On Space-Time Block Codes from Coordinate Interleaved Orthogonal Designs

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Introduction

- Among the proposed space-time coding techniques, orthogonal space-time block codes (OSTBC) [1] are one of the most attractive methods to exploit the spatial diversity of the multiple input multiple output (MIMO) fading channels.
- While OSTBC enjoy the minimal maximum likelihood (ML) detection complexity, their code rate is rather low when there are more than 2 transmit (Tx) antennas.
- Two new STBC have been proposed to increase the code rate and keep the decoding-complexity minimal: (1) *Minimum decoding complexity quasi-orthogonal space-time block code* by Yuen, Guan, and Tjhung [2]; and (2) *coordinate interleaved orthogonal designs* (CIOD) due to Khan and Rajan [3].
- The code rates of OSTBC, MDC-QSTBC, and CIOD codes are compared in Table 1.

Table 1: Code Rates of Single-Symbol Decodable STBC

Codes	$M=2$	$M=3,4$	$M=5,6$	$M=7,8$
OSTBC	1	3/4	2/3	5/8
MDC-QSTBC		1	3/4	3/4
CIOD		1	6/7	4/5

CIOD codes offer equal or higher code rate compared with OSTBC and MDC-QSTBC.

Algebraic Constraints of Single-Symbol Decodable STBC

- The general linear dispersion representation of STBC is

$$C=\sum_{k=1}^{K}(a_kA_k+b_kB_k). \qquad (1)$$

- For OSTBC $\mathscr{O}_M$ designed for M antennas [4]:

$$A_i^{\mathsf{H}}A_i=B_i^{\mathsf{H}}B_i=I_M, \qquad i=1,2,\cdots,K \qquad (2a)$$
$$A_i^{\mathsf{H}}A_j+A_j^{\mathsf{H}}A_i=0_M, \qquad 1\le i\ne j\le K \qquad (2b)$$
$$B_i^{\mathsf{H}}B_j+B_j^{\mathsf{H}}B_i=0_M, \qquad 1\le i\ne j\le K \qquad (2c)$$
$$A_i^{\mathsf{H}}B_j+B_j^{\mathsf{H}}A_i=0_M, \qquad 1\le i,j\le K. \qquad (2d)$$

- The necessary and sufficient conditions to obtain single complex symbol decoding are [3]:

$$A_i^{\mathsf{H}}A_j+A_j^{\mathsf{H}}A_i=0_M, \qquad 1\le i\ne j\le K \qquad (3a)$$
$$B_i^{\mathsf{H}}B_j+B_j^{\mathsf{H}}B_i=0_M, \qquad 1\le i\ne j\le K \qquad (3b)$$
$$A_i^{\mathsf{H}}B_j+B_j^{\mathsf{H}}A_i=0_M, \qquad 1\le i\ne j\le K. \qquad (3c)$$

Comparing (3) and (2):

- The orthogonal constraint (2a) is removed.
- Constraint (2d) is applied for all pairs of A_i and B_j, while (3c) is applied for the pairs of A_i and B_j with sub-indexes $i\ne j$.

Single real-symbol decoding complexity is possible with OSTBC, but pair-wise real-symbol decoding is the minimum decoding complexity that can be obtained for the STBC satisfying (3).

Constructions of CIOD Codes

There are two constructions of CIOD codes: delay-optimal and rate-optimal.

1. Delay-optimal CIOD codes

- The CIOD code matrix for $M=2m$ Tx antennas is:

$$\mathscr{C}_M=\begin{bmatrix}\mathscr{O}_{m,1} & 0\\ 0 & \mathscr{O}_{m,2}\end{bmatrix} \qquad (4)$$

