Diversity-Multiplexing Tradeoff for Practical MIMO Channels

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Diversity-multiplexing tradeoff (DMT)

- There exists fundamental tradeoff b/w diversity and multiplexing (Zheng and Tse, 2003)

  - Diversity gain
    \[ \lim_{{\text{SNR} \to \infty}} \frac{R}{\log(\text{SNR})} = r \]

  - Multiplexing gain
    \[ \lim_{{\text{SNR} \to \infty}} \frac{\log P_e}{\log(\text{SNR})} = -d \]

- Fundamental DMT for i.i.d. Rayleigh MIMO channels
  \[ d^*(r) = (n_T - r)(n_R - r) \]
  \( n_T \): # of Tx antennas, \( n_R \): # of Rx antennas, \( l \): codeword length
Practical consideration

- Assuming i.i.d. Rayleigh fading can be pessimistic!
- More realistic scenarios include
  - Rician channels, which can be a model for a channel having partial CSIT
  - Channel model with spatial correlation among antennas
  - Rank-deficient channel in poor scattering environments

- Our objective
  - Analysis of the outage performance and DMT for three types of practical MIMO channel models
Practical MIMO systems – Rician channels

- Rician fading and partial feedback channel
  - Channel and system model

\[
y = \sqrt{\frac{\rho}{n_T}} H x + n \quad \Rightarrow \quad H = \sqrt{\frac{K}{K+1}} \bar{H} + \sqrt{\frac{1}{K+1}} H_w \quad (K: \text{Rician factor})
\]

- Can be a model for a channel with mean feedback

\[
H \sim CN\left(\sqrt{\frac{K}{K+1}} \bar{H}, \frac{1}{K+1} I\right)
\]
Asymptotic SNR gap (ASG)

- **Outage probability**
  \[
  P_{out}(R, \rho, K, \tilde{\Phi}) = \Pr \left\{ \log \det \left( I_{n_R} + \frac{\rho}{n_T} \mathbf{H}\mathbf{H}^H \right) < R \right\}
  \]
  where \( \tilde{\Phi} = \text{diag} \{ \phi_i \} \) whose diagonal elements are the \( \min\{n_T, n_R\} \) eigenvalues of \( \overline{\mathbf{H}} \)

- **Definition of the ASG**

\[
\Delta_a (R, K, \Phi) \Delta \lim_{\rho \to \infty} \frac{P^I_o (R, P_{out} (R, \rho, K, \tilde{\Phi}), 0, \tilde{\Phi})}{\rho}
\]
where \( P^I_o \) is the inverse function of \( P_{out} \)
Comparison with analytical results (ASG-Rician)

- The diversity order, the slope of the outage probability, is same at high SNR
- The ASG b/w Rayleigh and Rician channels exists at high SNR
Maximum diversity order (MDO) and finite-SNR gap (FSG)

- Definition of the MDO and FSG

\[
d_{\text{max}}(R, K, \bar{O}) \equiv d_o(R, \rho^*, K, \bar{O})
\]

where
\[
d_o(R, \rho, K, \bar{O}) = -\frac{d \log P_{\text{out}}(R, \rho, K, \bar{O})}{d \log \rho}
\]

\[
\Delta_f(R) \equiv \lim_{K \to \infty} \left[ 2^{\beta_{\text{op}}(R, K, \bar{O}) - \beta_G(R)} \right]
\]

where
\[
\beta_{\text{op}}(R, K, \bar{O}) = \arg \max_{\rho} d_o(R, \rho, K, \bar{O})
\]
Comparison with analytical results (MDO and FSG)

- The MDO shows that there exists an SNR where diversity order is maximized.
- It is shown that $\beta_{op}(R, K)$ is irrelevant to $K$.
- The linearity of the MDO with respect to $K$ is shown.
- Although the analytical results of the MDO are asymptotic, they are quite accurate, even for small $K$.

\[
\gamma_{\text{max}} = \frac{1}{2} K + \frac{5}{4} + O\left(\frac{1}{K}\right)
\]
Differential DMT (DDMT) for Rayleigh channels

- Definition 1
  - The DDMT of Rayleigh channels is characterized for different operating regions

  Approaches are based on throughput-reliability tradeoff (TRT) (Azarian and El Gamal, 2005)
Differential DMT (DDMT) for Rician channels

- Definition 2
  - Conventional DMT fails to explain the transient behavior such as the MDO and FSG → we need to formulate the DDMT!

