Diversity-Multiplexing Tradeoff for Practical MIMO Channels

Won-Yong Shin, Woohyuk Chang,
Sae-Young Chung, and Yong H. Lee

Korea Advanced Institute of Science and Technology

IEEE Communication Theory Workshop
Dorado, Puerto Rico
May 22, 2006

Diversity-multiplexing tradeoff (DMT)

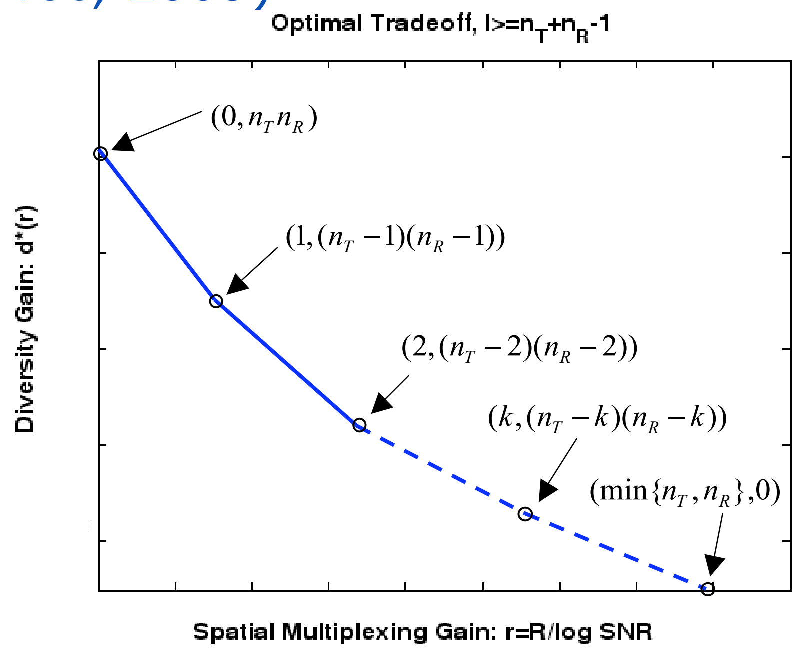
- There exists fundamental tradeoff b/w diversity and multiplexing (*Zheng and Tse, 2003*)

  - Diversity gain

$$\lim_{SNR\to\infty} \frac{R}{\log(SNR)} = r$$

  - Multiplexing gain

$$\lim_{SNR\to\infty} \frac{\log P_e}{\log(SNR)} = -d$$



  - Fundamental DMT for i.i.d. Rayleigh MIMO channels

$$d^*(r) = (n_T - r)(n_R - r)$$

n_T: # of Tx antennas, n_R: # of Rx antennas, l: codeword length

Practical consideration

- Assuming i.i.d. Rayleigh fading can be pessimistic!
- More realistic scenarios include
  - Rician channels, which can be a model for a channel having partial CSIT
  - Channel model with spatial correlation among antennas
  - Rank-deficient channel in poor scattering environments

- Our objective
  - Analysis of the **outage** performance and **DMT** for three types of practical MIMO channel models

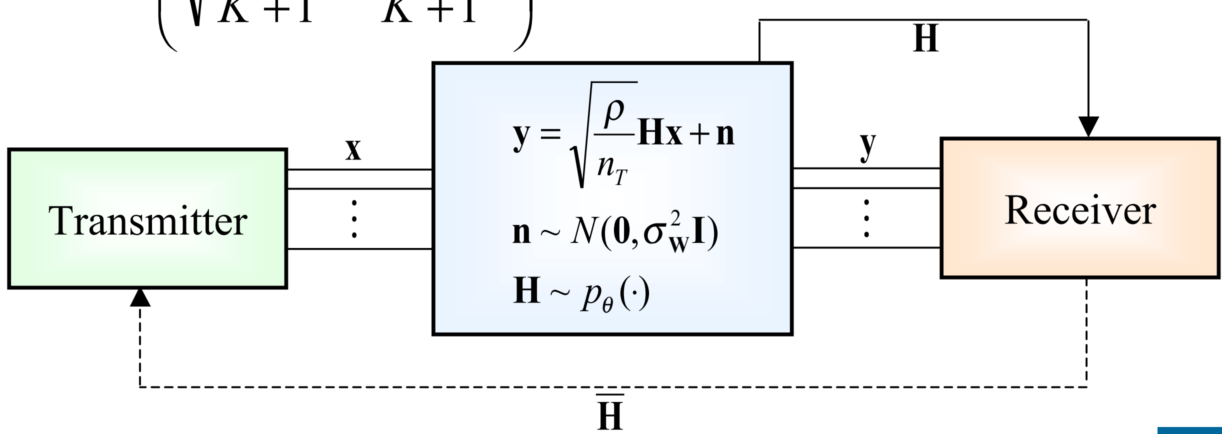
Practical MIMO systems – Rician channels

- Rician fading and partial feedback channel
  - Channel and system model

$$\mathbf{y} = \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{x} + \mathbf{n} \quad \Rightarrow \quad \mathbf{H} = \sqrt{\frac{K}{K+1}}\overline{\mathbf{H}} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w \quad (K: \text{Rician factor})$$