  where 0 is all-zero matrix of proper size, and $\mathscr{O}_{m,1}$ and $\mathscr{O}_{m,2}$ are code matrices of an OSTBC designed for m Tx antennas. There are K symbols $s_1,\ldots,s_K$ embedded in $\mathscr{O}_{m,1}$ and other K symbols $s_{K+1},\ldots,s_{2K}$ embedded in $\mathscr{O}_{m,2}$.
- The structure of the example CIOD code in (4) guarantees single complex symbol decoding. However, to achieve full diversity, the transmitted symbols s_i are generated as follows.
  - Let $d_i=a_i+\mathrm{j}b_i$ $(i=1,\ldots,2K, \mathrm{j}^2=-1)$ be the data (information) symbols drawn from QAM, PSK, Hex constellations. The real and imaginary parts of d_i are jointly rotated by a unitary matrix $R=\begin{bmatrix}\sin\alpha & \cos\alpha\\ \cos\alpha & -\sin\alpha\end{bmatrix}$ to generate intermediate symbols $\left[x_i\ y_i\right]^{\mathsf{T}}=R\left[a_i\ b_i\right]^{\mathsf{T}}$.
  - The transmitted symbols $s_k(k=1,\ldots,K)$ are formulated as $s_k=x_k+\mathrm{j}y_{K+k}, s_{K+k}=x_{K+k}+\mathrm{j}y_k$.
- Example: The CIOD code for 4 antennas.

$$\begin{bmatrix}a_1+\mathrm{j}b_3 & a_2+\mathrm{j}b_4 & 0 & 0\\ -a_4+\mathrm{j}b_4 & a_1-\mathrm{j}b_3 & 0 & 0\\ 0 & 0 & a_3+\mathrm{j}b_1 & a_4+\mathrm{j}b_2\\ 0 & 0 & -a_4+\mathrm{j}b_2 & a_3-\mathrm{j}b_1\end{bmatrix} \qquad (5)$$

- By the above encoding rule, the data symbols a_i and b_i are transmitted over all the Tx antennas. Thus symbol-wise diversity can be achieved. At the receiver, the ML detection of data symbols d_i can be decoupled such that only two real symbols a_i and b_i are required to be jointly detected.

2. Rate-optimal CIOD codes

To improve the rate of CIOD codes, the code matrices $\mathscr{O}_{m,1}$ and $\mathscr{O}_{m,2}$ in (4) of the same OSTBC are replaced by $\mathscr{O}_{M_1}$ and $\mathscr{O}_{M_2}$ (of two different OSTBC), respectively. Some necessary modifications are also required. See [3] for full details.

Open Problems

Khan and Rajan derive the optimal rotation R in terms of coding gain, which is actually minimizing the worst-case pair-wise error probability (PEP); the optimal rotations are obtained for QAM only.

1. How to find the optimal rotation in terms of minimizing symbol error rate (SER)?
2. Among QAM, HEX (or TRI), and PSK, which signal yield the best performance?

Our Results

1. The equivalent channels are derived.
2. A new maximum likelihood decoding metric is proposed, which is simpler than that proposed by Khan-Rajan.
3. Closed-form pair-wise SER is obtained.
4. The union bound on SER can be numerically evaluated. The bound is in the proximity of 0.1 dB from the simulated SER, when SER $<10^{-2}$.
5. Optimize the rotation R based on the union bound for an arbitrary constellation.
6. For signals with unbalanced powers of real and imaginary parts, we propose a new method combining power re-allocation and signal rotation. The new method performs better than the ones proposed in [3, 5].

Numerical Examples and Discussion

Fig. 1 shows that the calculated union bound is only becomes tight when SER $<10^{-1}$ and is only 0.1 dB from the simulated SER when SER $<10^{-2}$.

