Joint Source-Channel Coding for MIMO Block Fading Channels

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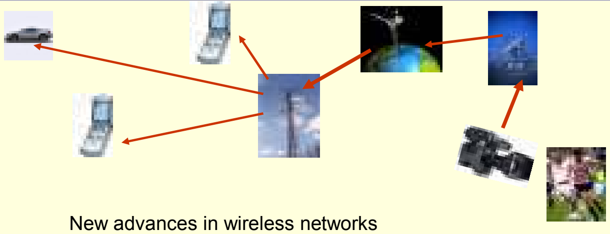
Motivation



New advances in wireless networks provide services with rich multimedia content (voice, video, digital TV, …)

- High data rate requirements,
- Error sensitive,
- Delay intolerant.

While wireless channels are

- Unreliable due to channel fluctuations,
- Bandwidth and power limited.

We need:

Meaningful end-to-end performance metric

+

Cross-Layer Optimization

System Model

SOURCE

- Memoryless, complex Gaussian source with unit variance (generalization later…)

CHANNEL

- M_t x M_r MIMO system,
- Slow i.i.d. Rayleigh L-block fading,
- Channel state known only to tx.

K source samples in N channel uses ⇨
Bandwidth expansion ratio: b = N / K

Due to delay limitations, both end-to-end mutual information and end-to-end distortion are random variables

⇨ our performance metric is expected end-to-end distortion (ED)

- Very hard problem for general SNR,
- We are interested in the high SNR behavior of expected distortion
  - Define distortion exponent:

$$\Delta = -\lim_{SNR\to\infty} \frac{\log(ED(R,SNR))}{\log(SNR)}$$

i.e., ED ~ SNR $^{-\Delta}$

Easy to do: Single Layer Transmission

Fix channel coding rate: R bits/channel use

⇨ Source coding rate = NR/K = bR bits/sample

Expected distortion (ED):

$$ED = (1 - P_{out}(R,SNR))\underbrace{D(bR)}_{\text{Distortion-rate function}} + \underbrace{P_{out}(R,SNR)}_{\text{Outage probability}}$$

For vanishing average distortion at high SNR, scale rates as: $R = r \log SNR$

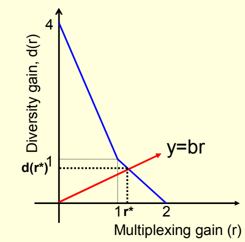
From diversity-multiplexing trade-off curve: $P_{out}(R,SNR) = SNR^{-d(r)}$

Distortion-rate function: $D(br\log SNR) = 2^{-br\log SNR} = SNR^{-br}$

High SNR approximation $ED \approx SNR^{-br} + SNR^{-d(r)}$

For highest decay rate, we need equal exponentials

$$\Delta = br^* = d(r^*)$$



Upper Bound

Assume transmitter has channel state information,

- at each state separation applies,
- transmit at instantaneous channel capacity,

$$R = C(\mathbf{H}) \le \frac{1}{L}\sum_{i=1}^{L} \log\det\left(\mathbf{I} + L\cdot \mathrm{SNR}\,\mathbf{H}_i\mathbf{H}_i^*\right)$$

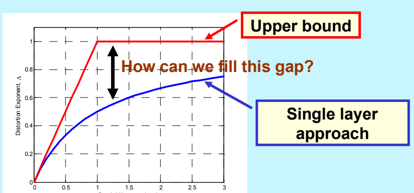
- corresponding distortion :

$$D(\mathbf{H}) = 2^{-bC(\mathbf{H})} \ge \prod_{i=1}^{L}\left[\det\left(\mathbf{I} + L\cdot SNR\,\mathbf{H}_i\mathbf{H}_i^*\right)\right]^{-b/L}$$

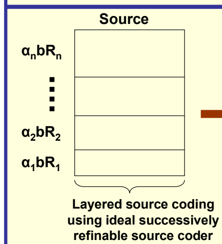
- average distortion over channel states:

$$ED = E_{\mathbf{H}}[D(\mathbf{H})] \Rightarrow \Delta^{UB} = L\sum_{i=1}^{\min(M_t,M_r)} \min\left(\frac{b}{L}, 2i-1+|M_t-M_r|\right)$$

Even for SISO



Layered Source with Progressive Transmission



Channel

Layers transmitted progressively over the channel with varying rates

N ch. uses			
$\alpha_1 N, R_1$	$\alpha_2 N, R_2$	……	$\alpha_n N, R_n$

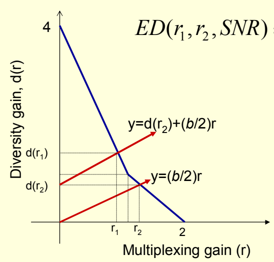
The fading state of the channel determines the number of layers that will be successfully decoded

for 2 layers

$$ED = (1-P_{out}^2)D(\alpha_1 bR_1 + \alpha_2 bR_2) + (P_{out}^2 - P_{out}^1)D(\alpha_1 bR_1) + P_{out}^1$$

