Image enhancement by nonlinear wavelet processing

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Abstract

In this paper we describe how the theory of wavelet thresholding introduced by Donoho and Johnstone can successfully be applied to two distinct problems in image processing where traditional linear filtering techniques are insufficient. The first application is related to speckle reduction in coherent imaging systems. We show that the proposed method works well for reducing speckle in SAR images while maintaining bright reflections for subsequent processing and detection. Secondly we apply the wavelet based method for reducing blocking artifacts associated with most DCT based image coders (e.g., most notably the Joint Photographic Experts Group (JPEG) standard at high compression ratios). In particular we demonstrate an algorithm for post-processing decoded images without the need for a novel coder/decoder. By applying this algorithm we are able to obtain perceptually superior images at high compression ratios using the JPEG coding standard. For both applications we have developed methods for estimating the required threshold parameter and we have applied these to large number of images to study the effect of the wavelet thresholding. Our main goal with this paper is to illustrate how the recent theory of wavelet denoising can be applied to a wide range of practical problems which does not necessarily satisfy all the assumptions of the developed theory.

1 Introduction

Recent developments in wavelet theory [10, 11, 15, 12, 14, 13, 8] have made a significant contribution towards the potential for finding new and exciting applications of wavelets in one dimensional as well as multidimensional signal processing. Donoho and Johnstone [13, 15] showed that wavelets, which are unconditional bases for a large number of function spaces (smoothness spaces included) are optimal bases for compression, estimation (noise reduction) and recovery [8]. In all its simplicity the discovery was to observe that signals corrupted with additive noise satisfy the following heuristics [30].

- Signals are represented with a few large wavelet coefficients
- Noise is evenly distributed across wavelet coefficients and is generally small

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Without oversimplifying the results these two observations are the key elements of the theory of wavelet based noise reduction popularized by Donoho and Johnstone's proof of optimality. The significant contribution of Donoho and Johnstone was to show that under certain assumptions on the signal and it's estimate is was possible to obtain a wavelet based noise reduction algorithm which is optimal in a minimax sense [10, 8]. In practice noise reduction by wavelet theory is an $O(N)$ operation achieved by taking the wavelet transform of the noisy signal, apply a nonlinear threshold to the wavelet coefficients (shrink or keep the large wavelet coefficients, they are signal, and discard small wavelet coefficients, they are noise) and finally take the inverse wavelet transform to get an estimate of a signal with reduced noise level.

This paper describes two applications of the above mentioned wavelet based noise reduction algorithm. Both examples deal with two dimensional signals (images) corrupted by statistically different noise. Several one dimensional examples can be found in [9].

The two problems are:

- Speckle reduction in SAR (Synthetic Aperture Radar) images for applications in target tracking, detection and recognition.
- Reduction of blocking artifacts introduced by decoding images encoded by the JPEG (Joint Photographic Experts Group) standard. At high compression ratios (quantized too coarsely for baseline JPEG acceptable quality) the quality of the decoded image is significantly deteriorated and often perceptually unacceptable.

The general model for the problems we consider is as follows. Let $y = x + e$ be the observed image of the ideal image x corrupted by noise e. Most observed images typically suffer from artifacts due to the process associated with formation/processing/compression. The objective is then to remove as much of the corruption as possible without destroying relevant information.

2 Nonlinear wavelet-based noise reduction

Given that the observed signal, y, is modeled as $y = x + e$ and if the error e is Gaussian and uncorrelated (both spatially and with x), then finding x from y is a classical statistical estimation problem. Since the estimation procedure, $\hat{x}(y)$, should work for a variety of signals (images) $x \in \mathcal{X}$ (where $\mathcal{X}$ is the space/set of all images belonging to the application considered), the goal is to find $\hat{x}$ such that $\sup_{x\in\mathcal{X}} \|\hat{x}(y) - x\|$ is minimized. Let W and W^{-1} denote the wavelet and inverse wavelet transform operators respectively. Then Donoho [10] showed that spurious oscillations or equivalently smoothness of the signal x can be maintained by imposing the shrinkage condition