  - Can be a model for a channel with mean feedback

$$\mathbf{H} \sim CN\left(\sqrt{\frac{K}{K+1}}\overline{\mathbf{H}}, \frac{1}{K+1}\mathbf{I}\right)$$



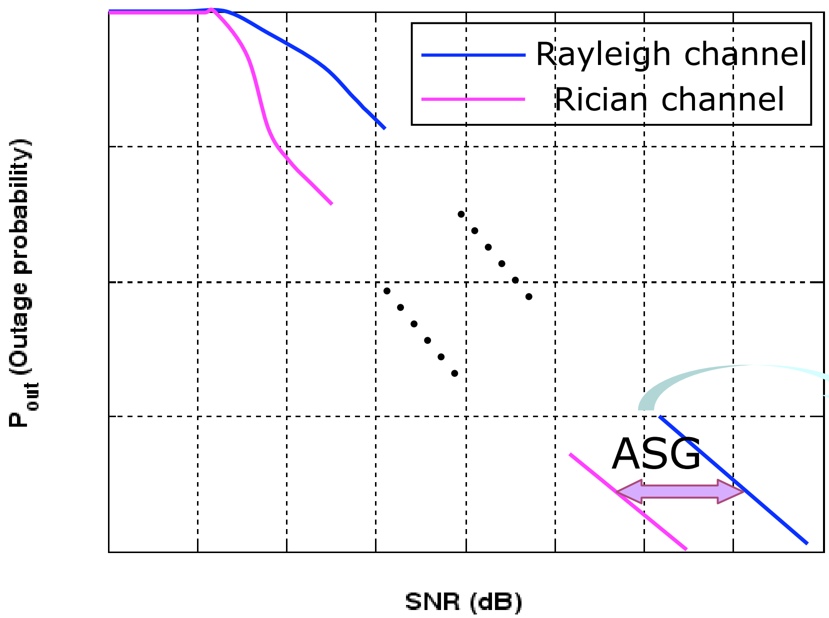
Asymptotic SNR gap (ASG)

- Outage probability

$$P_{out}(R,\rho,K,\mathbf{\ddot{O}}) = \Pr\left\{\log\det\left(\mathbf{I}_{n_R} + \frac{\rho}{n_T}\mathbf{H}\mathbf{H}^H\right) < R\right\}$$

where $\mathbf{\ddot{O}} = diag\{\phi_i\}$ whose diagonal elements are the $\min\{n_T, n_R\}$ eigenvalues of $\overline{\mathbf{H}}$

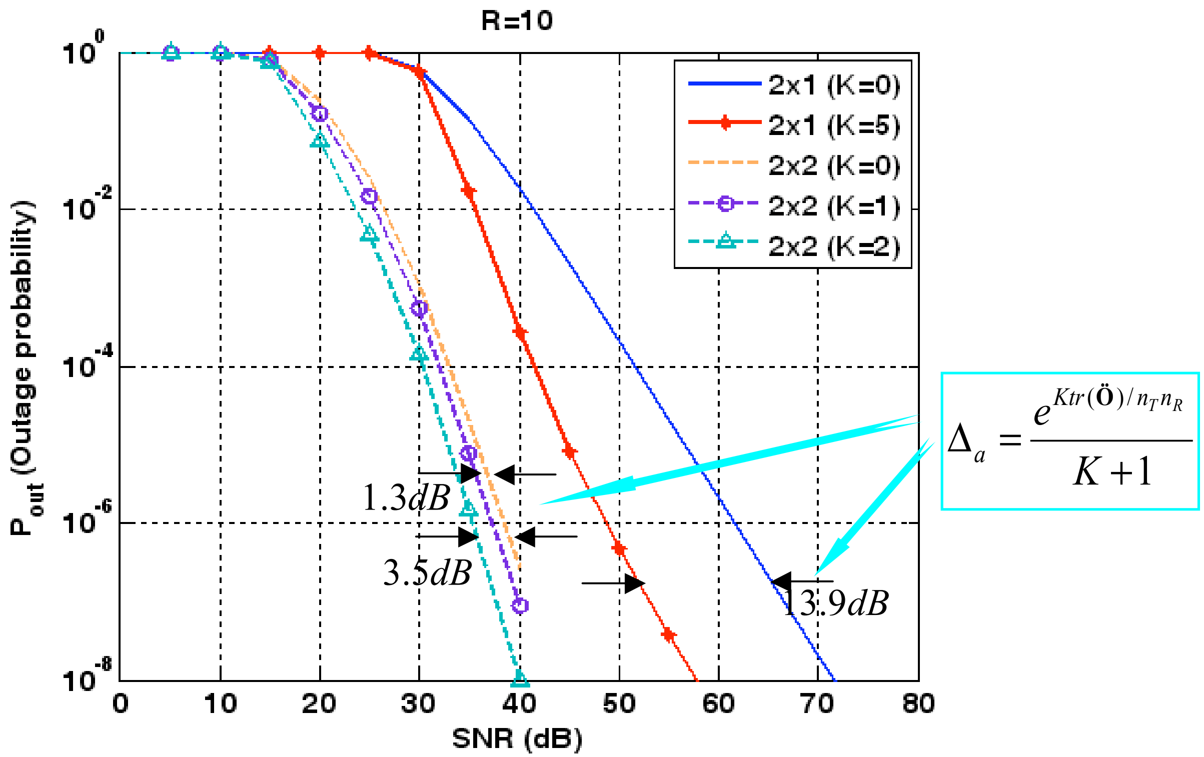
- Definition of the ASG



$$\Delta_a(R,K,\Phi) \triangleq \lim_{\rho\to\infty} \frac{P_o^I(R, P_{out}(R,\rho,K,\mathbf{\ddot{O}}), 0, \mathbf{\ddot{O}})}{\rho}$$

where P_o^I is the inverse function of P_{out}

Comparison with analytical results (ASG-Rician)