$SNR \to \infty$

$$ED(r_1, r_2, SNR) \approx SNR^{-(\alpha br_1 + (1-\alpha)br_2)} + SNR^{-(d(r_2)+\alpha br_1)} + SNR^{-d(r_1)}$$



- **The more layers we have, the more we can climb up the trade-off curve, and the higher the distortion exponent is.**
- **Equal channel allocation is asymptotically optimal, i.e., α=1/n as n→∞.**
- **For MISO/SIMO systems**

$$\Delta^{LS} = \max(M_t, M_r)\left(1 - e^{-b/\max(M_t,M_r)}\right)$$

Hybrid Digital-Analog Strategy (HLS)

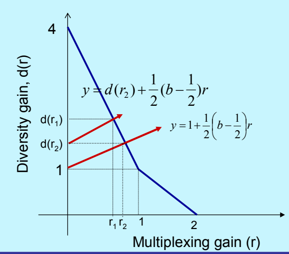
- Analog is optimal for AWGN channels (for b=1)
- Provides graceful performance degradation with channel quality
- Analog still optimal in SISO for b≥1 in terms of distortion exponent, but it cannot exploit higher diversity *(Gunduz-Erkip, SPAWC 2005)*
- Combine analog + digital with multiple layers: Hybrid LS (HLS)

$b \ge \min(M_t, M_r)$ Case:

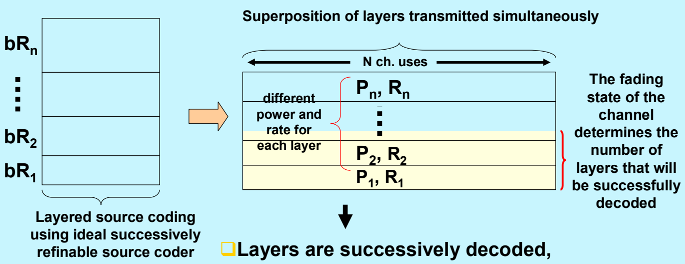
- Similar to LS, send the error signal without coding in the end.
- We start climbing trade-off curve from 1.

$b < 1/\min(M_t, M_r)$ Case:

- Not enough channel uses to transmit one signal for all samples
- Superposition of analog and digital signals *(Caire, Narayanan, 2005)*
  - Achieves the upper bound for $b<1/\min(M_t, M_r)$



Broadcast Strategy with Layered Source



- Layers are successively decoded, considering the rest as noise.
- For vanishing distortion, we need power allocation that varies with average SNR.
- Successive decoding not necessarily optimal.

In the limit of infinite layers:

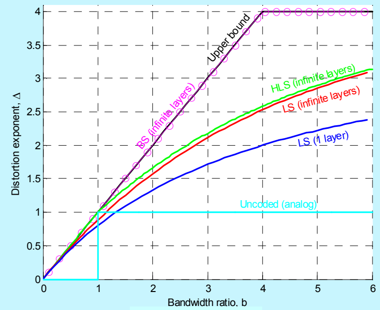
$$\Delta = \begin{cases} M_t M_r & if \quad b \geq M_t M_r \\ b & if \quad b < M_t M_r \end{cases}$$

Meets the upper bound for MISO/SIMO systems at any bandwidth ratio!

- ***Improvement in BS over LS comes at the expense of a more complex encoder-decoder pair, as BS requires SNR-dependent power allocation among layers, superposition of codewords and sequential decoding.***

Distortion Exponent vs. Bandwidth Expansion

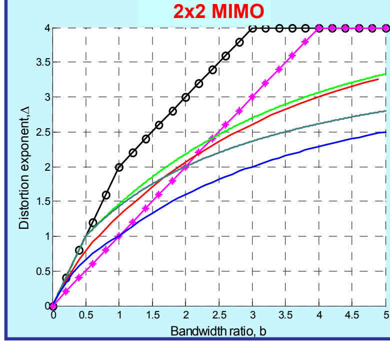
4x1 MIMO



BS is optimal

HLS optimal for b≤1

2x2 MIMO



BS optimal only for b≥4$_r$

HLS optimal for b≤1/2

Conclusions

- Results can be easily generalized to multiple blocks (i.e., L>1) scenario,
- L-block fading scenario with bandwidth ratio b is equivalent to L parallel fading channels with bandwidth ratio b/L. (*Gunduz-Erkip, ISIT 2006*)
- Gaussian distortion-rate function is a tight upper bound in the high SNR/ high rate regime, and "All sources are nearly successively refinable" (Lastras, Berger 2001) ⇒ Our high SNR results valid for almost all sources.
- Results apply to high rate quantization as well,
- Optimizing end-to-end average distortion over fading channels is an open problem for general SNR's and bandwidth ratios, but we have obtained optimality results for certain cases in the high SNR regime in terms of distortion exponent.
- Already applied to relay channels (Gunduz-Erkip, SPAWC2005), ongoing work for extending to multiple access channel with correlated sources.

http://wireless.poly.edu http://pages.poly.edu/~dgundu01/ http://eeweb.poly.edu/~elza/