Capacity of Broadcast Channels with Relative Fading Knowledge

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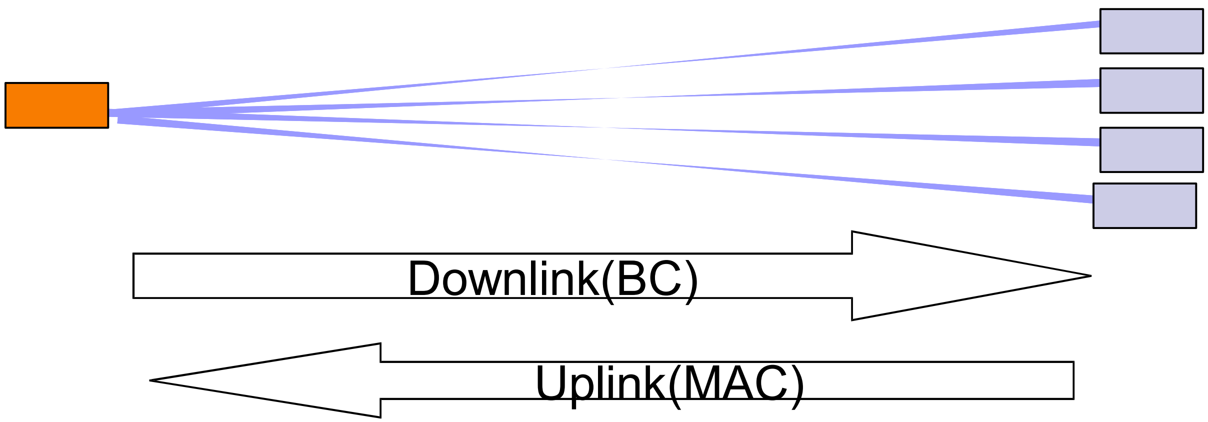
Motivation(1)

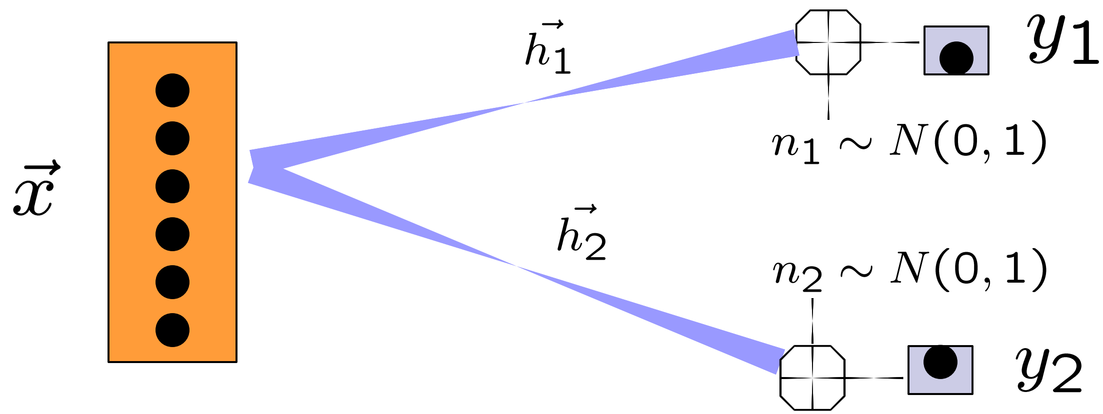


- Getting an exact estimate of channel state is very hard
  - Feedback : expensive, hard to implement & noisy
  - TDD : correlation between channels is not perfect.

Motivation (2)

- Easier to get relative channel information
  - Relative channel information defined as which channel is stronger
- For a downlink, relative information obtained from uplink
  - Relative information is related to distance (path loss)
  - Error probability in determining relative information is much less than that in exact channel estimate

System Model



$$y_1 = \vec{h_1} \cdot \vec{x} + n_1$$

$$y_2 = \vec{h_2} \cdot \vec{x} + n_2$$

Assumptions

- MISO : multiple antennas at Tx, single antenna at Rxs, two receivers
- Assumption
  - $\|\vec{h_i}\|$'s are known to receiver i where $\|\cdot\|$ is Frobenius Norm. (though a general norm works)
  - $S = \mathrm{I}\{\|\vec{h_1}\| \geq \|\vec{h_2}\|\}$ is known to Tx.

MIMO BC Capacity

- Perfect channel state knowledge at Tx
  - Non-degraded channel
- No channel knowledge at Tx
  - Non-degraded in general
  - Degraded for special cases
    - $p(\vec{h_1}) = p(\vec{h_2})$ where p is p.d.f
    - $\vec{h_1} \geq \vec{h_2}$ ($\geq$ in the semi-definite sense) for all $(\vec{h_1}, \vec{h_2})$ pairs

MIMO BC with Relative Info(1)

- Relative information S known to Tx
  - Degraded if $\vec{h_1}$ and $\vec{h_2}$ are such that

$$p\left(\frac{\vec{h_1}}{\|\vec{h_1}\|}\right) = p\left(\frac{\vec{h_2}}{\|\vec{h_2}\|}\right)$$

    where p is p.d.f
  - Scalar channel with relative info is *always* degraded

    since $p\left(\frac{h_1}{h_1}\right) = p\left(\frac{h_2}{h_2}\right) = p(1)$.