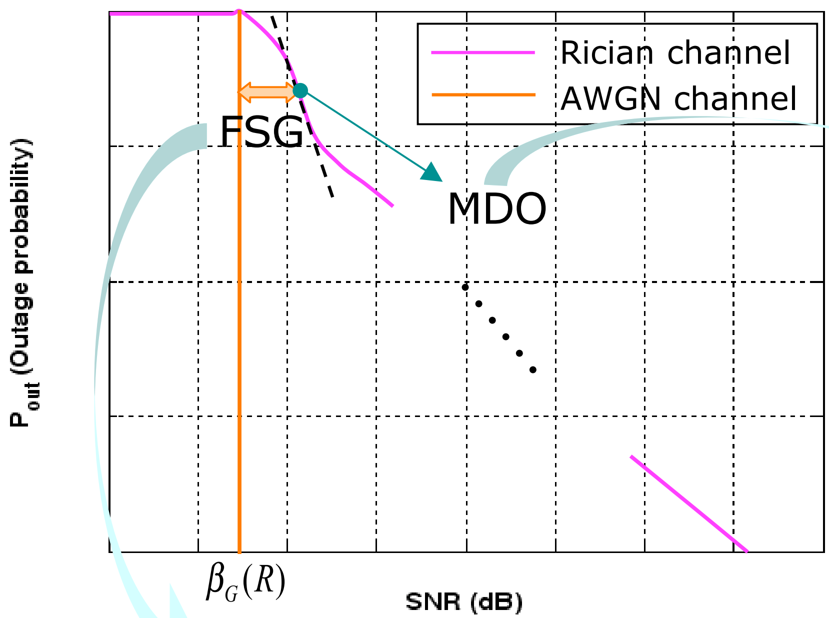


- The diversity order, the slope of the outage probability, is same at high SNR
- The ASG b/w Rayleigh and Rician channels exists at high SNR

KAIST

Maximum diversity order (MDO) and finite-SNR gap (FSG)

- Definition of the MDO and FSG



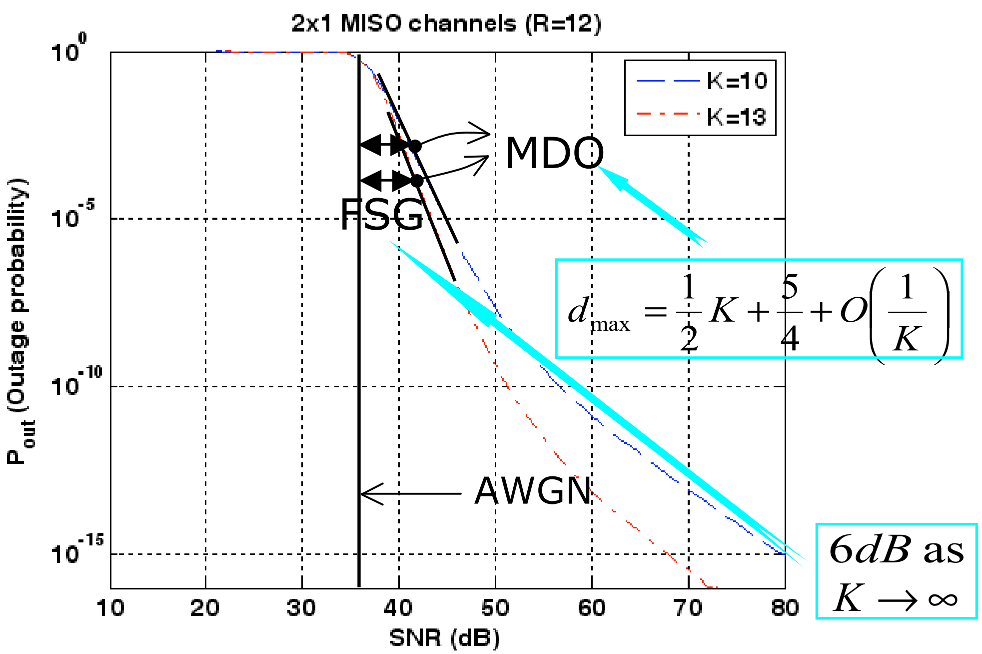
$$d_{\max}(R,K,\mathbf{\ddot{O}}) \triangleq d_o(R,\rho^*,K,\mathbf{\ddot{O}})$$