$$|W\hat{x}(y)| \leq |Wx| \,. \tag{1}$$

This problem has a solution which is asymptotically near optimal (simultaneously) for a wide variety of classes $\mathcal{X}$ [10]. A procedure, $\hat{x}(y)$, satisfying the above minimax problem with the given smoothness condition is given by soft-thresholding ("shrink or kill") in the wavelet domain where the threshold depends on the variance of e (the noise). Then the nonlinear wavelet procedure is given by

$$\hat{x}(y) = W^{-1} T_\delta(Wy) \tag{2}$$

and

$$T_\delta(x) = \begin{cases} x - \operatorname{sgn}(x)\delta & \text{for } |x| > \delta \\ 0 & \text{for } |x| \leq \delta \end{cases} \tag{3}$$

is the proposed soft-thresholding rule (function). The threshold δ is obtained by an estimation procedure from the observed image data y.

Imposing the shrinkage condition results in two important qualitative features: (i) the reconstruction is noise-free in the sense that no spurious oscillations are introduced (other than in the data y) and (ii) relatively sharp features in y are maintained.

2.1 Thresholding

Donoho [10] showed theoretically that soft-thresholding (see Eqn. 3) is the optimal nonlinear function to apply if smoothness of the estimate is important. The optimality of the soft-threshold is in terms of mean squared error subject to smoothness. However, it is well known that for many practical applications mean squared error is not a good measure of performance (e.g., perceptual image quality is not well measured in terms of mean squared error).

Hence, one should not restrict oneself to only consider soft-thresholding since particular problems might provide additional information about the noise which should be incorporated into the nonlinear wavelet coefficient manipulation process. This can be achieved either through the choice of the nonlinear rule or by altering the method for computing of the threshold parameter. An obvious alternative for a threshold rule would be to apply hard-thresholding ("keep or kill") defined by

$$T_\delta(x) = \begin{cases} x & \text{for } |x| > \delta \\ 0 & \text{for } |x| \leq \delta. \end{cases} \tag{4}$$

However, regardless of which thresholding rule one chooses to apply one has to determine at least one threshold parameter (possibly several depending on the complexity of the thresholding rule and whether or not thresholding should be scale/level adaptive) from the noisy data. Donoho and Johnstone [12, 11], Johnstone and Silverman [26], Donoho et al. [15], Nason [32], Weyrich and Warhola [43] Saito [37, 38] and Vidakovic [42] have proposed various methods for estimating the threshold from the observed noisy data. Each of these papers discusses one or several methods for picking the optimal threshold (for both hard and soft-thresholding rules as well as variations thereof) for various problem formulations. We will not describe any of these methods here other than say that most of these methods for obtaining the threshold, δ, can be formulated as

$$\delta = \lambda \hat{\sigma}(y) \tag{5}$$

where λ is a constant and $\hat{\sigma}(y)$ is an estimate of the noise standard deviation. Donoho and Johnstone [12] showed that in fact if $\lambda = \sqrt{2\log(n)}$ then δ was the optimal minimax threshold subject to the assumptions previously mentioned (they referred to this as the *universal* threshold). Hence the challenge is to design a robust noise variance estimator for any given problem. Furthermore, our experience shows that each problem seems to require a (slightly) different method for picking the threshold and in fact if one can not find a "robust" noise variance estimate then choosing $\lambda < \sqrt{2\log(n)}$ will help preventing over-smoothed estimates. In fact for the two problems considered here we found that we could use the same variance estimator (to be described later) and were left with having to optimize λ to achieve the desired performance for each application.