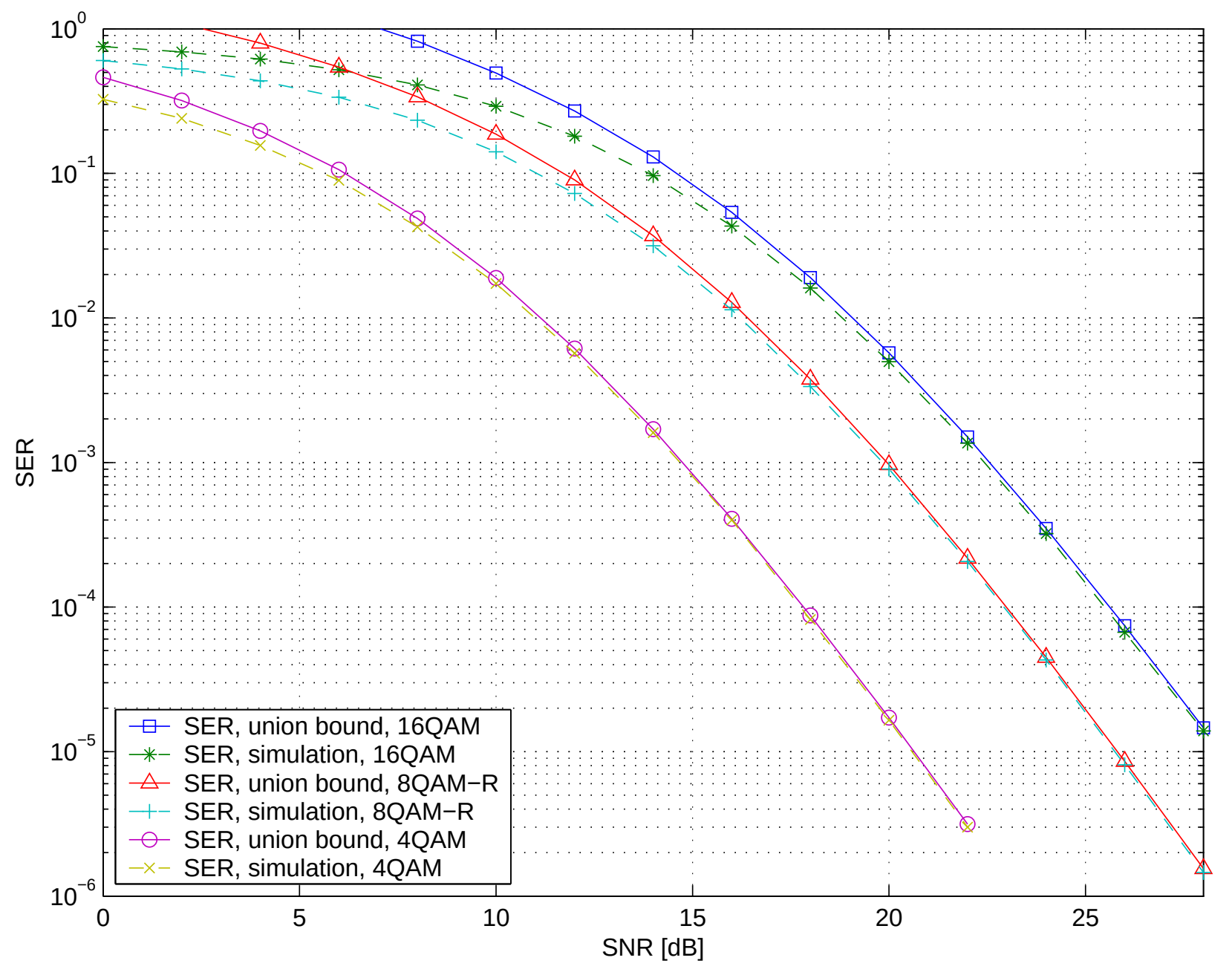

Figure 1: SER and the union bound of a rate-one CIOD code for 4 Tx/1 Rx antennas.

Union bounds on SER for different types of signals are compared in Fig. 2. Obviously, QAM produces the best performance compared with HEX and PSK signals.