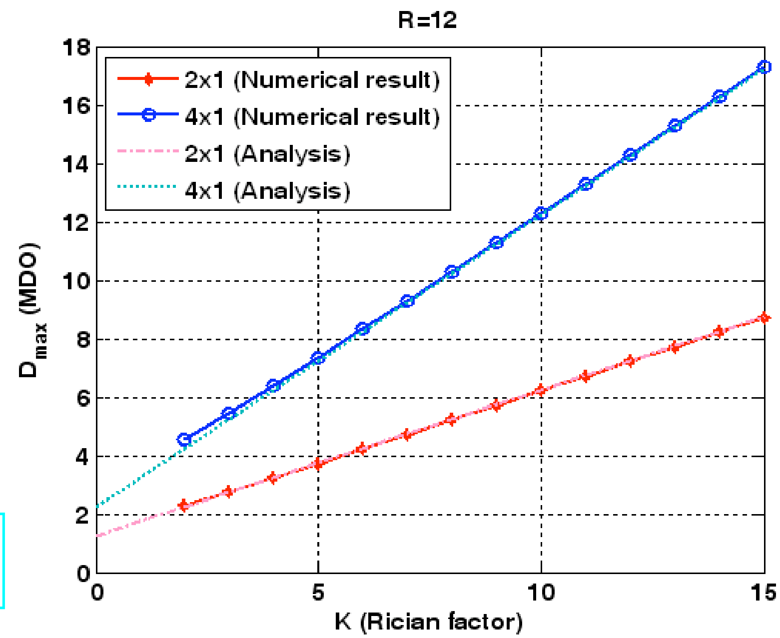
where $d_o(R,\rho,K,\mathbf{\ddot{O}}) = -\dfrac{d \log P_{out}(R,\rho,K,\mathbf{\ddot{O}})}{d \log \rho}$

$$\Delta_f(R) \triangleq \lim_{K\to\infty} \left[2^{\beta_{op}(R,K,\mathbf{\ddot{O}})-\beta_G(R)} \right]$$

where $\beta_{op}(R,K,\mathbf{\ddot{O}}) = \log \arg\max_{\rho} d_o(R,\rho,K,\mathbf{\ddot{O}})$

Comparison with analytical results (MDO and FSG)

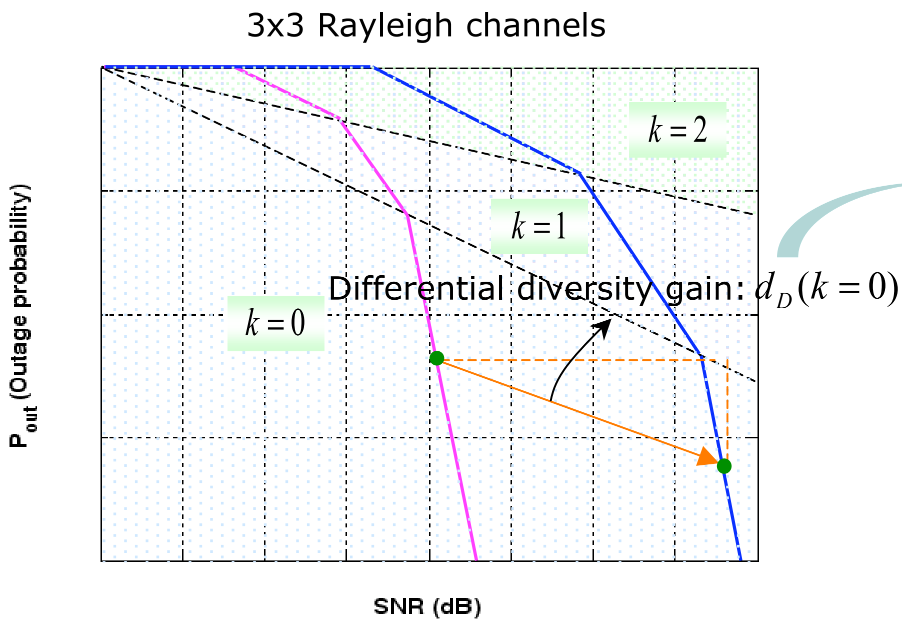


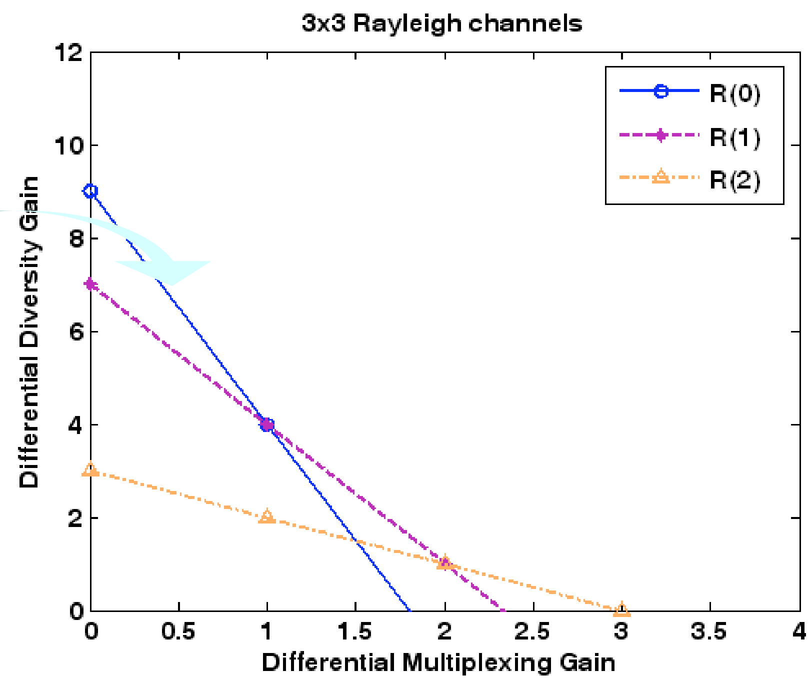


- The MDO shows that there exists an SNR where diversity order is maximized
- It is shown that $\beta_{op}(R,K)$ is irrelevant to *K*
- The linearity of the MDO with respect to *K* is shown
- Although the analytical results of the MDO are asymptotic, they are quite accurate, even for small *K*