Given the above *universal* type threshold parameter and assuming White Gaussian Noise (WGN) the obvious variance estimator, also proposed by Donoho and Johnstone, is to estimate the noise variance by computing the variance of the high frequency band at the first level of the wavelet transform of the noisy signal. That is, for images compute the variance from the high-high subband at the first level of the two dimensional wavelet transform of the noisy image. The reason for this is that at the Nyquist rate the energy in the high-high frequency band will be mostly affected by noise and not much signal energy will leak into this band. Hence computing the variance from high-high band makes intuitive sense. Since this estimate might be inflated by the presence of some signal energy in high-high band we compensate for that by optimizing λ based on the application at hand. That is, experience shows that for most problems we have to choose $\lambda < \sqrt{2\log(n)}$ (see Table 1 Section 4.1). For more details on this see Gopinath et al. [18, 17] and Guo et al. [22, 21]. In the next two sections we will discuss each of the two applications in more detail and give examples of the performance for each case.

2.2 Wavelet parameter optimization

So far we have only been concerned with discussing methods for nonlinear thresholding rules and threshold estimators. However, in implementing the wavelet based noise reduction algorithm for most applications one would want to consider a whole range of wavelet related parameters to optimize over such as: (*i*) error measure, (*ii*) type of transform (2-band [6], M-band [39], wavelet packets [4], multiwavelets [40] etc.), (*iii*) properties of the wavelet filters (vanishing moments [6, 25], splines [3], smoothness [23, 24], stopband attenuation [33], signal dependent optimal filters [19, 41], etc.) (*iv*) number of levels to threshold in the wavelet coefficient domain.

Although it is infeasible to optimize over all of these parameter one needs to consider each of these parameters since they might contribute to improving the noise reducing capabilities for an application. Often one will find that some of the parameters will naturally be determined based on the application (e.g., still image processing typically works better with short filters etc.) and hence the search space is reduced. For a more complete discussion of the significance of each of these see Gopinath et al. [18].

3 SAR speckle reduction

Speckle is a physical phenomenon that can be found in a lot of observed imaged data such as SAR, acoustic imagery such as sonar, electronic speckle pattern interferometry, laser range data etc. When an object is illuminated by a coherent source and the object has a surface structure that is roughly on the order of a wavelength of the incident radiation, the wave reflected from such a surface consists of contributions from many independent scattering points. Interference of these dephased but coherent waves result in the granular pattern known as *speckle*. Thus, speckle tends to obscure image details and hence speckle reduction is important in most detection and recognition systems. It can be shown and simply verified by measurement that the WGN model serves as a good approximation for speckle [16].

Our goal is to show that wavelet based noise reduction can be used for minimizing the effects of speckle when the observed image y is a digitized SAR image. Several algorithms, not based on wavelet theory, for reducing speckle in SAR images have been proposed [7, 34]. However, no single method has been found (possible with the exception of the polarimetric whitening filter (PWF) [34] which uses multiple observations to reduce speckle) that does a good job of removing speckle without a significant loss of image resolution or extensive knowledge of ground truth. The results as presented here are not new and have independently been reported by Moulin [31] and Guo et al. [21, 22]. The difference between the two approaches is that Moulin only considers hard-thresholding while Guo et al. considers both hard and soft-thresholding. Furthermore, Guo et al. applied the method to both single channel SAR images as well as combined the wavelet based denoising algorithm with the PWF for fully polarimetric SAR image speckle reduction and achieved superior performance in terms of speckle statistics. Extensive testing of the resulting fully polarimetric wavelet based speckle reduction algorithm has been performed at Lincoln Laboratory for applications in automatic target detection and recognition (ATD/R) and is reported in [35].

Table 1: Standard deviation to mean (std/m) and log standard deviation (log-std) for SAR HH clutter data. The table gives a comparison between original unprocessed HH SAR image and the wavelet despeckled HH SAR image.

	std/m		log-std (dB)	
	Original	Wavelet despeckled	Original	Wavelet despeckled
Trees	1.7639	0.8995	6.8750	3.6006
Grass	1.2061	0.3933	5.8213	1.6358

A classic measure of speckle size is the standard-deviation-to-mean (s/m) ratio [16, 7] and log standard deviation [34]. Table 1 shows these two values for two regions of an original and a processed SAR image (more detailed results can be found in Guo et al. [22, 21]). In Figs. 1 and

2 we have plotted the original and the processed version of a SAR image scene. We can see by comparing appropriate images in Fig. 1 and Fig. 2 that speckle is greatly reduced while image details such as bright reflectors are well preserved and hence the improved image quality might provide advantages for further processing such as detection and recognition.

