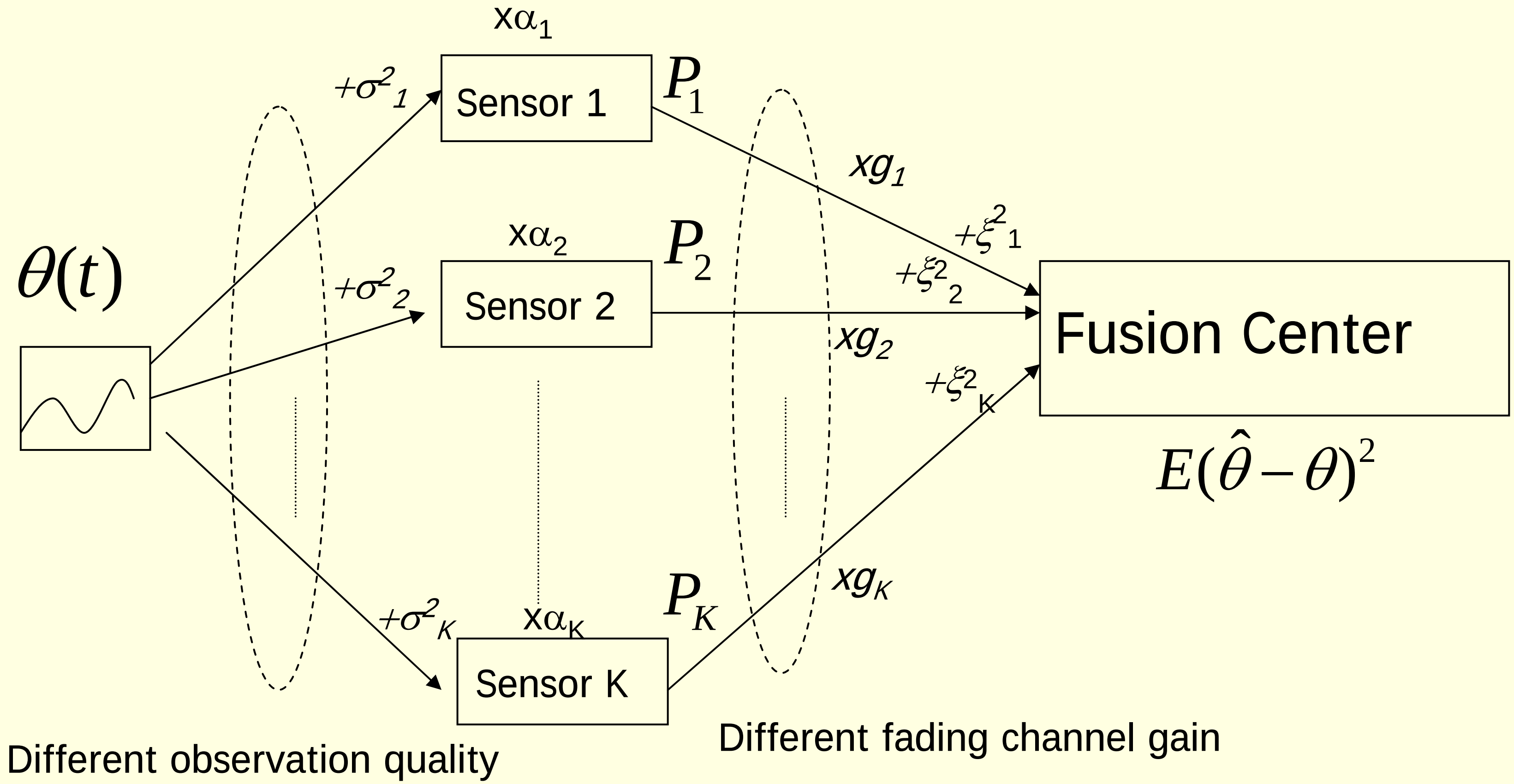
Joint Estimation With Slow Fading Channels (Othorgonal MAC)



