On the Finite-SNR Diversity-Multiplexing Tradeoff of Half Duplex Protocols in Fading Relay Channels

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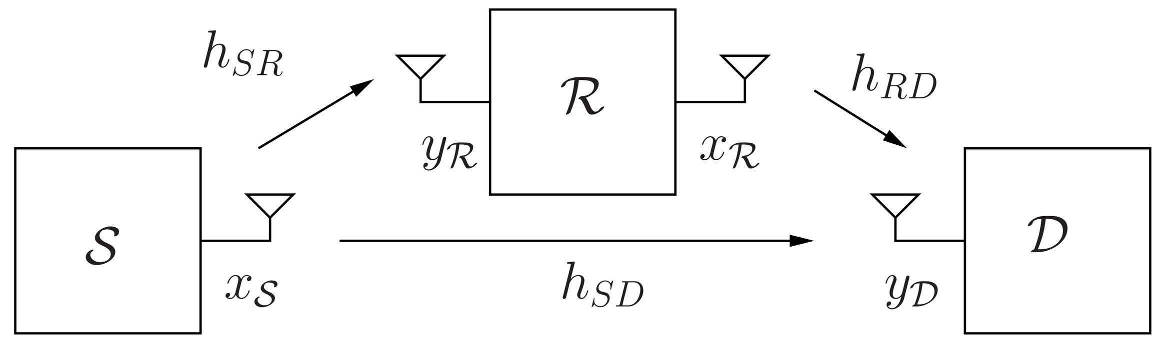
Abstract

We analyze the diversity-multiplexing tradeoff in a fading relay channel at finite signal-to-noise ratios (SNRs) when the data rate increases with SNR. In this framework, the rate adaptation policy is such that the target system data rate is a multiple of the capacity of an additive white Gaussian noise (AWGN) channel. The proportionality constant determines how aggressively the system scales the data rate and can be interpreted as a finite-SNR multiplexing gain. The diversity gain is given by the negative slope of the outage probability versus SNR curve. The finite-SNR diversity-multiplexing tradeoff is characterized for three practical half-duplex cooperative protocols. We derive closed-form expressions and estimates on the achievable diversity and multiplexing gains as a function of SNR under a system-wide power constraint on the source and relay transmissions and quantify performance improvement with relay cooperation over direct transmissions in terms of the diversity-multiplexing tradeoff. Finally, we verify our analytical results by numerical simulations.

Channel and Signal Models



-A fading relay channel. $\mathcal{S}$ is the source node, $\mathcal{R}$ is the relay node, and $\mathcal{D}$ is the destination node.

Signal Statistics (assuming two system time slots):

$$\mathcal{E}\{x_{\mathcal{S},i}\} = 0 \qquad \mathcal{E}\left\{\sum_{i=1}^{2} |x_{\mathcal{R},i}|^2\right\} = \alpha P$$
$$\mathcal{E}\{x_{\mathcal{R},i}\} = 0 \qquad \mathcal{E}\left\{\sum_{i=1}^{2} |x_{\mathcal{S},i}|^2\right\} = \beta P$$
$$\alpha + \beta = 1 \text{ and } \alpha, \beta \geq 0$$

- α and β describe the power split between the relay and the source terminals.

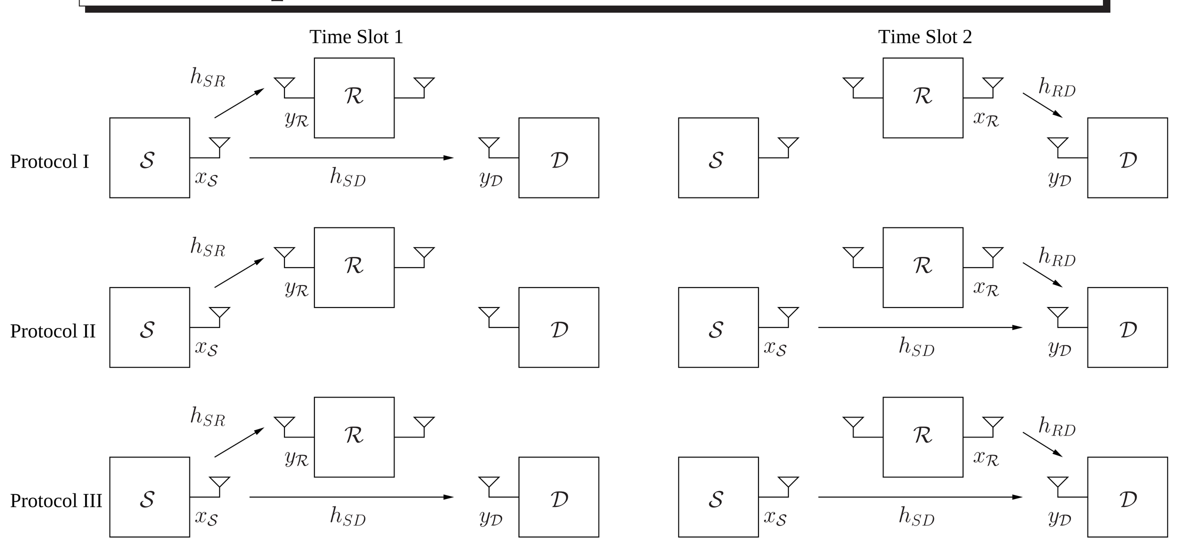
Channel input-output relations:

$$y_{\mathcal{R},i} = h_{SR}x_{\mathcal{S},i} + n_{\mathcal{R},i}$$
$$y_{\mathcal{D},i} = h_{SD}x_{\mathcal{S},i} + h_{RD}x_{\mathcal{R},i} + n_{\mathcal{D},i}$$

- $n_{\mathcal{R},i} \sim \mathcal{CN}(0, \sigma_n^2)$ and $n_{\mathcal{D}} \sim \mathcal{CN}(0, \sigma_n^2)$.

Define the network SNR as: $\rho = \frac{P}{\sigma_n^2}$

Half Duplex Decode and Forward Protocols



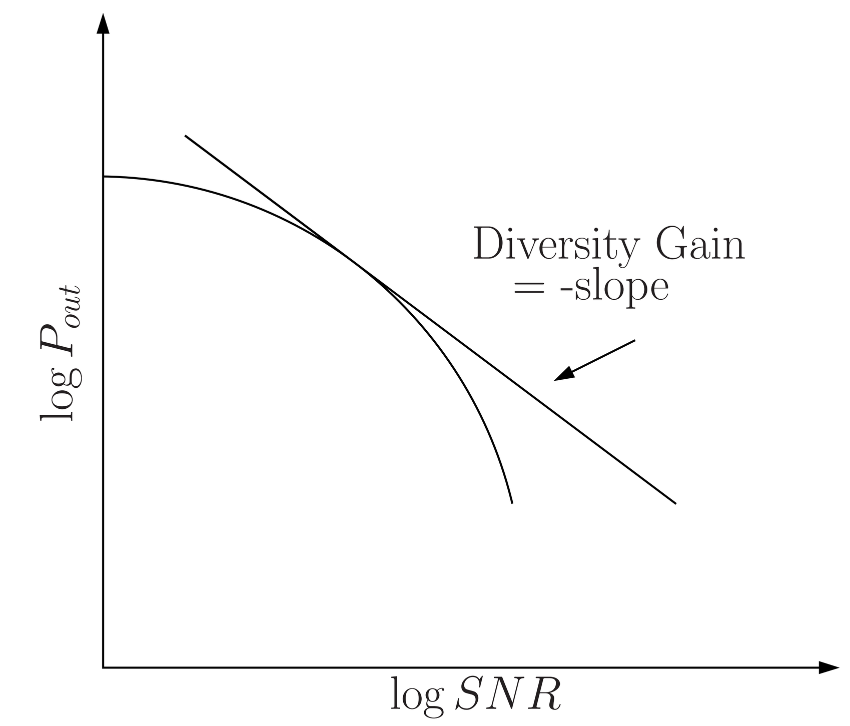
Three half duplex decoded and forward protocols described in [1] are considered:

Finite-SNR Diversity-Multiplexing Tradeoff

Finite SNR multiplexing gain [2–4]:

$$r = \frac{R}{\log_2(1+\rho)}$$

Finite SNR diversity is defined as the negative slope of outage probability:



-Illustration of diversity gain at finite SNR.

$$d(r,\rho) = -\frac{\rho}{P_{out}(r,\rho)} \frac{\partial P_{out}(r,\rho)}{\partial \rho}$$

Outage Probability

Consider the probability of outage for channels with mutual information of the following form:

$$I = \log_2(1 + \gamma_1^2 \rho X_1 + \gamma_2^2 \rho X_2),$$

Where X_1 and X_2 are exponential random variables with mean one.

$$P_{out} = P(I < R)$$
$$= 1 - \frac{1}{\gamma_2\rho - \gamma_1\rho}\left[\gamma_2\rho \exp\left(\frac{-((1+\rho)^r - 1)}{\gamma_2\rho}\right) - \gamma_1\rho \exp\left(\frac{-((1+\rho)^r - 1)}{\gamma_1\rho}\right)\right]$$

Where $2^R = (1+\rho)^r$.