Differential DMT (DDMT) for Rayleigh channels

- Definition 1
  - The DDMT of Rayleigh channels is characterized for different operating regions

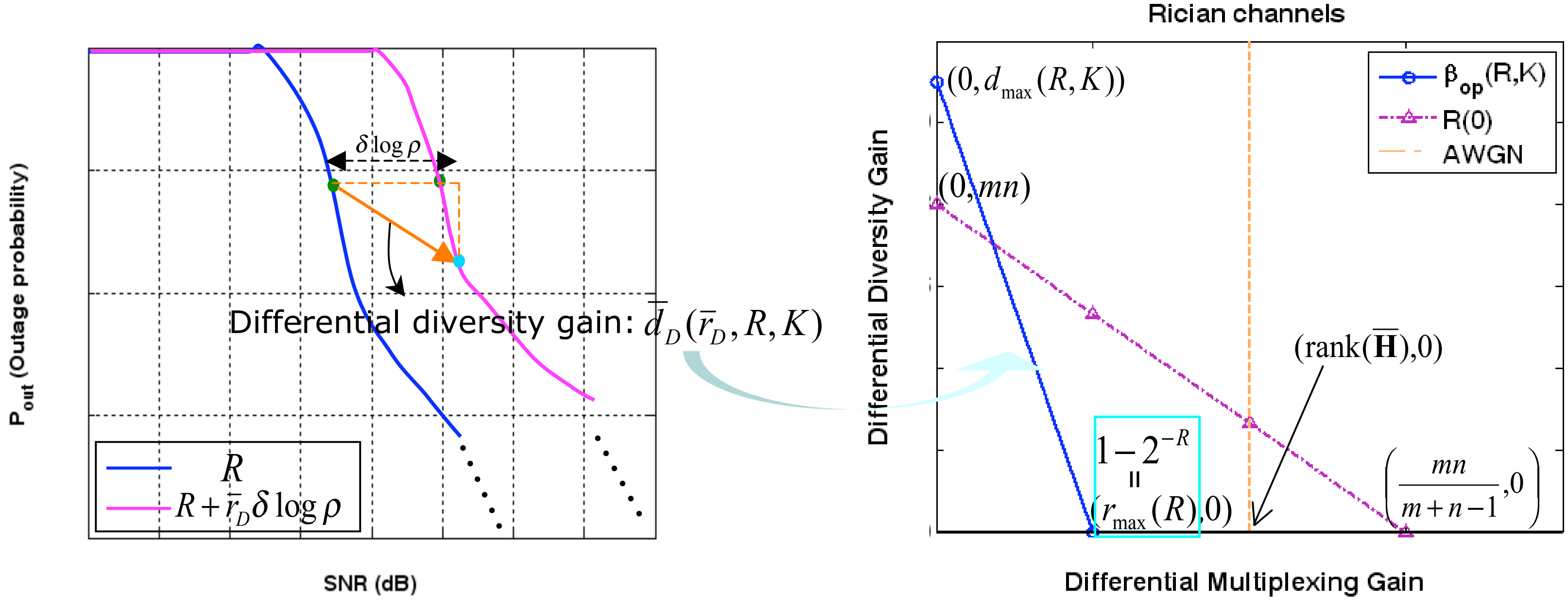




°ÿApproaches are based on throughput-reliability tradeoff (TRT) (*Azarian and El Gamal, 2005*)

Differential DMT (DDMT) for Rician channels

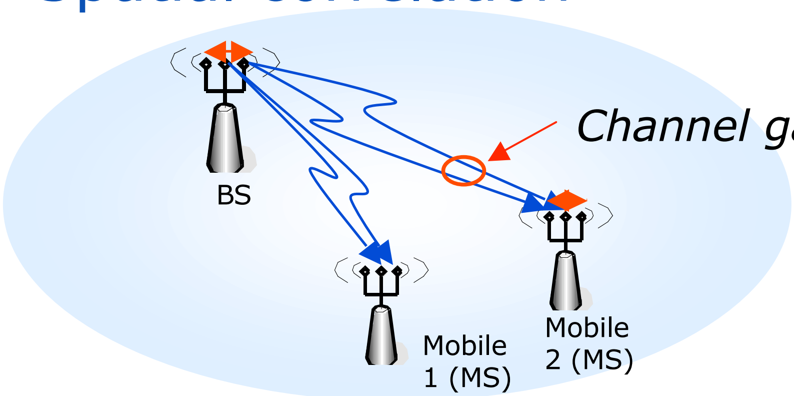
- Definition 2
  - Conventional DMT fails to explain the transient behavior such as the MDO and FSG → we need to formulate the DDMT!



