Spectral Analysis
ELEC 532
Assignment 1

Due: Wednesday 2 February 2000

Note: Homework, tests, and solutions from previous offerings of this course are off limits, under the honor code.

1. Using Matlab, compute the rank, determinant, eigenvalues, and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 1 & 1 & 3 & 0 \\ 1 & 1 & 3 & 0 \\ -2 & 0 & 0 & 2 \end{bmatrix}.$$

2. (a) Show that the power spectral density of a real WSS random process is a real, even function of frequency.

   (b) Show that the power spectral density of any WSS random process is non-negative at all frequencies.

   HINT: Use the properties of the auto-correlation function for both parts.

3. Show that the random process

$$x[n] = A\cos(2\pi f_0 n + \phi),$$

   with ϕ uniformly distributed on $[0, 2\pi)$, is WSS by finding its mean and auto-correlation function. Repeat for the single complex sinusoid

$$x[n] = A\exp[j(2\pi f_0 n + \phi)].$$

4. Consider an iid $N(0, 2)$ random process.

   (a) In Matlab, plot one realization of this process. (You will find the command `randn` useful.)

(b) Compute a time average of this realization.

(c) Compute (approximate) and plot an ensemble average for this process.

(d) Explain the difference between the time average and the ensemble average and when you can substitute one for the other.

(e) In Matlab, plot one realization of a process that is iid uniform on the interval $[1, 3]$. How does it differ from the Gaussian process generated above? (You will find the command `rand` useful.)

5. **Window functions in time and frequency**[1]

   (a) Plot several 64-point windows in the time domain (use `stem` if your plots are large enough) and in the frequency domain (use an fftshifted, zero-padded FFT and `semilogy`). There are Matlab commands for the Hamming, Hanning, Boxcar, Kaiser, and Bartlett windows, among others. Note the tradeoff of rate of decay in time vs. frequency main lobe width vs. sidelobe height vs. sidelobe decay.

   (b) For one of these windows, show that as you make the window longer in time, the performance in frequency improves. What should happen in frequency as the window becomes infinitely long?

Do this exercise with your group and turn in one set of plots per group.
Save trees — use subplots and overlays!

6. With your group, discuss and synthesize the major *themes* of this week of lectures. Turn in a one page summary of your discussion. You need turn in only one summary, but include the names of all group members. Please do not write up just a "table of contents".

[1] The book *Computer-based Exercises for Signal Processing*, by C. S. Burrus et al, has a nice section on windows, pp. 83–92.