Spectral Analysis
ELEC 532
Assignment 6

Due: Wednesday 12 April 2000

1. **Array Processing**

   You are chief astrologer to the four rain gods *Chacs* in the ancient Mayan city of Tikal. The year is 931 A.D. This morning your ruler, *Chizén Ampol*, decreed that a new age will dawn when the seven (7) recently discovered pulsar stars[1] line up in a grand harmonious coincidence at a set of sacred angles with the sacred mountain peak *Emdal* on the southern horizon.

   Your task today is to check the world's progress towards the new age by estimating the angles that the seven pulsars currently make with *Emdal*. To this end, you have constructed (out of tree trunks, woven grass, and titanium threads) 64 large, omnidirectional radio telescopes and placed them in a linearly spaced array along a north-south line in the suburbs of Tikal, pointing at *Emdal*. The spacing between the individual antennae is set to one-half the wavelength of the pulsars, which you assume emit identical and constant sinusoidal pulsations.

   By treating the parallel outputs of the telescopes at time $t = t_0$ as a temporal signal, you have realized that a spectral estimate of this signal at frequency $f_0 \in [-1/2, 1/2]$ will estimate the astral radio emissions in a cone at roughly $\theta = \frac{\pi}{2} - \pi f_0$ radians above the southern horizon.

   Chizén Ampol is pleased with the array; it is even larger than the 17 sensor array built at Uaxactun in 299 A.D. But will it estimate the angles accurately enough to please the rain gods?

   Using a periodogram spectral estimate of your choice, estimate the strengths and angles of the pulsars. Repeat your estimates using the AR, Capon, Bartlett, Owsley, Eigenvector, and MUSIC estimates. Use the modified covariance method to estimate the autocorrelation matrix in the model-based techniques. Try also *Rich's secret technique*, which multiplies the Owsley estimate by the Eigenvector estimate. The numerator of this estimate uses the signal subspace, while the denominator uses the noise subspace.

   Make sure you keep

$$p < M < N$$

[1] The Mayans were pretty advanced in astronomy.

with N the number of signal samples, M the size of the estimated autocorrelation matrix, and p the "model order" (number of assumed sinusoids). At least a factor of 2 in each case is a good idea.

In each case, **comment** on the quality of the estimates and your confidence in them.

The data is available on the Elec 532 www page. You can do this problem in your group.

2. With your project gang, write a quick 1/2 page progress report on your project and include it on your WWW page.

3. With your study group, discuss and synthesize the major *themes* of this week of lectures. Turn in a one page summary of your discussion. You need turn in only one summary, but include the names of all group members.