SIMULTANEOUS NOISE REDUCTION AND SAR IMAGE DATA COMPRESSION USING BEST WAVELET PACKET BASIS

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ABSTRACT

We propose a novel method for simultaneous noise reduction and data compression based on shrinking, quantizing and coding the wavelet packet (WP) coefficients. A dynamic programming and fast pruning algorithm is used to efficiently choose the best basis from the entire library of admissible WP bases, and jointly optimize the bit allocation strategy and the quantization scheme in the rate-distortion framework. Soft-thresholding in the wavelet domain can significantly suppress noise, e.g., the speckles of the synthetic aperture radar images, while maintaining bright reflections for subsequent detection and recognition. Optimal bit allocation, quantization and entropy coding achieve the goal of compression while maintaining the fidelity of the image.

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1. INTRODUCTION

Synthetic aperture radar (SAR) is an active coherent all-weather imaging system that operates in the microwave region of the spectrum. There are two problems in the practical SAR applications. One is the speckle phenomena. Speckle results from the necessity of creating the image with coherent radiation. A fully developed speckle pattern appears chaotic and unordered. When the detail information in the image is important, speckle can be viewed as a noise that causes degradation of the image. Therefore, speckle reduction is an essential procedure before the procedures of automatic target detection and recognition. Another problem is the large amount of SAR data. The data collected and processed by a SAR system are inherently complex. Thus, data compression is desirable for quick transmission of the collected information.

The wavelet transform is a relatively new technique for multi-resolution decomposition of images and is widely used in both noise reduction and data compression of SAR images [1, 2, 3]. Thus, it is very efficient to combine the procedures of noise reduction and data compression in a single process of decomposition and reconstruction. As a generalization of the wavelet basis, the WP, which is a rich family of orthonormal bases, can be expected to be more suited to match the non-stationary statistics of the images. Therefore, it is desirable to fast select the optimal WP basis under some criterion and to achieve better performances of de-noising and compression.

We apply a dynamic programming and fast pruning algorithm to efficiently choose the best basis that jointly optimize the bit allocation strategy and the quantization scheme in the rate-distortion sense. Our method can be viewed as not only a generalization but also a combination of the methods in [1] and [3]. Our method is different from that in [4], which chooses the optimal WP basis using the minimum description length criterion and only optimizes the thresholding of the WP coefficients rather than quantization and entropy coding.

Section 2 introduces fundamental background of image coding based on the WP decomposition. In Section 3, denoising via wavelet shrinkage is described. We discuss our novel algorithm for de-speckling and coding of SAR images in Section 4. Section 5 describes our experimental results. Finally, summary follows in Section 6.

2. IMAGE CODING USING WAVELET PACKET TRANSFORM

The WP bases [5] were introduced recently as a collection of orthonormal bases for discrete functions of $\mathbf{R}^N$. This library contains the well-known wavelet basis, short-time-Fourier-transform-like basis, Walsh functions, and smooth versions of Walsh functions. The library of WP bases organizes itself into a homogeneous tree (e.g., for two-dimensional signals such as images, this library has a structure of a complete quad-tree.), which can be efficiently searched for a best basis under some optimality criterion. The entire WP tree can be obtained by recursively decomposition of both the low-pass subband and the high-pass subband using a pair of wavelet filters. Each admissible basis appears as a unique subtree formed by pruning the whole WP tree in some way, corresponding to a unique subband decomposition. Due to the non-stationary behavior of images, any particular choice of subband decompositions, including the widely used wavelet decomposition, does not provide the optimal WP basis that best adapt to a given image. On the other hand, it can be shown that there are more than 2^N WP bases for a given signal of size N. Therefore, fast search algorithm is essential. A key point for this class of "best-basis" algorithms is that the cost functional $M(\cdot)$ should be additive, i.e., $M(0) = 0$ and $M(\sum_i X_i) = \sum_i M(X_i)$, so that it can split nicely across the Cartesian product. Thus the search is a fast divide-and-

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conquer.

An entropy-based algorithm for best-basis selection was proposed [5]. It chooses the basis with the minimum Shannon-Weaver entropy, which is a measure of the energy distribution of an unquantized vector and is not directly related to quantization and coding. Therefore, this criterion does not guarantee the optimality in the rate-distortion sense, which is the measure of the "true" performance of data compression. In [6] a generalized algorithm using the rate-distortion framework was proposed. Their algorithm is indeed a combination of the concept of orthonormal tiling of the spatial-frequency plane using the WP and the discipline of rate-distortion optimal bit allocation [7]. However, the computational complexity is much higher than the entropy-based algorithm.

A major concern of a subband coding system is bit allocation, the task of distributing a given quota of bits to subbands to optimize the overall coding performance. Let $W_i(b)$ denote the distortion incurred in optimally quantizing the ith subband with b bits of resolution. Let B be the given fixed quota of available bits. We define the overall distortion, D, and the total bit rate, R as functions of the bit allocation vector, $\mathbf{b} = (b_1, b_2, \ldots, b_k)$. The bit allocation problem is to find $\mathbf{b}$ that minimizes $D(\mathbf{b}) = \sum_{i=1}^{K} W_i(b_i)$ subject to the constraint that $R(\mathbf{b}) = \sum_{i=1}^{K} b_i \leq B$.