Protocol Outage Probability

- E_1: occurs if there is an outage between the source and the relay.
- E_2: occurs if there is an outage between the source / relay and the destination.
- E_3: occurs if there is an outage between the source and the destination without the help of the relay.

Protocol outage probability: $P_{out} = P\left((E_1 \cup E_2) \cap E_3\right)$.

Protocol Power Allocation

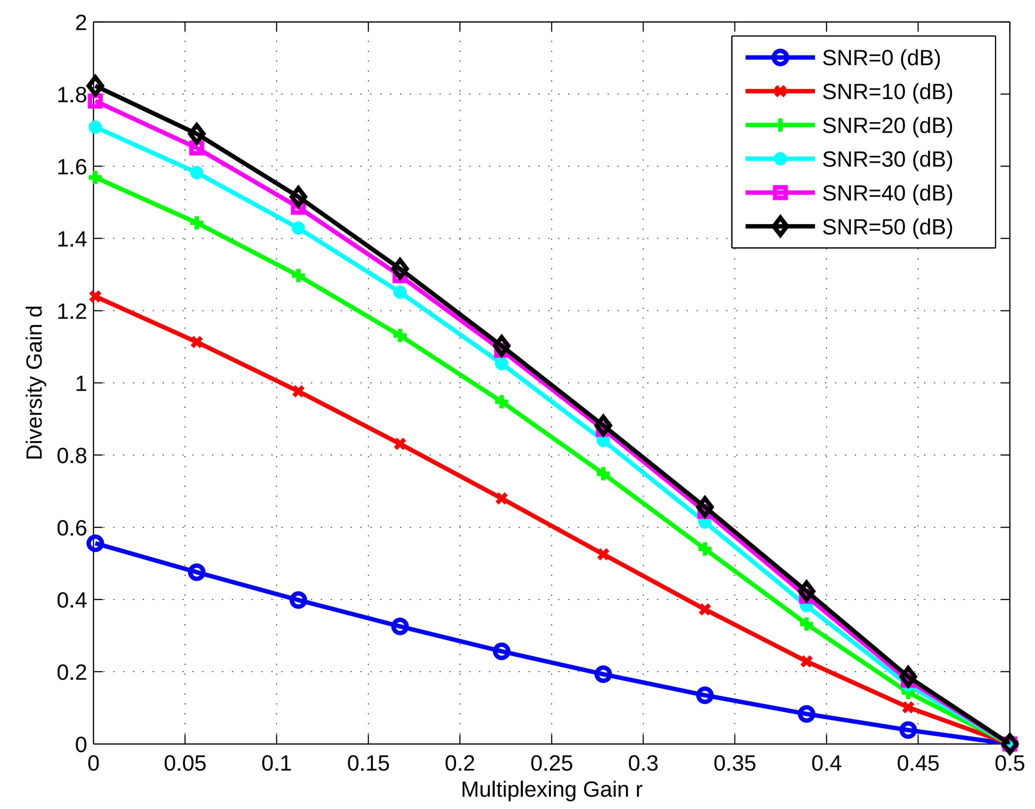
Protocol I

$$\mathcal{E}\left\{|x_{\mathcal{S},1}|^2\right\} = 2\beta P \qquad \mathcal{E}\left\{|x_{\mathcal{R},1}|^2\right\} = 0$$
$$\mathcal{E}\left\{|x_{\mathcal{S},2}|^2\right\} = 0 \qquad \mathcal{E}\left\{|x_{\mathcal{R},2}|^2\right\} = 2\alpha P$$