4 JPEG post-processing

Highly compressed still images typically suffer from unacceptable coding artifacts when reconstructed. In particular the JPEG standard introduces objectionable blocking artifacts due to the coarse quantization of AC coefficients in each 8×8 DCT block at high compression ratios. The blocking artifacts introduced are "weakly" correlated spatially and can have rather significant signal correlation. Hence, removing these coding artifacts for perceptual image enhancement has been considered difficult if not impossible without significantly smoothing the resulting image.

The noise in images corrupted by coding artifacts such as blocking can not be satisfactorily modeled by an uncorrelated (white) additive noise process and hence does not satisfy the stochastic assumptions on the error assumed in the proof of optimality by Donoho and Johnstone [12]. Hence, it is not obvious that wavelet based noise reduction should work for this problem. As we will see, the process of wavelet noise reduction does an exceptional job of reducing coding artifacts without perceivable loss of image detail and without redesigning the JPEG decoder. Although there does not exist a rigorous proof of why the wavelet based noise reduction algorithm works for the correlated noise case we have developed some intuition for why it works. Firstly the wavelet transform tends to whiten the data and hence while the error, e, might not be white, We might be. Secondly, Donoho [10] showed that if the error is bounded (which it clearly is for most signal processing applications) then soft-thresholding is optimal in the L^2 sense if smoothness is desired.

Our experience with JPEG compressed data supports the above intuition and we notice significant improvements in perceptual quality as well as mean-squared error. Although no extensive comparative study has been performed between various alternative methods for blocking artifact reduction/removal the wavelet based method seems more promising than other methods in the literature (see [36, Chapter 16] and [44, 45, 29]). Besides,

the methods in [45, 29] requires the use of a novel decoder, while the methods proposed here is only $O(N)$ and can be used with a conventional JPEG coder/decoder for which VLSI chips are available.

Finally it has been proposed that by applying scale adaptive thresholding one can adapt to the correlation of the error signal and possibly achieve even better performance for strongly correlated noise. The adaptive thresholding method was first proposed by Donoho and Johnstone [11] (*SureShrink*) for an approach to recover a function of unknown smoothness from noisy, sampled data. Furthermore recent work by Johnstone and Silverman [26] makes a first attempt to show