MIMO BC with Relative Info (2)

- Vector channel case
  - $$y_1' = \frac{\vec{h}_1}{\|\vec{h}_1\|} \cdot \vec{x} + \frac{n_1}{\|\vec{h}_1\|} \quad , \quad y_2' = \frac{\vec{h}_2}{\|\vec{h}_2\|} \cdot \vec{x} + \frac{n_2}{\|\vec{h}_2\|}$$
  - If S=1, then in the capacity sense this channel has same capacity region as

    $$y_1' = \vec{u} \cdot \vec{x} + \frac{n_1}{\|\vec{h}_1\|}, \qquad y_2'' = \vec{u} \cdot \vec{x} + \frac{n_1}{\|\vec{h}_1\|} + n_3$$

    where $n_3 \sim N\left(0, \mathrm{Var}\left(\frac{n_1}{\|\vec{h}_1\|}\right) - \mathrm{Var}\left(\frac{n_2}{\|\vec{h}_2\|}\right)\right)$

Achievability and Converse

- Achievability: One code book per state $S = \{0,1\}$. Super position coding in each state
- Converse: Gallager's approach when $S=1^n$

$$
\begin{aligned}
nR_1 &= H(W_1|S^n = 1^n) \le I(W_1; Y_1^n, H^n|S^n = 1^n) \\
&\le I(W_1; Y_1^n, W_2|S^n = 1^n, H^n) \\
&= I(W_1; Y_1^n|S^n = 1^n, H^n, W_2) \\
&= \sum_{i=1}^{n} I(W_1; Y_{1i}|H^n, S^n = 1^n, W_2, Y_1^{i-1}) \\
&= \sum_{i=1}^{n} I(W_1; Y_{1i}|H^n, U_i) \\
&\le \sum_{i=1}^{n} (h(Y_{1i}|H_i, U_i) - h(Y_{1i}|H_i, U_i, W_1, X_i)) \\
&= \sum_{i=1}^{n} I(Y_{1i}; X_i|H_i, U_i)
\end{aligned}
$$

$H \overset{\text{def}}{=} [\vec{h_1} \vec{h_2}]$

$y_{1i} = \vec{h_1} \cdot x + n_1$

$\Rightarrow Y_{1i} \overset{X_i, H_i}{\perp} H^{i-1}, H_{i+1}^n, U_i$

Converse Continued

$$
\begin{aligned}
nR_2 &= H(W_2|S^n = 1^n) \leq I(W_2; Y_2^n, H^n|S^n = 1^n) \\
&= I(W_2; Y_2^n|S^n = 1^n, H^n) \\
&= \sum_{i=1}^{n} I(W_2; Y_{2i}|S^n = 1^n, H^n, Y_2^{i-1}) \\
&= \sum_{i=1}^{n} \Big(h(Y_{2i}|S^n = 1^n, H^n, Y_2^{i-1}) - h(Y_{2i}|S^n = 1^n, H^n, Y_2^{i-1}, W_2)\Big) \\
&\leq \sum_{i=1}^{n} \Big(h(Y_{2i}|H^n) - h(Y_{2i}|H^n, S^n = 1^n, Y_2^{i-1}, W_2, Y_1^{i-1})\Big) \\
&= \sum_{i=1}^{n} \Big(h(Y_{2i}|H^n) - h(Y_{2i}|H^n, S^n = 1^n, W_2, Y_1^{i-1})\Big) \\
&\leq \sum_{i=1}^{n} (h(Y_{2i}|H_i) - h(Y_{2i}|H_i, U_i)) \\
&= \sum_{i=1}^{n} I(Y_{2i}; U_i|H_i)
\end{aligned}
$$

$Y_{2i} \perp H^{i-1}, H_{i+1}^n$

The Capacity Region

- Results (we can apply similar method when S=0)

S=1	S=0
$R_1 \leq I(Y_1;X\|H,U) = I_{1,1}(H)$ $R_2 \leq I(Y_2;U\|H) = I_{2,1}(H)$	$R_1 \leq I(Y_1;V\|H) = I_{1,0}(H)$ $R_2 \leq I(Y_2;X\|V,H) = I_{2,0}(H)$