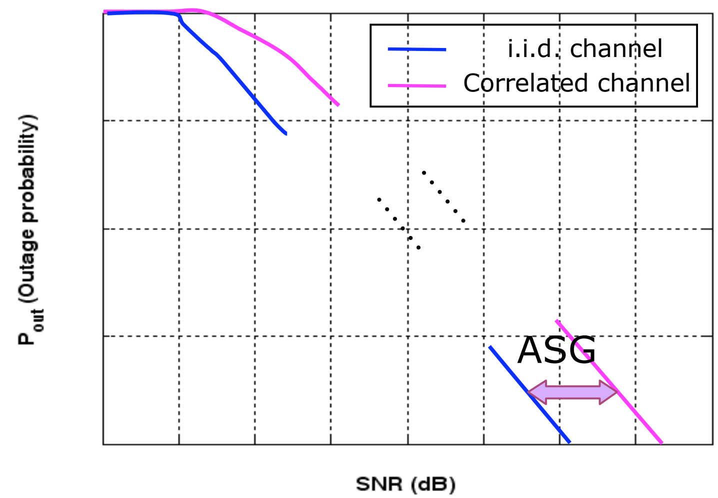
$$\bar{d}_D(\bar{r}_D, R, K) \triangleq \lim_{\delta \log \rho \to 0} \left[\frac{\log P_{out}(R, \rho^*, K) - \log P_{out}(R + \bar{r}_D \delta \log \rho, 2^{\log \rho^* + \delta \log \rho}, K)}{\delta \log \rho} \right]$$

Practical MIMO systems – spatially-correlated channels

- Spatial correlation



Channel gains are correlated!



- Channel and system model

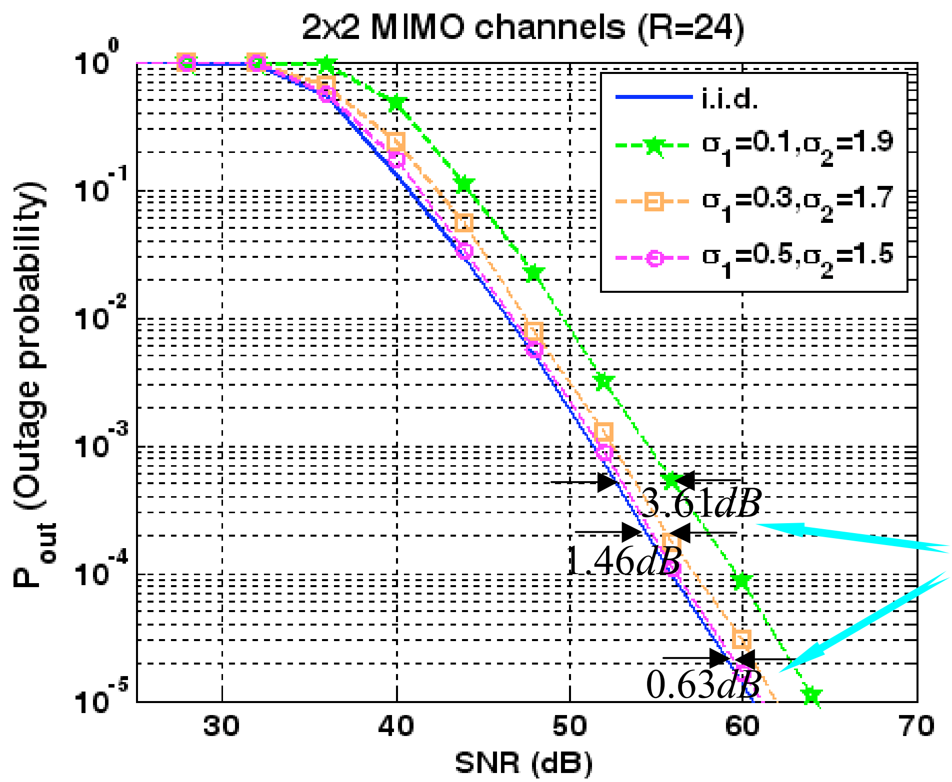
$$\mathbf{y} = \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{x} + \mathbf{n} \quad \Rightarrow \quad \mathbf{H} = \boldsymbol{\Sigma}^{1/2}\mathbf{H}_w$$

where $\boldsymbol{\Sigma} = E\left[\mathbf{h}_j \mathbf{h}_j^H\right]$, $\mathbf{h}_j$ is the jth column of $\mathbf{H}$

- ASG

$$\tilde{\Delta}_a(R, \boldsymbol{\Sigma}) \triangleq \lim_{\rho \to \infty} \frac{P_o^I(R, P_{out}(R, \rho, \mathbf{I}), \boldsymbol{\Sigma})}{\rho}$$

Comparison with analytical results (ASG-corr)