Best Linear Unbiased Estimator (BLUE)

- Why BLUE?
  - Universal (for any joint distribution)
  - Simple processing
  - Optimal in Gaussian case
- With ideal channels:
  - Infinite bandwidth (no need for lossy quantization)
  - Infinite power (or no channel distortion)
  - BLUE gives:

$$\overline{\theta}_K = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1} \sum_{k=1}^{K} \frac{x_k}{\sigma_k^2} \qquad \mathrm{E}\left(|\overline{\theta}_K - \theta|^2\right) = \left(\sum_{k=1}^{K} \frac{1}{\sigma_k^2} \right)^{-1}$$

Joint Estimation in Fading Environment

- Channel gain with exponential distribution: $g_k = \frac{G_0}{d_k^2}|r_k|^2$
- Slow fading: random but static within each observation period
- Multiple nodes (say, K nodes):
  - Reduce observation noise by linear combination
  - Diversity over independent fading channels
- To compare the performance for different K's
  - Let us fix the total power budget
  - Each node also has an individual power limit (later)

Average vs. Outage Performance

Average distortion in fast fading channels:

$$E\{Var[\hat{\theta}]\} = \int Var[\hat{\theta}] f_{\mathbf{g}}(\mathbf{g}) d\mathbf{g}$$

Distortion outage probability in slow fading channels:

$$\mathcal{P}_{D_0} = \mathrm{Prob}\{\mathrm{Var}[\hat{\theta}] > D_0\}$$

Equal Power Allocation

- We may not know the channel at Tx side
- We may just be lazy
- We may just want to investigate the performance lower bound

Observation SNR: $\gamma_k = W^2/\sigma_k^2$ Channel SNR: $s_k = g_k/\xi_k^2$

$$\mathrm{Var}[\hat{\theta}] = W^2 \left(\sum_{k=1}^{K} \frac{P s_k}{\gamma_k^{-1} P s_k + K} \right)^{-1}$$

Outage Performance

With a distortion target D_0

Theorem: $\mathcal{P}_{D_0} \sim \exp(-KI_s(a))$

where $I_s(a)$ is the rate function of s_k

Proof based on Large Deviation Theorem: for i.i.d. β_k

$$\text{Prob}\left\{\frac{1}{K}\sum_{k=1}^{K}\beta_k < a\right\} \leq \exp(-KI_\beta(a))$$

Where $I_\beta(a) = \sup_{\theta \in \mathbb{R}}(\theta a - \log M_\beta(\theta))$

Estimation Diversity With Rayleigh Fading

pdf of channel SNR: $$f_s(x) = \frac{1}{2\eta^2} \exp\left\{-\frac{x}{2\eta^2}\right\}$$