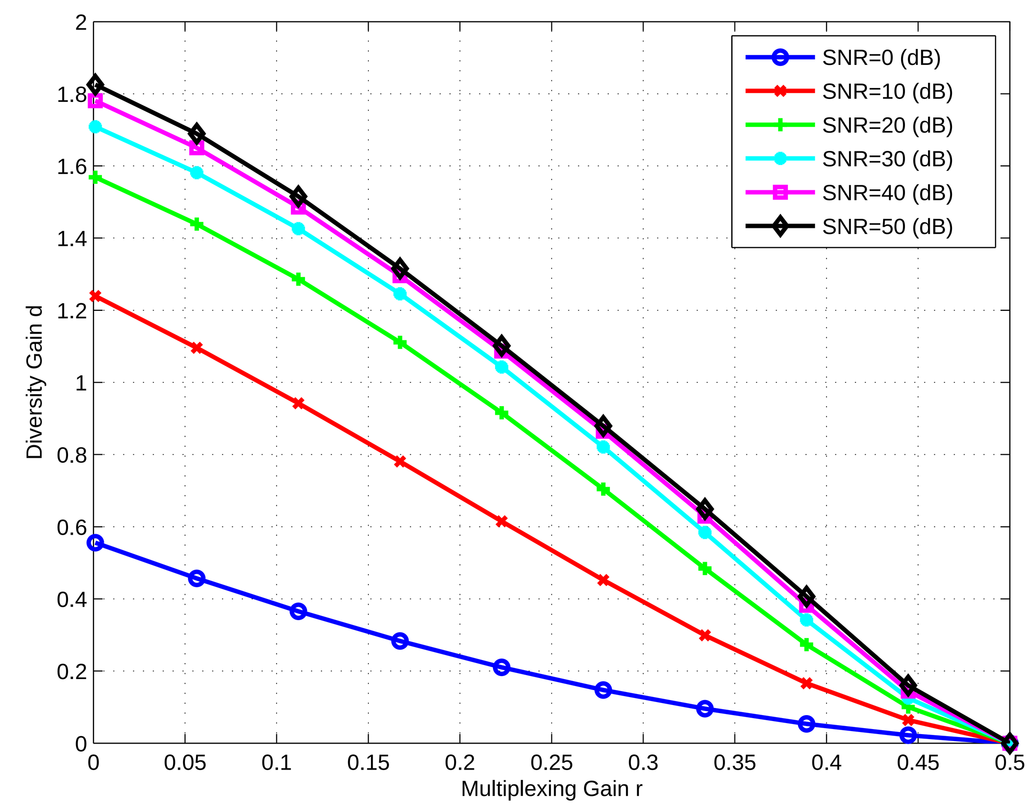
Protocol II and III

$$\mathcal{E}\left\{|x_{\mathcal{S},1}|^2\right\} = \beta P \qquad \mathcal{E}\left\{|x_{\mathcal{R},1}|^2\right\} = 0$$
$$\mathcal{E}\left\{|x_{\mathcal{S},2}|^2\right\} = \beta P \qquad \mathcal{E}\left\{|x_{\mathcal{R},2}|^2\right\} = 2\alpha P$$