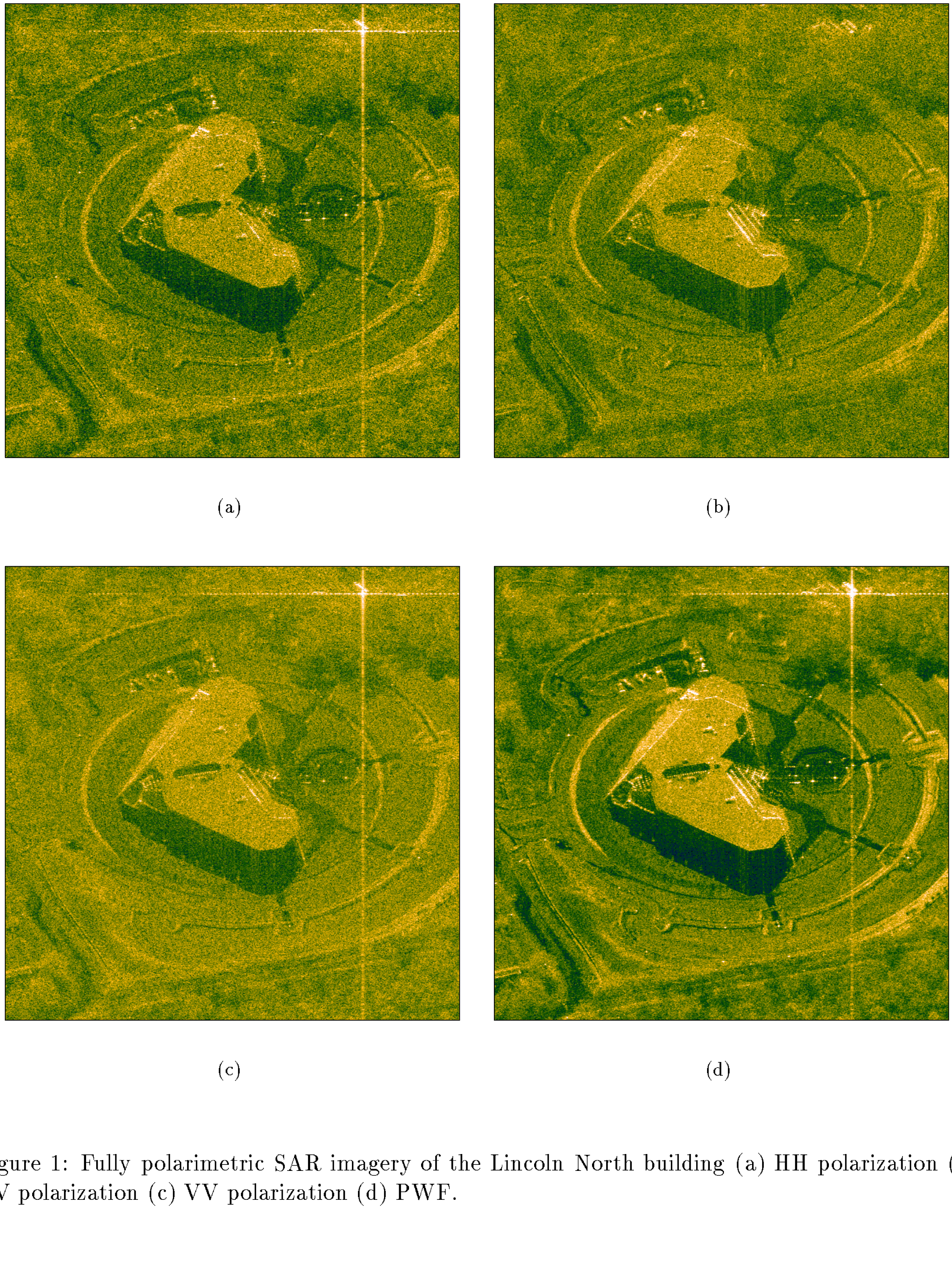

Figure 1: Fully polarimetric SAR imagery of the Lincoln North building (a) HH polarization (b) HV polarization (c) VV polarization (d) PWF.