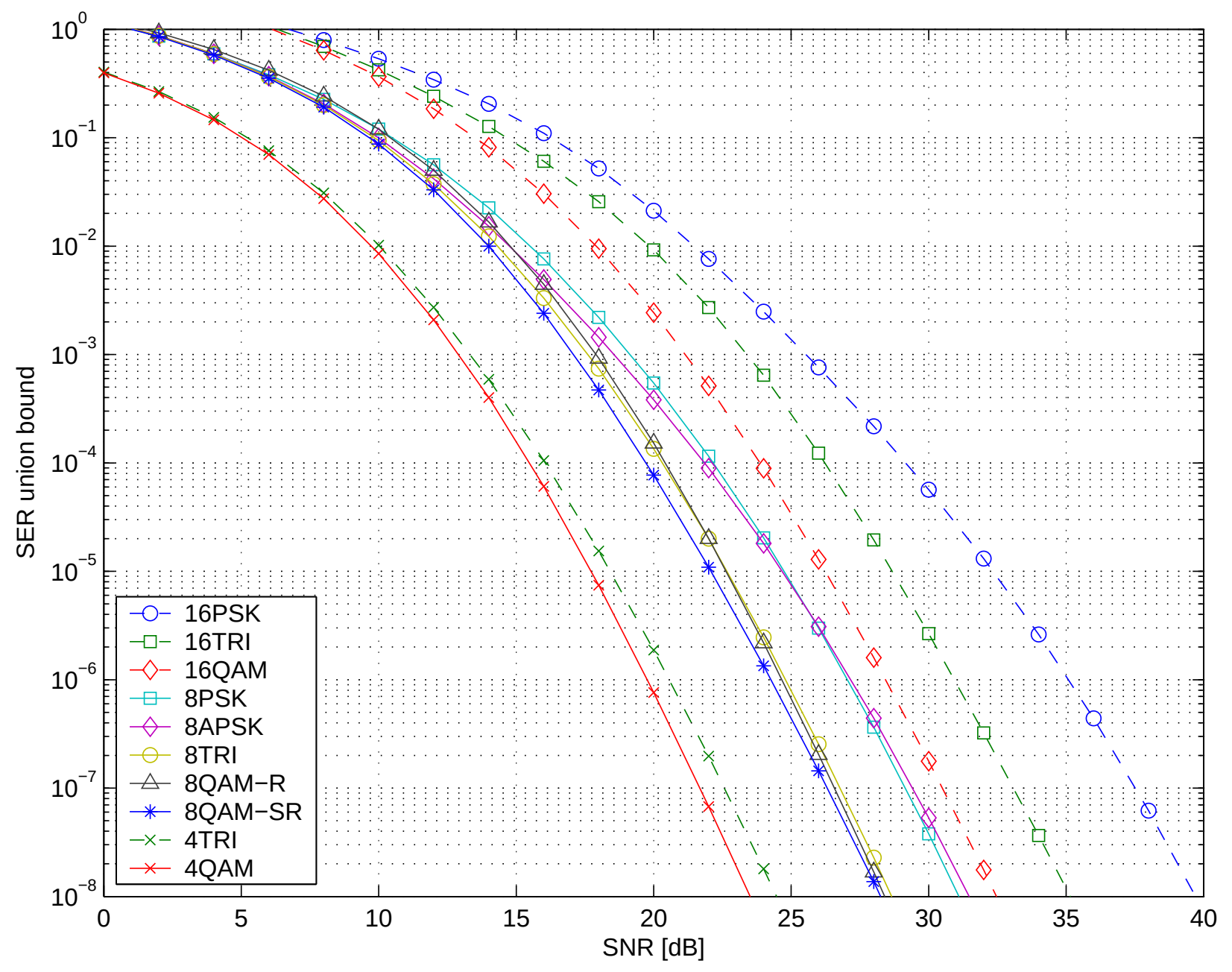

Figure 2: SER union bound of CIOD codes with rate-6/7 for 6 Tx/1 Rx antennas.

In Fig. 3, performance of CIOD codes with rectangular QAM using our newly proposed signal design is compared with the existing signal transformations in [3, 5]. Our new method yield 0.2 and 0.4 dB SNR gains compared with the results due to Khan-Rajan [3] and Wang-Wang-Xia [5], respectively.