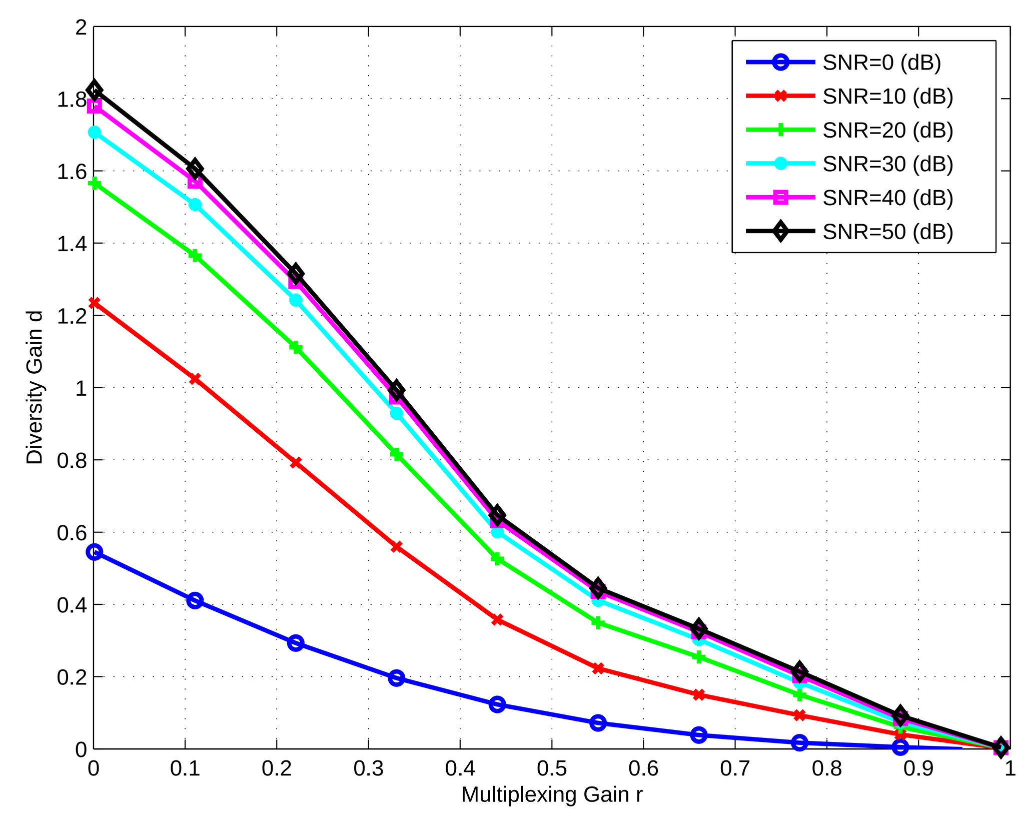
Protocol Performance



-Protocol I. $\alpha = \beta = 1/2$.



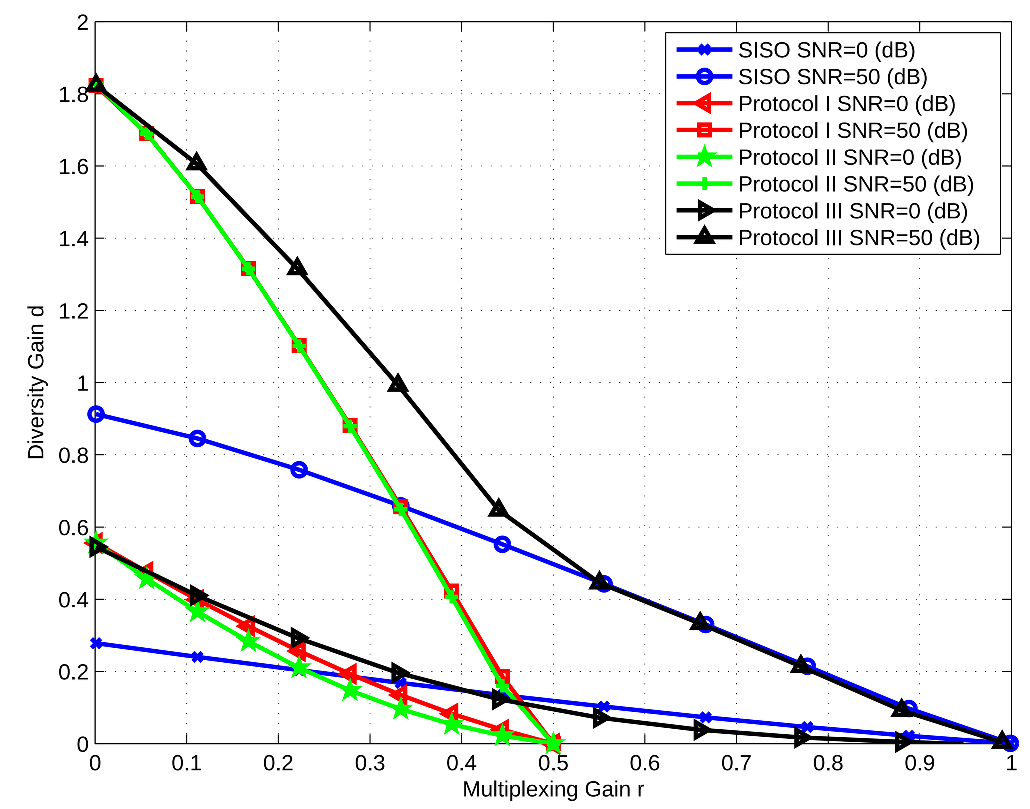
-Protocol II. $\alpha = \beta = 1/2$.