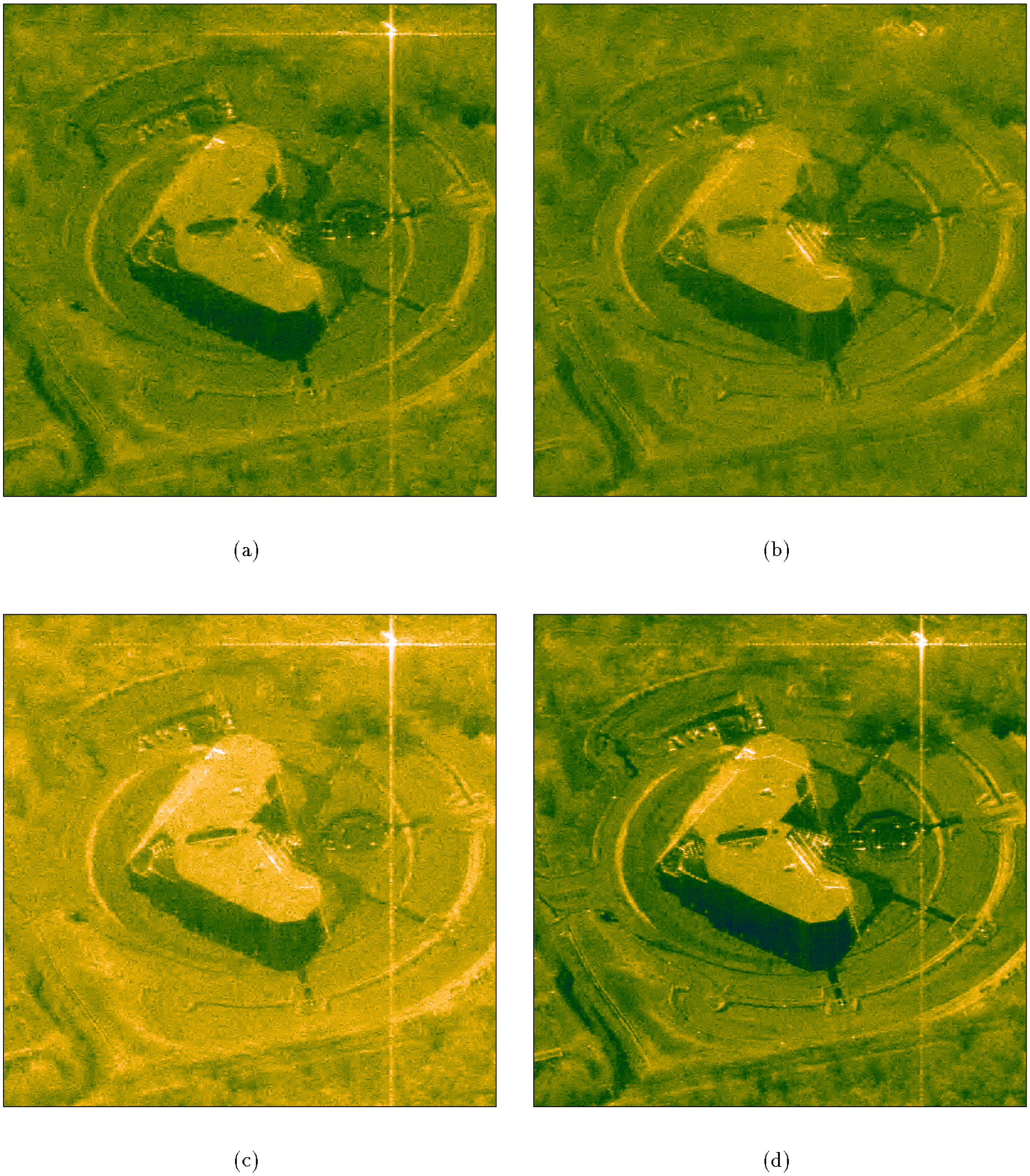

Figure 2: Wavelet-based despeckled SAR images of the Lincoln North building (a) HH polarization (b) HV polarization (c) VV polarization (d) PWF.

that under certain conditions level dependent thresholding is optimal in a minimax way.

4.1 JPEG enhanced performance

To evaluate the performance of the post-processing algorithm we have considered several different images from the standard gray scale image processing collection. For each image considered we computed the optimal parameters (filter length (N), number of levels (L), threshold factor (λ)) assuming a 2-band Daubechies type wavelet transform. The optimality criteria is maximal peak signal to noise ratio (PSNR) defined by

$$\mathrm{PSNR} = 20 \log_{10} \frac{255}{\mathrm{RMSE}} \tag{6}$$

where for an $N \times M$ image, $x(i,j)$, the RMSE (root mean squared error) is defined by

$$\mathrm{RMSE} = \sqrt{\frac{1}{NM} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \{x(i,j) - \hat{x}(i,j)\}^2} \tag{7}$$

and $\hat{x}(i,j)$ is the pixel values of the reconstructed or enhanced image. For each of the 4 images (Lenna 512×512, Camera 256×256, Mandrill 480×480, Boats 576×720) considered we choose three different JPEG quality levels (50, 10 and 5) corresponding to three different bits per pixel (bpp) values. The results are tabulated in Table 2 together with the PSNR for the plain JPEG. Notice furthermore that for each of the 4 images, JPEG quality levels 10 and 5 were intentionally chosen to correspond to significant distortion (i.e., the JPEG code issues a warning that the quantization tables is too coarse for baseline JPEG).