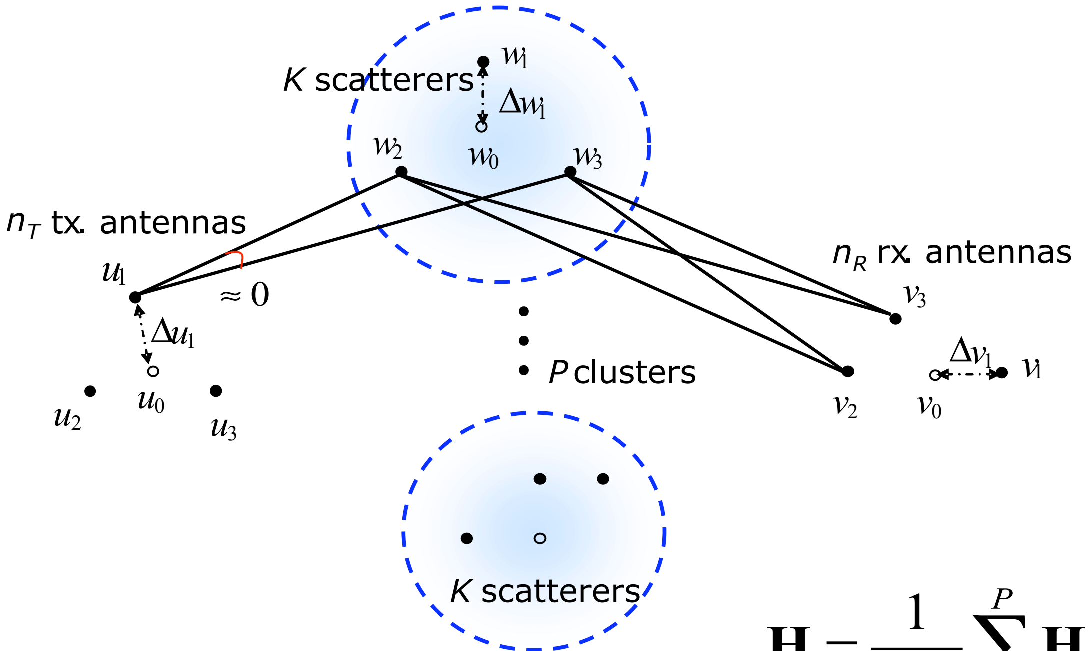
$$\widetilde{\Delta}_a = |\Sigma|^{-1/\min\{n_T, n_R\}}$$

- The DMT, i.e., diversity order at high SNR, is same
- The degradation appears only as a penalty in SNR gap

KAIST

Practical MIMO systems – rank-deficient channels

- Channel and system model

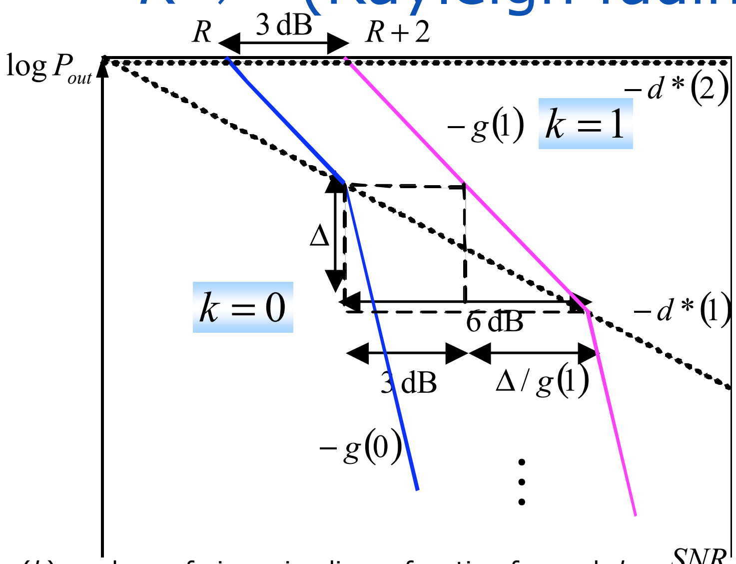


$$\mathbf{H} = \frac{1}{\sqrt{P}} \sum_{p=1}^{P} \mathbf{H}_p = \frac{1}{\sqrt{P}} \sum_{p=1}^{P} \mathbf{u}_p \cdot h_p \cdot \mathbf{v}_p^H$$

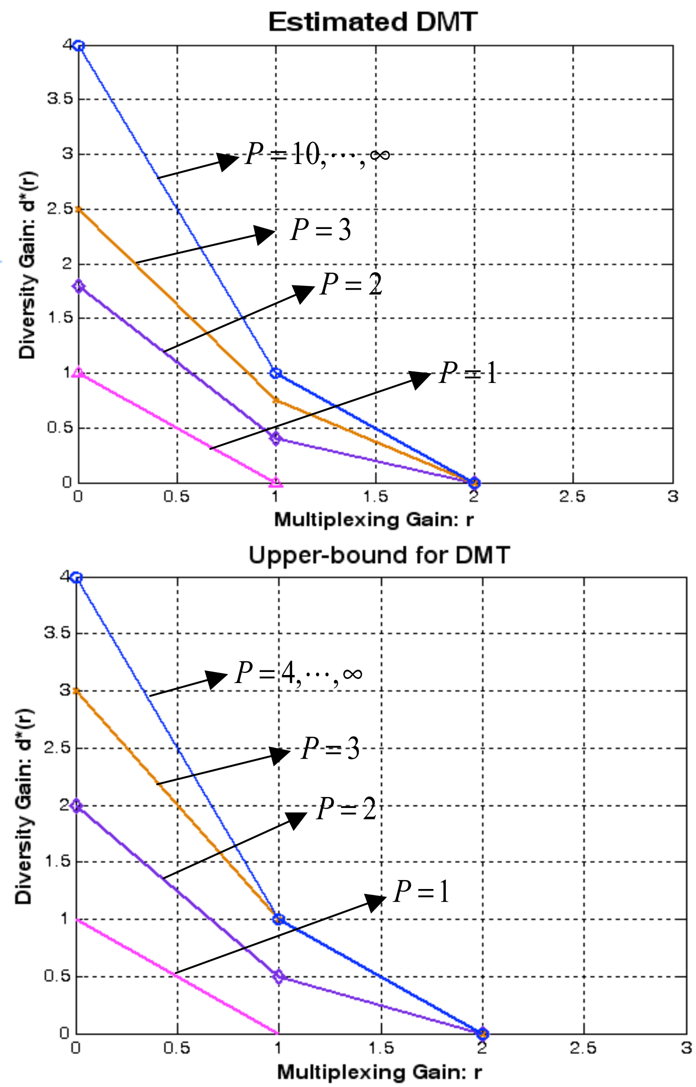
- $\boldsymbol{u}_p$, $\boldsymbol{v}_p$: column vectors whose elements are i.i.d.
- $K \to 1$: h_p is deterministic, $K \to \infty$: h_p is a complex Gaussian

