BER Performance Evaluation of a Continuous Transmission Cooperative Relaying Protocol

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Cooperative Relaying

Pros

- Usage of energy savings due to nonlinear pathloss
- Exploitation of large scale spatial diversity
- Simple (adaptive) protocols available with low complexity
- Simple roll-out, coverage extension and avoidance of shadowed areas

Cons

- Increase of necessary spectral efficiency due to **orthogonality constraint**
- Poor performance in low SNR/high rate regime

System and Protocol Description

System model

- All channels are modeled as Rayleigh fading channels ($h_{i,j}[n]$ is $\mathcal{CN}(0, \sigma_{i,j}^2)$)
- The effective E_b/N_0 is $\gamma[n] = |h_{i,j}|^2 \overline{\gamma}$ ($\overline{\gamma}$ is the average E_b/N_0 of direct transmission)
- Evaluation considers M-QAM with an approximated BER of $aQ\left(\sqrt{b\overline{\gamma}}\right)$.
- Interference cancellation is modeled by $0 < \eta \leq 1$ ($\eta \cdot E_b/N_0$ cannot be canceled out)
- Only a fraction $0 < \kappa_s \leq 1$ of the overall energy is assigned to the source (comparability)

Protocol description

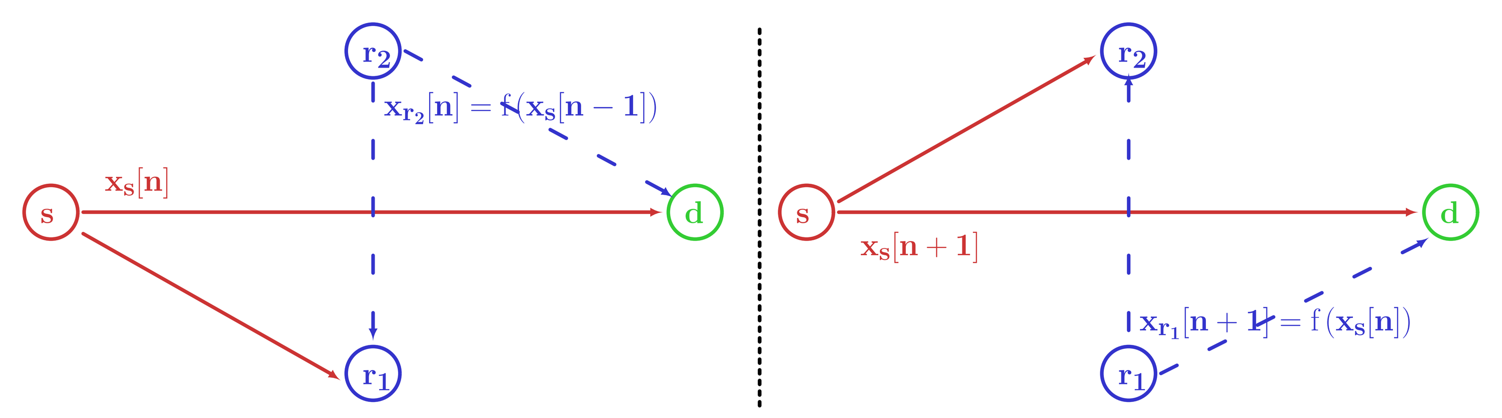

FIGURE 1: Example situation for YARP (assuming r_2 and r_1 successfully decoded).

- YARP – **Y**ARP is an **A**dvanced **R**elaying **P**rotocol
- At even time instances k:
  - Source broadcasts $x_s[k]$ to destination and currently receiving relay $\mathbf{r_1}$
  - Relay r_2 broadcasts $x_r[k] = f(x_s[k-1])$ if $|h_{s,r}[k-1]|^2 > \epsilon$
  - $x_r[k]$ is considered as interference at currently receiving r_1
- At odd time instances $k+1$:
  - Source broadcasts $x_s[k+1]$ to destination and currently receiving relay $\mathbf{r_2}$
  - Relay $\mathbf{r_1}$ now broadcasts $x_r[k+1] = f(x_s[k])$ depending on $|h_{s,r}[k]|^2$
  - Using $x_r[k+1]$ and $x_s[k]$ the destination now decodes $x_s[k]$
  - The information about $x_s[k]$ is used to cancel $x_r[k+1]$ out of $y_d[k+1]$

BER Analysis

The basic idea of the derivation is to devide the set of all constellation points S into the subsets S_i whereas each element $x \in S_i$ has equal energy $\Gamma(S_i)$ and $\Gamma(S) = \overline{\gamma} \log_2 M$. The BER for a link which is interfered by another one is averaged over all sets (of the interfering link) whereas the interference of one set can be seen as additional AWGN.