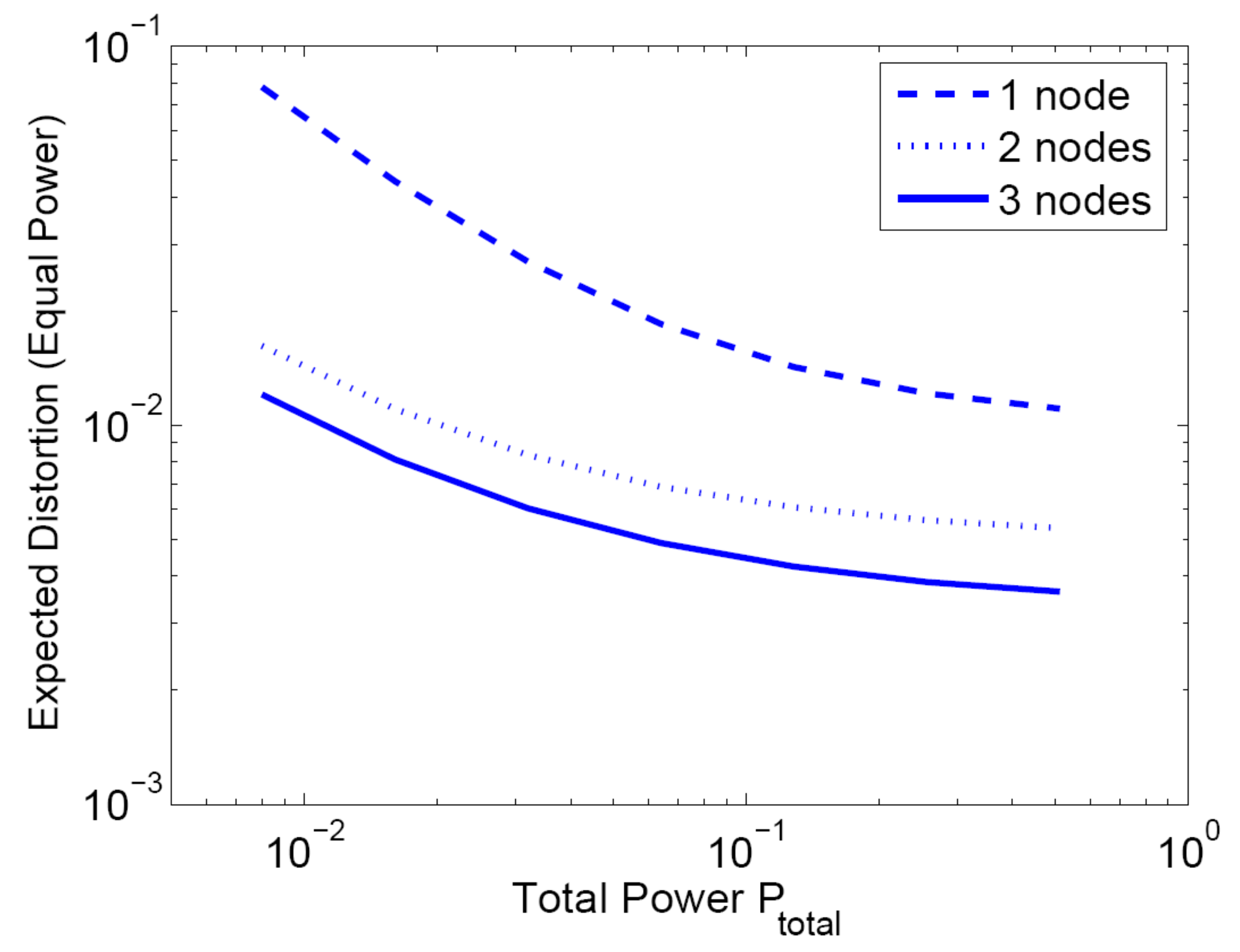
$$I(a) \sim \log P$$ When P is large

Outage Probability:

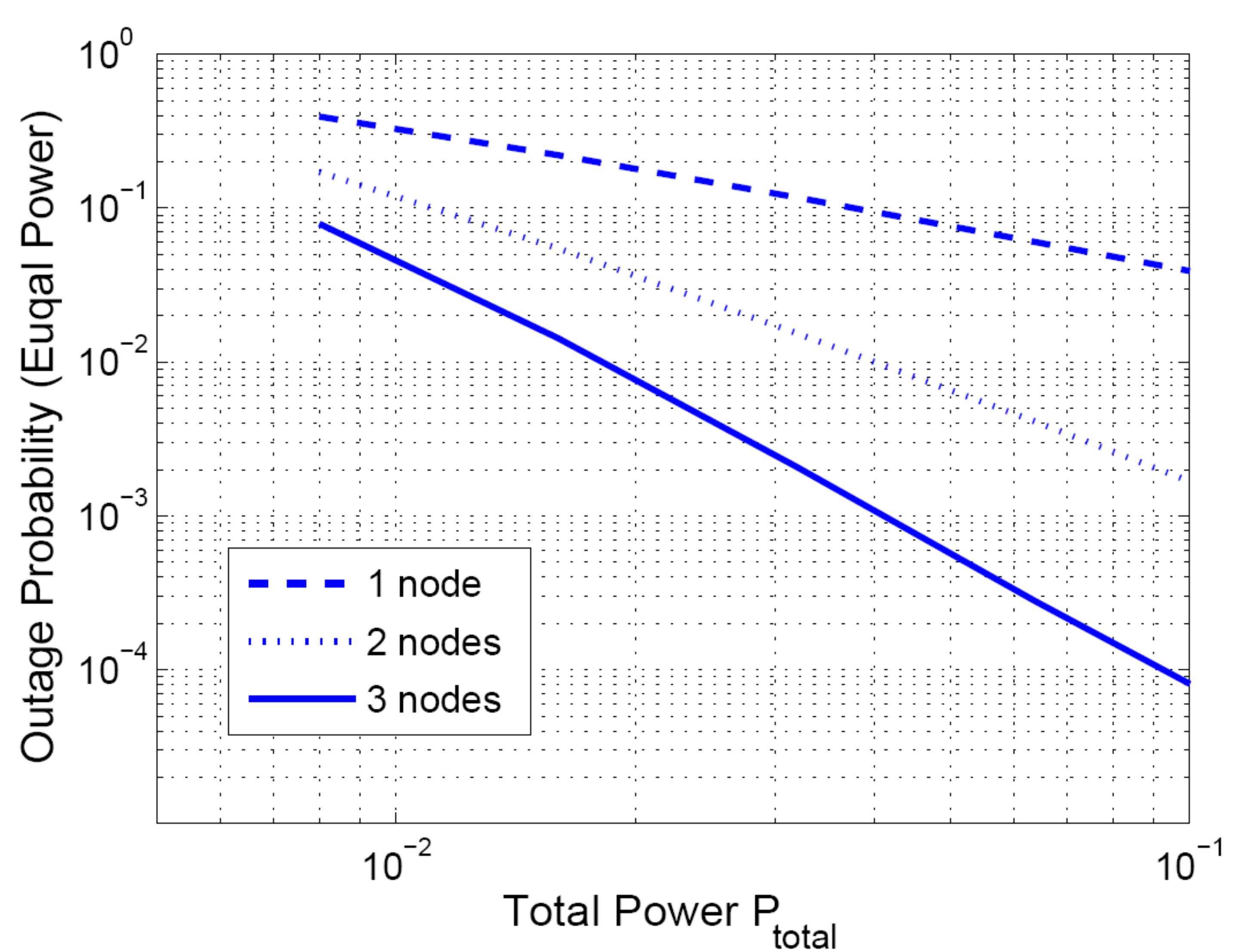
$$\log \mathcal{P}_{D_0} \sim -K \log P$$

K is the slope of the probability curve at high P: *Diversity order*

Average Performance



Outage Performance



Minimizing Distortion with Sum Power Constraint

$$\min_{\alpha_k} \quad \left(\sum_{k=1}^{K} \frac{\alpha_k s_k}{\sigma_k^2 \alpha_k s_k + 1} \right)^{-1}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} 4W^2 \alpha_k \leq P_{total}$$

$$\alpha_k \geq 0, \quad k = 1, \cdots, K.$$

Equivalent: $$\min_{\alpha_k} \quad - \sum_{k=1}^{K} \frac{\alpha_k s_k}{\sigma_k^2 \alpha_k s_k + 1}$$