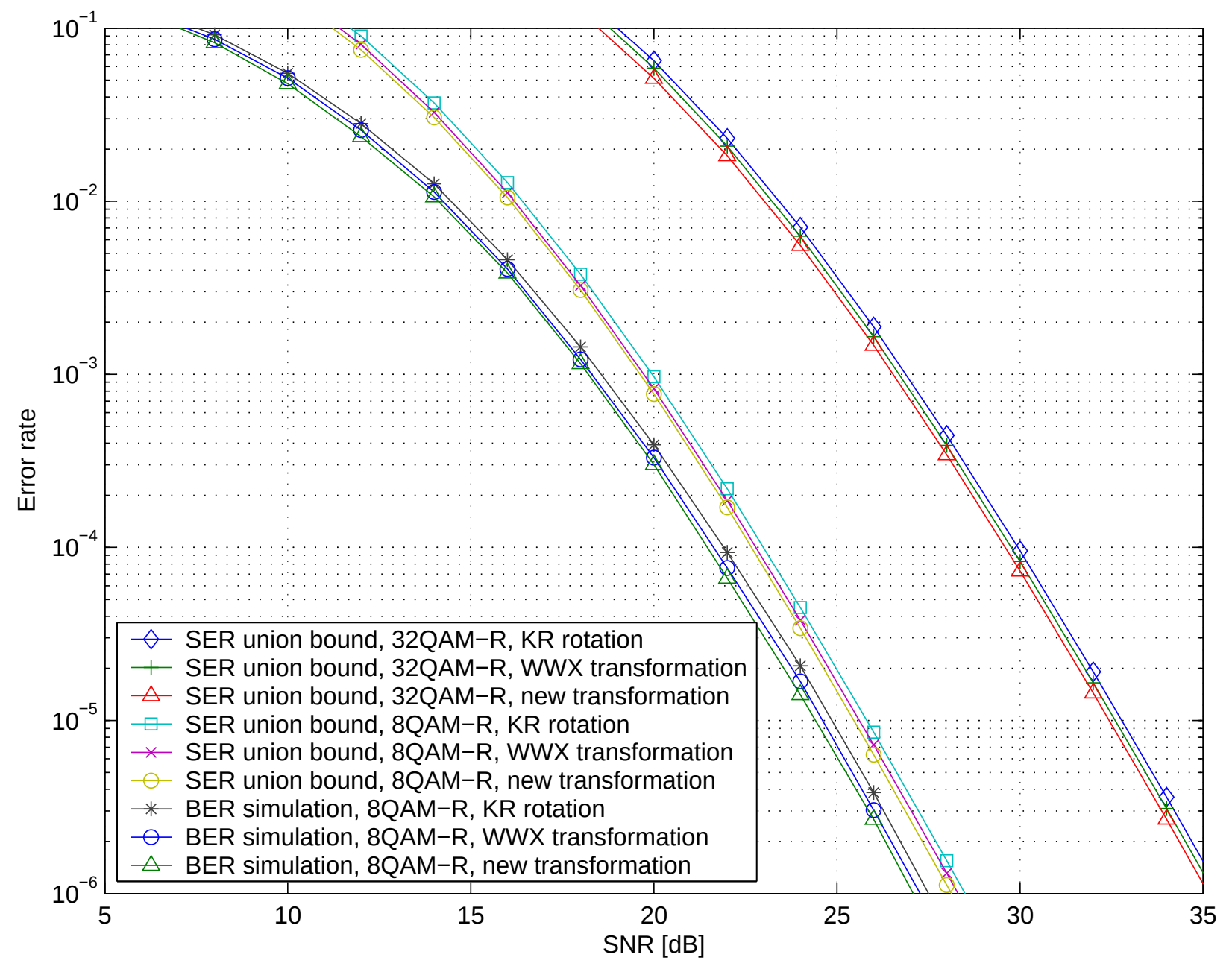

Figure 3: BER and Union bound on SER of the rate-one CIOD code with rectangular 8QAM and 32QAM for 4 Tx/1 Rx antennas.

Conclusion

- We have presented a new method to optimize the signal transformation for CIOD codes based on minimizing the tight union bound on SER. The method can be applied for any constellation with an arbitrary geometrical shape.
- A new signal transformation is proposed for signals with uneven powers of the real and imaginary parts, such as rectangular QAM.
- The results of this paper can be extended to solve other problems, e.g. antenna selection and beamforming.
- More details of our work can be found in [6].

References

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