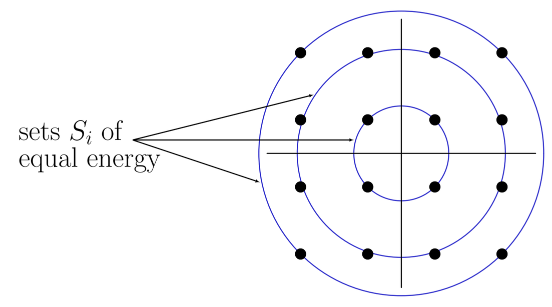

FIGURE 2: The three different energy radi of a 16-QAM.

Let R_k denote the event that the currently receiving relay decodes message $x_s[k]$ and D_{k-1} the event $\hat{x}_s[k-1] \neq x_r[k]$ where $\hat{x}[n]$ is the destination's estimation of $x[n]$. Using the definition of both events and the AWGN approximation of the interference we can divide the BER in four cases:

1. $\overline{\mathbf{R}}_\mathbf{k} \wedge \overline{\mathbf{D}}_\mathbf{k-1}$: Since $\hat{x}_s[k-1] = x_r[k]$, $x_r[k]$ can be perfectly canceled out using the (perfect) knowledge of $h_{r,d}[k]$. The BER for this case is given by $p_e^1 = p_{e,1}\left(\kappa_s \sigma_{s,d}^2, 0\right)$ with

$$p_{e,1}(\sigma^2, \sigma_0^2) = a \cdot Q\left(\sqrt{b\sigma_0^2 \overline{\gamma}}\right) - a \cdot \exp\left(\frac{\sigma_0^2}{\sigma^2}\right) \sqrt{\frac{1}{\frac{2}{b\overline{\gamma}\sigma^2} + 1}}\, Q\left(\sqrt{2\left(\frac{1}{\sigma^2} + \frac{b}{2}\overline{\gamma}\right)\sigma_0^2}\right),$$

where $\sigma_0^2 = 0$ denotes the decoding threshold (used in case 3) for the definition of $p_{e,R}$).

2. $\overline{\mathbf{R}}_\mathbf{k} \wedge \mathbf{D}_\mathbf{k-1}$: as 1) but $x_r[k]$ cannot be canceled out: $p_e^2 = \sum_i \frac{|S_i|}{M} p_{e,1}\left(\frac{\kappa_s \sigma_{s,d}^2 \overline{\gamma}}{1 + \eta \kappa_r \sigma_{r,d}^2 \Gamma(S_i)}, 0\right)$.

3. $\mathbf{R}_\mathbf{k} \wedge \overline{\mathbf{D}}_\mathbf{k-1}$: In this case we receive the relayed version of $x_s[k]$ at time instance $n = k+1$. Since we have no knowledge of $x_s[k+1]$ it must be considered as interference for $x_r[k+1]$:

$$p_e^3 = (1 - p_{e,R})\, p_{e,2,i} + p_{e,R} p_{ex,2,i},$$
$$p_{e,R} = \sum_i \frac{|S_i|}{M} p_{e,1}\left(\frac{\kappa_s \sigma_{s,r}^2 \overline{\gamma}}{1 + \eta \kappa_r \sigma_{r,r}^2 \Gamma(S_i)}, \epsilon\right), \ \kappa_r = 1 - \kappa_s.$$

$p_{e,2,i} = p'_{e,2,i}(-1)$, $p_{ex,2,i} = p'_{e,2,i}(1)$ denote the BER for two-path diversity with one interfered path and $p_{e,R}$ defines the error at the decoding relay. $p_{ex,2,i}$ considers the MRC of two contradicting signals (in the case of a decoding error at the relay). Both probabilities are defined using

$$p'_{e,2,i}(\nu) \approx \sum_i \frac{|S_i|}{M} p_{e,2}\left(\kappa_s \sigma_{s,d}^2 \overline{\gamma}, \frac{\kappa_r \sigma_{r,d}^2 \overline{\gamma}}{1 + \eta \kappa_s \sigma_{s,d}^2 \Gamma(S_i)}, \nu\right),$$
$$p_{e,2}(\overline{\gamma}_1, \overline{\gamma}_2, \nu) = \frac{a}{2}\left[1 + \nu \frac{\sqrt{2b}}{2(\overline{\gamma}_2 + \nu\overline{\gamma}_1)}\left(\overline{\gamma}_2 \sqrt{\frac{\overline{\gamma}_2}{\frac{b}{2}\overline{\gamma}_2 + 1}} - \overline{\gamma}_1 \sqrt{\frac{\overline{\gamma}_1}{\frac{b}{2}\overline{\gamma}_1 + 1}}\right)\right].$$