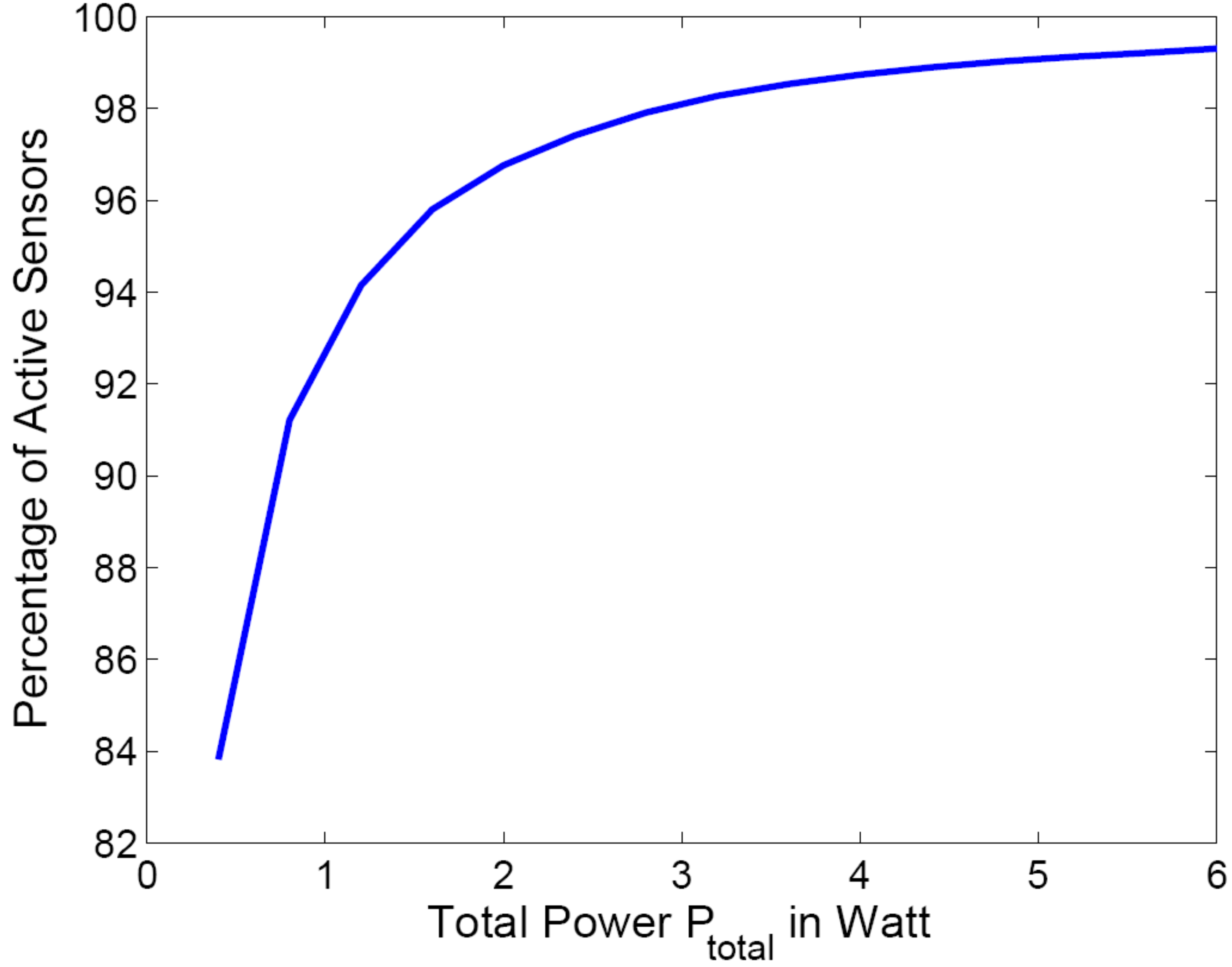
Closed-form Solution

Assume: $s_1 \geq s_2 \geq \ldots \geq s_K.$

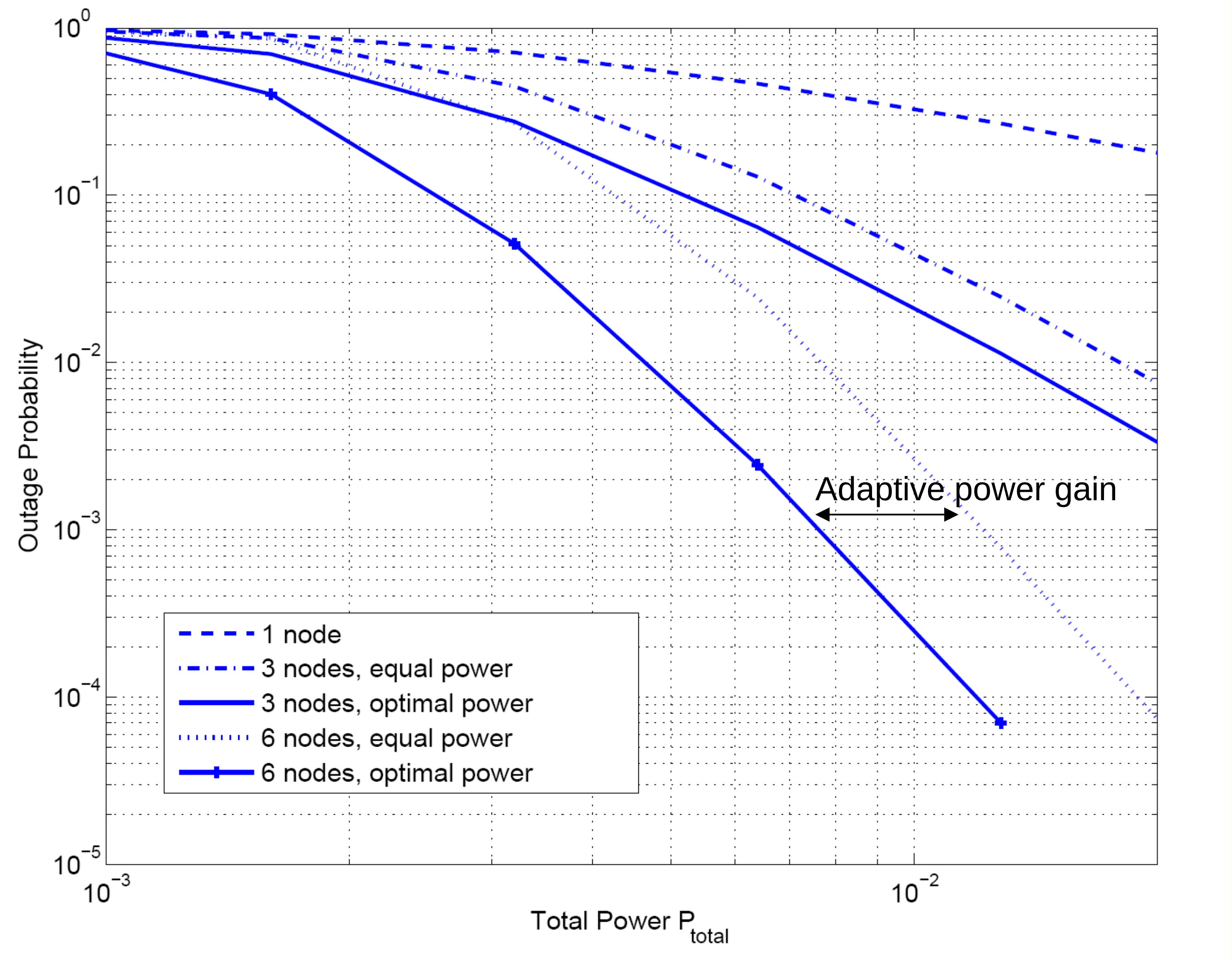
$$\alpha_k^{opt} = \begin{cases} 0, & k > K_1 \\ \frac{1}{s_k \sigma_k^2} \left(\sqrt{s_k} \eta_0 - 1 \right), & k \leq K_1 \end{cases}$$

$$\mathrm{Var}[\hat{\theta}] = \left(\sum_{k=1}^{K_1} \frac{1}{\sigma_k^2} \left(1 - \frac{1}{\eta_0} \sqrt{\frac{1}{s_k}} \right) \right)^{-1}$$