Estimation and upper-bound for DMT

- $K \to \infty$ (Rayleigh fading), $n_T = n_R = 2$



$g(k)$: a slope of piecewise-linear function for each k

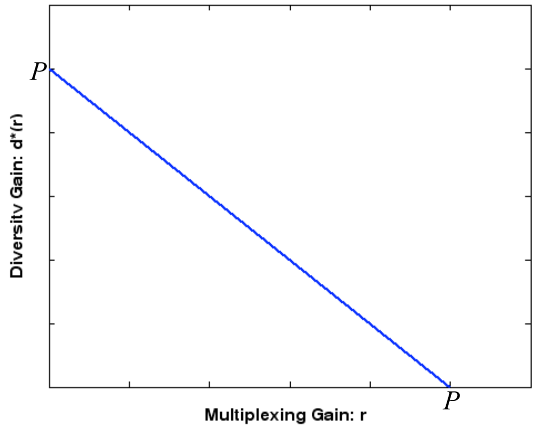


P	$g(1)$	$d^*(1)$	$g(0)=d^*(0)$
1	0	0	1
2	0.8	0.4	1.8
3	1.5	0.75	2.5
10	2	1	4
∞	2	1	4

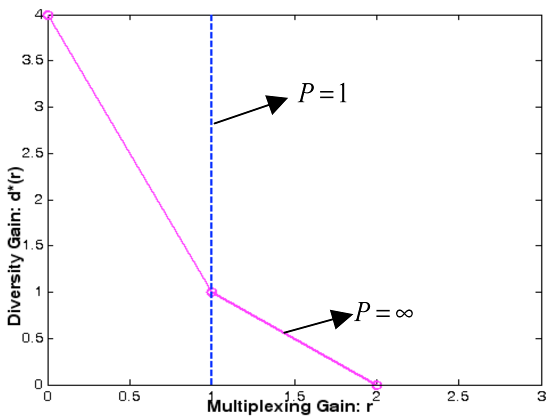
°ÿApproaches are based on TRT (*Azarian and El Gamal, 2005*)

Other DMT curves
for poor scattering channels

- $K \to \infty$ (Rayleigh fading), $n_T, n_R \to \infty$



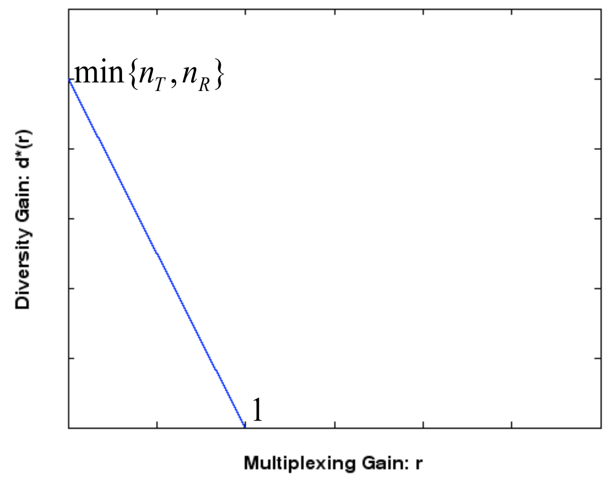
♣ K=1 (deterministic), $n_T = n_D$=2Æ•



- Keyhole model

  $\mathbf{H} =$ **âá** ^{H}

  - •; •; column vectors whose el ements are i.i.d. complex G aussian r.v.'s



Discussions and conclusion

- The results of our work
  - The effect of Ricianness and spatial correlation
    - They can change the outage performance by a constant dB gap
    - They cannot change the DMT, i.e., diversity order at high SNR
  - The MDO for Rician channels
    - There exists an SNR where the diversity order is maximized, whic h can be a desired operating point
  - The analysis of the DDMT
    - It is suitable for capturing the DMT for Rician
  - The DMT for rank-deficient channels
    - DMT curves are lowered
    - DMT of rank-deficient channels approach to that of the i.i.d. Rayle igh channels as scattering becomes rich