4. $\mathbf{R}_\mathbf{k} \wedge \mathbf{D}_\mathbf{k-1}$: As 3) but $x_r[k]$ is considered as interference: $p_e^4 = (1 - p_{e,R})\, p_{e,2,i_2} + p_{e,R} p_{ex,2,i_2}$ where $p_{e,2,i_2} = p'_{e,2,i_2}(-1)$ and $p_{ex,2,i_2} = p'_{e,2,i_2}(1)$ denote the BER for two-path diversity with two interfered paths but $p_{ex,2,i_2}$ defines the BER for a MRC of two contradicting signals. Both probabilities utilize

$$p_{e,2,i_2}(\nu) \approx \sum_i \frac{|S_i|}{M} p_{e,2}\left(\kappa_s \sigma_{s,d}^2 \overline{\gamma}', \frac{\kappa_r \sigma_{r,d}^2 \overline{\gamma}'}{(1 + \eta\psi\Gamma(S_i))^2}, \nu\right),$$
$$\overline{\gamma}' = \frac{\overline{\gamma}}{1 + \eta\psi\Gamma(S_i)} \text{ and } \psi = \max\left(\kappa_s \sigma_{s,d}^2, \kappa_r \sigma_{r,d}^2\right).$$

Using $\Pr(D_k) = (1 - p_e)\, p_{e,R} + p_e\,(1 - p_{e,R})$, the decoding probability $p_R = \exp\left(-\epsilon/(\kappa_s \sigma_{s,r}^2)\right)$ and $p_{e,\overline{R}} = 1 - p_{e,R}$ the overall BER can easily shown to be

$$p_e = \frac{p_{e,\overline{R}}\left(p_e^1 + p_R\left(p_e^3 - p_e^1\right)\right) + p_{e,R}\left(p_e^2 + p_R\left(p_e^4 - p_e^2\right)\right)}{1 - \left[\left(2p_{e,R} - 1\right)\left[(1 - p_R)\left(p_e^1 - p_e^2\right) + p_R\left(p_e^3 - p_e^4\right)\right]\right]}.$$

Results, Conclusions and Further Work

Conclusions

- \+ Reduced BER at low SNR
- \+ RN-interference can be canceled using last decoded message
- \+ No 'fancy' signaling/initialization necessary → adaptive behavior
- – Interference limited (noise floor)
- – Increased BER at high SNR
- – Additional relay necessary

Further Work

- Application on (MC-)CDMA based system
- Application on different coding schemes (LDPC, CC, ...) and usage of their FEC ability instead of a SNR threshold
- Investigation of YARP with (Hybrid-)ARQ
- System Level analysis, e.g. regarding increased interference, existence of suitable relays, ...
- Routing/Scheduling

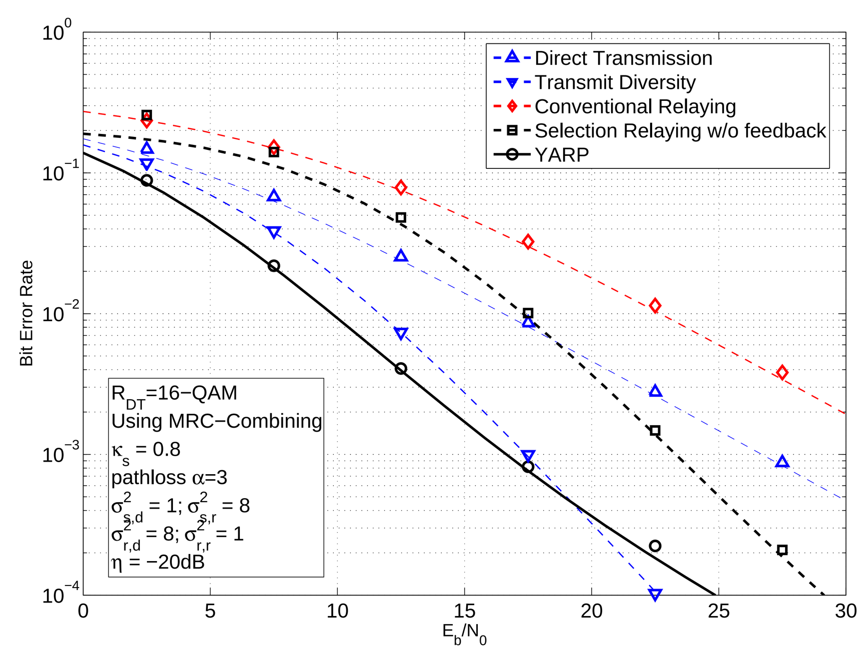

FIGURE 3: End-to-end BER. Lines denote analytical results and symbols denote simulation results for $\epsilon = \frac{M-1}{\Gamma(S)}\left(1 + \kappa_r \sigma_{r,r}^2 \eta \Gamma(S)\right)$.

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