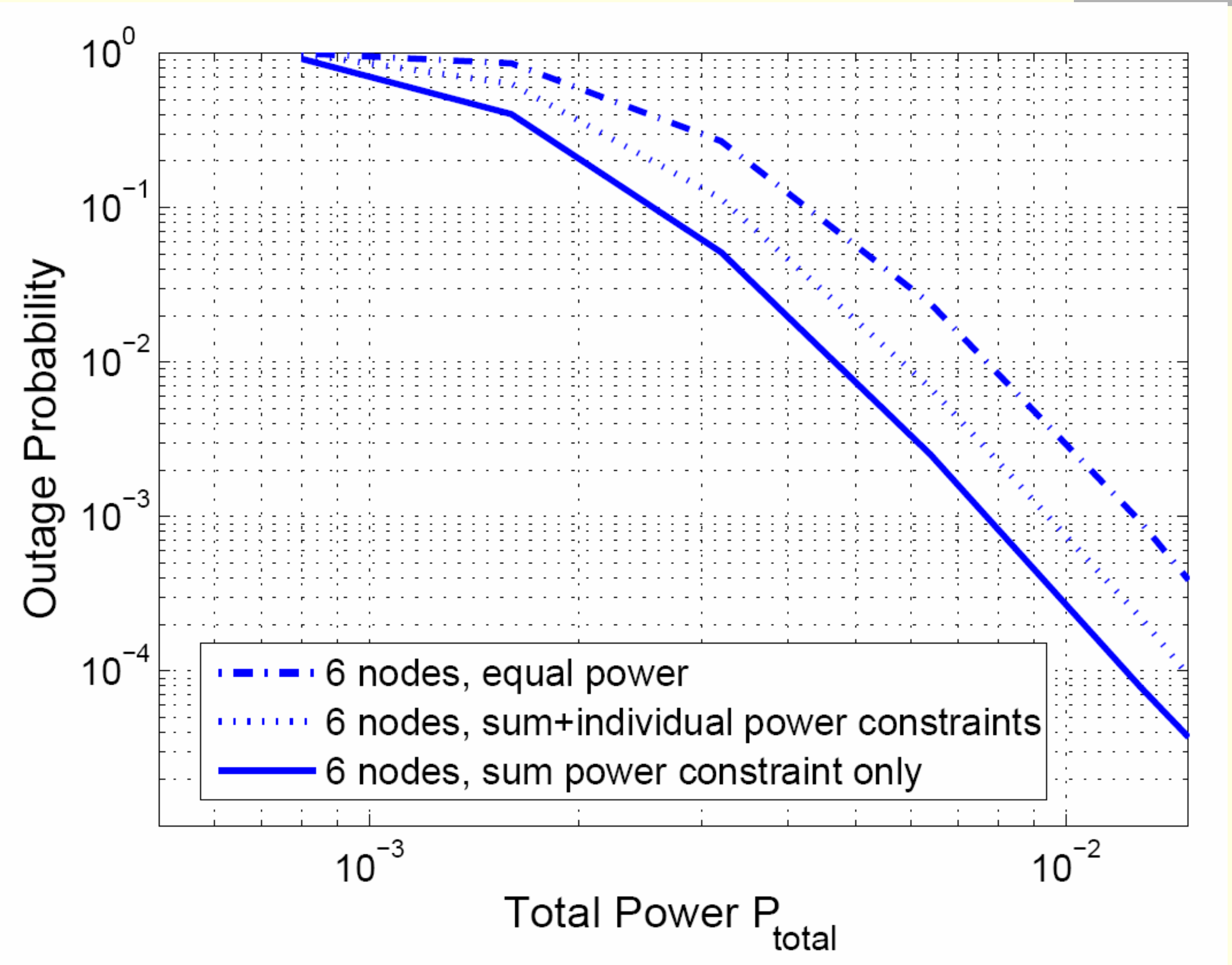
Bad Sensors Won't Help



Outage Performance with Adaptive Power Gain



Performance Loss with Additional Individual Power Constraint



Conclusions

- Joint design of the application layer and the link layer is beneficial
- Achievable estimation diversity gain + power gain in slow fading environment
- Bad sensors won't help