\[
\tilde{D}(R,D,R,K) = \lim_{\delta \log \rho \to 0} \left[ \log P_{\text{out}}(R,\rho^*,K) - \log P_{\text{out}}(R+\tilde{D}\delta \log \rho,2^{\log \rho^*+\delta \log \rho},K) \right] / \delta \log \rho
\]

Rician channels

Differential diversity gain: \( \tilde{D}(R,D,R,K) \)

\( P_{\text{out}}(\text{Outage probability}) \)

SNR (dB)

Differential Multiplexing Gain

(0,0)

(0,d_{\text{max}}(R,K))

\( R \)

\( R + \tilde{D}\delta \log \rho \)

\( (0,mn) \)

\( (\text{rank} (\mathbf{H}),0) \)

(1-2^{-R})

\( r_{\text{max}}(R),0 \)

(\frac{mn}{m+n-1},0)
Practical MIMO systems – spatially-correlated channels

- Spatial correlation
  - Channel and system model
    \[ y = \sqrt{\frac{\rho}{n_T}} H x + n \quad \Rightarrow \quad H = \hat{O}^{1/2} H_w \]
    where \( \hat{O} = E[h_j h_j^H] \) and \( h_j \) is the \( j \)th column of \( H \)
  - ASG
    \[ \bar{\Delta}_a (R, \hat{O}) \Delta \lim_{\rho \to \infty} \frac{P_o^I (R, P_{out} (R, \rho, I), \hat{O})}{\rho} \]
Comparison with analytical results (ASG-corr)

- The DMT, i.e., diversity order at high SNR, is same
- The degradation appears only as a penalty in SNR gap

\[ \tilde{\Delta}_a = |\hat{\Omega}|^{-1/\min\{n_T, n_R\}} \]
Practical MIMO systems – rank-deficient channels

- Channel and system model

\[
H = \frac{1}{\sqrt{P}} \sum_{p=1}^{P} H_p = \frac{1}{\sqrt{P}} \sum_{p=1}^{P} u_p \cdot h_p \cdot v_p^H
\]

- \(u_p, v_p\): column vectors whose elements are i.i.d.
- \(K \rightarrow 1\): \(h_p\) is deterministic, \(K \rightarrow \infty\): \(h_p\) is a complex Gaussian
Estimation and upper-bound for DMT

- \( K \to \infty \) (Rayleigh fading), \( n_T=n_R=2 \)

\[
\log P_{out} = -g(l) - d*(2) \quad \text{for} \quad k = 1
\]

\[
\log P_{out} = -g(l) - d*(1) \quad \text{for} \quad k = 0
\]

\[
\Delta / g(l)
\]

\[
-g(0)
\]

\[
\Delta
\]

\[
R + 2
\]

\[
R
\]

\[
3 \text{ dB}
\]

\[
6 \text{ dB}
\]

\[
3 \text{ dB}
\]

\[
\Delta / g(l)
\]

\[
g(k): \text{a slope of piecewise-linear function for each } k
\]

<table>
<thead>
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<th>( P )</th>
<th>( g(1) )</th>
<th>( d^*(1) )</th>
<th>( g(0)=d^*(0) )</th>
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<tr>
<td>( \infty )</td>
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\( * \) Approaches are based on TRT (Azarian and El Gamal, 2005)
Other DMT curves for poor scattering channels

- $K \to \infty$ (Rayleigh fading), $n_T, n_R \to \infty$
- Keyhole model
  \[ H = \hat{\mathbf{a}} \mathbf{a}^T \]
  - $\hat{\mathbf{a}}$: column vectors whose elements are i.i.d. complex Gaussian r.v.'s

- $K=1$ (deterministic), $n_T = n_D = 2\hat{\mathbf{a}}$

\[ P = 1 \]

\[ P = \infty \]
Discussions and conclusion

- The results of our work
  - The effect of Ricianess and spatial correlation
    - They can change the outage performance by a constant dB gap
    - They cannot change the DMT, i.e., diversity order at high SNR
  - The MDO for Rician channels
    - There exists an SNR where the diversity order is maximized, which can be a desired operating point
  - The analysis of the DDMT
    - It is suitable for capturing the DMT for Rician
  - The DMT for rank-deficient channels
    - DMT curves are lowered
    - DMT of rank-deficient channels approach to that of the i.i.d. Rayleigh channels as scattering becomes rich