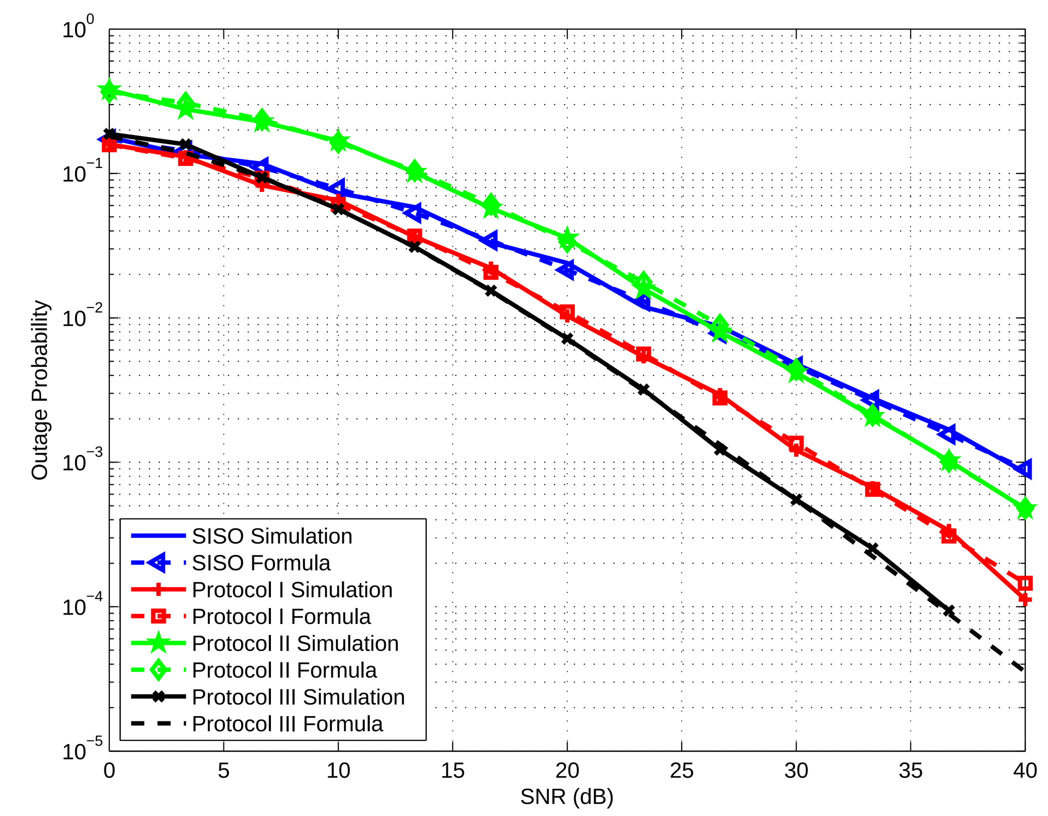
-Protocol III. $\alpha = \beta = 1/2$.

We can compute a lower bound on $P(E_2)$ using the optimization technique described in [3,5] and use this to estimate d.

Protocol Comparisons and Numerical Results



-Finite-SNR diversity-multiplexing comparison. $\alpha = \beta = 1/2$ and SNR values of 0 dB (low SNR) and 50 dB (high SNR) are shown.



-Comparison of analytical forms and bounds verses simulated performance. $r = 0.25$ and $\alpha = \beta = 1/2$.

The analytical results are verified through Monte Carlo simulations at a multiplexing gain of $r = 0.25$.

- All three protocols have superior diversity performance over that of the SISO system.
- Protocol I is superior to Protocol II, as the source in Protocol II must share power over two time slots.
- Protocol III is superior to Protocols I and II at high SNR due to the utilization of both time slots for the source to communicate with the destination.
- Notice that the lower bound for Protocol III is very close to the simulation performance.

Further details are discussed in [5].

References

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[2] L. Zheng and D. Tse, "Diversity and multiplexing: A fundemental tradeoff in multiple antenna channels," *IEEE Transactions on Information Theory*, vol. 49, May 2003.

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[4] R. Narasimhan, A. Ekbal, and J. M. Cioffi, "Finite-SNR diversity-multiplexing tradeoff of space-time codes," *IEEE International Conference on Communications*, vol. 1, pp. 458 – 462, 2005.

[5] E. Stauffer, O. Oyman, R. Narasimhan, and A. Paulraj, "Finite-SNR diversity-multiplexing tradeoffs in fading relay channels," *Submitted*.