- Ergodic capacity region

$$\begin{aligned} R_1 &\leq E_H[I_1] = E_S\left[E_H[I_1|S]\right] \\ &= \alpha E_{H|S=1}[I_{1,1}(H)] + \bar{\alpha} E_{H|S=0}[I_{1,0}(H)] \\ R_2 &\leq E_H[I_2] = E_S\left[E_H[I_2|S]\right] \\ &= \alpha E_{H|S=1}[I_{2,1}(H)] + \bar{\alpha} E_{H|S=0}[I_{2,0}(H)] \end{aligned}$$

where $p(S=1)=\alpha, p(S=0)=1-\alpha=\bar{\alpha}$

Gaussian inputs

- Conventional EPI techniques do not work
- Rate region for Gaussian inputs

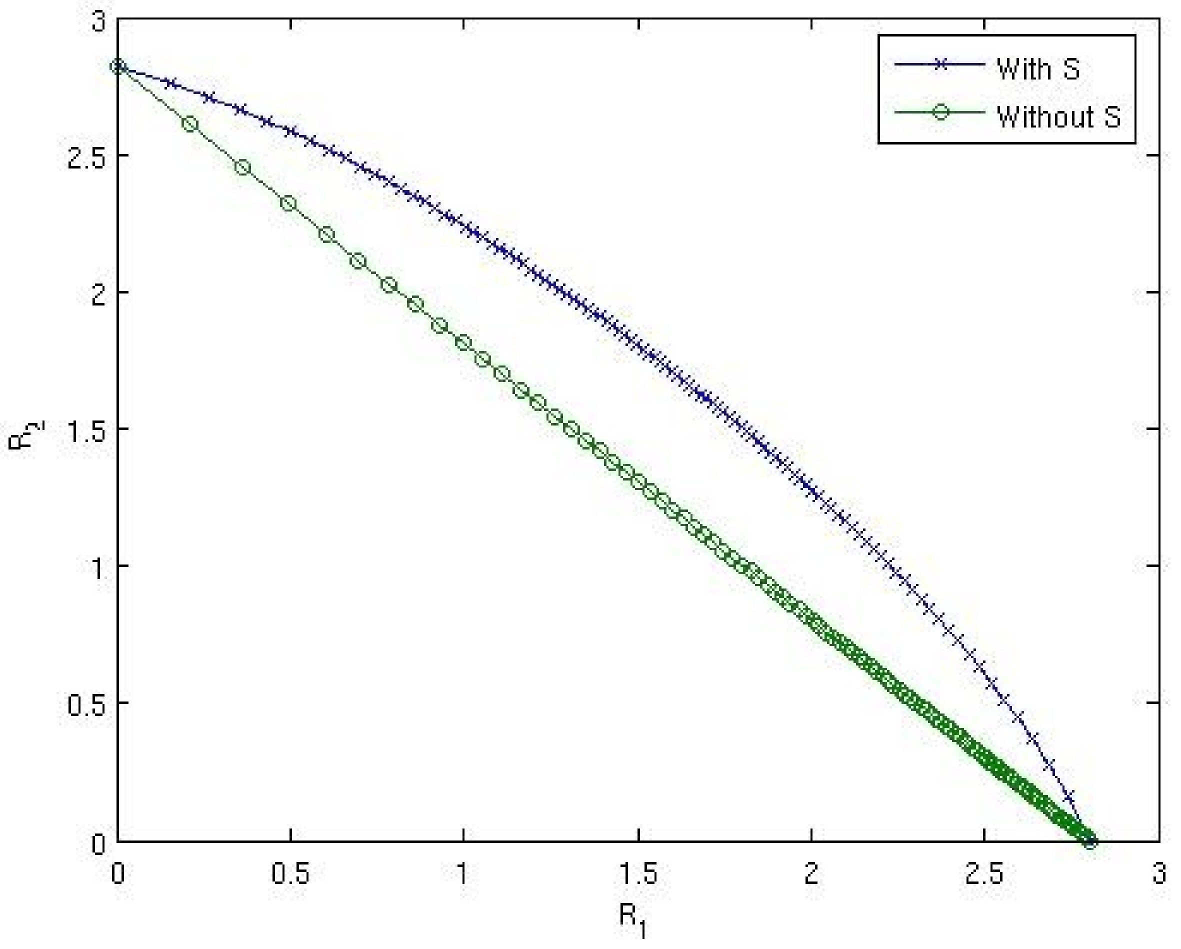
$$R_1 \leq \alpha E_{H|1}\left[\log(1+\vec{h_1}\Sigma_1(1)\vec{h_1}^\dagger)\right]+\bar{\alpha}E_{H|0}\left[\log\left(1+\frac{\vec{h_1}\Sigma_1(0)\vec{h_1}^\dagger}{1+\vec{h_1}\Sigma_2(0)\vec{h_1}^\dagger}\right)\right]$$

$$R_2 \leq \alpha E_{H|1}\left[\log\left(1+\frac{\vec{h_2}\Sigma_2(0)\vec{h_2}^\dagger}{1+\vec{h_2}\Sigma_1(0)\vec{h_2}^\dagger}\right)\right]+\bar{\alpha}E_{H|0}\left[\log(1+\vec{h_2}\Sigma_2(0)\vec{h_2}^\dagger)\right]$$

such that $\alpha \mathrm{Tr}(\Sigma_1(1)+\Sigma_2(1))+\bar{\alpha}\mathrm{Tr}(\Sigma_1(0)+\Sigma_2(0)) \leq P$

where $H \overset{\text{def}}{=} [\vec{h_1}\vec{h_2}]$

Rate Region



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Conclusion and future work

- 1 bit of information enough to
  - Achieve rates as if channel norm was known but power allocation fixed at transmitter
  - Allow for single letter capacity characterization
- Future Work: Are Gaussian inputs optimal?