In Figs. 3-5 we show the image "Boats" to further illustrate the improvement from wavelet based noise reduction. In Fig. 3 we see the original image, while Figs. 4 and 5 corresponds to JPEG and denoised JPEG respectively at 0.26bpp. Carefully studying these two images one observe clear perceptual improvements in the image in Fig. 5. Furthermore, also notice that the image detail (high frequency information not removed by JPEG) is remarkably well preserved (e.g., the image is not smoothed). The images as displayed here used the parameter from Table 2. The image quality could have been improved by perceptually adjusting the threshold parameter λ, however, we found this to be rather subjective and hence left the results corresponding to the the maximum PSNR.

The algorithm described here for improving distorted JPEG encoded images is entirely separate form the JPEG coder/decoder and hence no novel coder/decoder has to be designed. However, if one was willing to add a few bytes to the JPEG data-stream one could get exact estimates of the noise variance by subtracting the images at the encoder and computing the error variance explicitly. In most cases tested this did not give significant improvements. This added information would also be more important (possibly necessary) if a level dependent thresholding was to be used. In that case getting reliable estimates of the variance at course levels would be difficult due to signal leakage into the subbands.

Figure 3: Original image (8bpp).

Figure 4: JPEG encoded and decompressed image (0.26bpp).

Figure 5: Post-processed JPEG encoded and decompressed image (0.26bpp).

Table 2: Comparison of JPEG performance and wavelet denoised JPEG performance with corresponding optimal parameters. The parameter optimization was limited to considering the class of 2-band Daubechies (maximally vanishing moments) wavelet filters restricted to lengths $4 \leq N \leq 12$, and levels $3 \leq L \leq 5$ (WD = Wavelet Denoised).

						PSNR		
	bpp	N	L	λ	$\sqrt{2\log(n)}$	WD	JPEG	Δ
Lenna	0.65	12	3	1.0	3.3	36.10	35.80	0.30
	0.25	8	5	2.5		31.09	30.41	0.68
	0.18	8	5	4.8		28.31	27.33	0.98
Camera	0.88	4	3	0.4	3.1	32.02	31.75	0.27
	0.34	6	3	1.1		26.70	26.44	0.26
	0.24	6	5	2.2		24.58	24.30	0.28
Mandrill	1.84	12	3	0.2	3.3	25.86	25.64	0.22
	0.61	12	3	0.9		20.82	20.67	0.15
	0.35	8	5	1.3		19.24	19.01	0.23
Boat	0.67	6	3	0.8	3.4	36.27	35.91	0.36
	0.26	8	5	2.7		30.32	29.74	0.58
	0.17	8	5	5.1		27.49	26.66	0.83

5 Summary

In this paper we have described two applications of the recently published results on wavelet based noise reduction. Each of the two problems considered addresses two different aspects of the theory – additive white noise and additive "non-white" noise. To no surprise the theory works remarkably well in reducing speckle noise (well modeled as AWGN) from SAR images. This has significant consequences for both military applications of SAR (automatic target detection and recognition) as well as for scientific use of SAR imagery. The second application is the removal of coding artifacts (blocking) from JPEG encoded still images. At high compression ratios the blocking artifacts tends to be quit objectionable although the image detail might be sufficiently preserved for browsing etc. JPEG coding artifacts are clearly not modeled well by a AWGN process and hence the success of this problem illustrates the power of the wavelet base noise reduction algorithm. By applying the wavelet based noise reduction theory we have been able to obtain a perceptually superior image with a post processing algorithm. The key point is that this require no novel decoder and works with all existing JPEG implementations which are readily available on VLSI chips.

Recent results [20] indicates that the noise reduction performance can be further improved by combining the redundant wavelet transform [1, 2, 28] and nonlinear processing [27, 5].

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