The "hard" constrained problem of minimizing the total distortion D for a target bits budget R (or vice versa) can be converted to a relatively "easy" equivalent unconstrained problem by using Lagrange multiplier λ [7]. Thus the unconstrained problem becomes the minimization of the Lagrangian cost function defined as $J(\lambda) = D + \lambda R$.

It can be shown that the necessary condition for rate-distortion optimality is that all subbands have the same slope point λ. Based on this condition, we can sweep λ from zero. For a given λ, we compute all the bit rates b_i's of subbands resulting the minimum Lagrangian costs and sum them together to obtain the total bit rate. We iterate this process until we achieve the required bit rate.

In our method, we choose the uniform scalar quantizer (USQ). There are two advantages of using USQ: (i) it is very easy and fast to implement a USQ alleviating the computational complexity for the rate-distortion curves. The trade-off between computational complexity and performance is very crucial in the applications of compression of very large data set. (ii) it is easy to combine the procedure of wavelet domain soft-thresholding, which does good to both de-noising and data compression, with the procedure of uniform scalar quantization. In our algorithm, each subband is assigned a finite set of quantizers with different quantization steps. Hence, the rate-versus-distortion functions of subbands can be easily computed. We use mean-squared error (MSE) and first-order entropy as the measures of distortion and rate, respectively. The weighted MSE criterion related to the model of human vision system (HVS) can also be used to further improve the visual quality of the reconstructed images.

3. WAVELET THRESHOLDING

A nonlinear method was proposed for reconstructing an unknown signal from noisy data [8]. The method attempts to reject noise by damping or thresholding in the orthogonal wavelet domain and has been proved to work well in many applications.

Suppose we wish to recover an unknown signal $\mathbf{x}$ from noisy data $\mathbf{y}$, $y_i = x_i + \sigma e_i$, $i = 0, 1, \ldots, n-1$, where $e_i \overset{iid}{\sim} \mathcal{N}(0,1)$ is a white Gaussian noise, and σ is the noise level. Let $\hat{\mathbf{x}}$ be the estimate of $\mathbf{x}$. Our goal is to optimize the mean-squared error (MSE) $\frac{1}{n}E[\| \hat{\mathbf{x}} - \mathbf{x} \|_2^2]$. The simple wavelet-domain thresholding method has three steps:

1. Compute the orthonormal DWT of the noisy data $\mathbf{y}$, obtaining the wavelet coefficients;
2. Apply the soft-thresholding nonlinearity (shrinkage)
$$\eta_t(v) = \begin{cases} v - t & \text{for } v > t \\ 0 & \text{for } -t \leq v \leq t \\ v + t & \text{for } v < -t \end{cases}$$
to the wavelet coefficients (except the coarsest level) with a specially-chosen threshold $t = t_n = \sqrt{2\log(n)}\sigma$;
3. Perform the inverse orthonormal DWT on the thresholded wavelet coefficients, recovering the estimate $\hat{\mathbf{x}}^*$.

It has been shown that this method has three distinct features: (i) the estimate $\hat{\mathbf{x}}^*$ achieves almost the minimax MSE over every one of a wide range of smoothness classes, including many classes where traditional linear estimators do not achieve the minimax rate; (ii) this procedure maintains the sharp features of $\mathbf{x}$ (e.g., the edges in images), therefore, it provides better visual quality than procedures based on the MSE alone; (iii) the estimate does not exhibit any noise-induced structures, unlike most minimum MSE methods. This de-noising method can be extended to the WP transform [9].

The speckle noise in SAR images can be approximately modeled as a multiplicative i.i.d. Gaussian noise with unit mean [10]. Therefore, after the logarithmic operation, it becomes additive i.i.d. Gaussian noise so that it can be efficiently suppressed via the above wavelet-thresholding scheme.

A particular advantage of combining the de-noising procedure and the compression procedure is that in general, soft-thresholding does good to data compression. The wavelet shrinkage decreases the dynamic range of the coefficients and thresholds those insignificant coefficients. Therefore, the entropy of the quantized wavelet coefficients is lowered. Though we could put the de-noising procedure either in the coder or in the decoder, in our scheme, we put this procedure before the quantization according to the above argument.

4. BASIC IDEA OF THE ALGORITHM

Ideally, we want to jointly optimize the performance of compression and detection. However, this is an intractable optimization problem due to the non-existance of a good measure for both data compression and noise reduction. So, our goal here is to optimally compress the SAR image in rate-distortion sense while maintaining the performance of de-noising.

Assume that the optimal WP subtree from node n "onwards" to the full tree-depth is known. Then by Bellman's optimality principle [11], we know that all possible paths passing through node n must invoke this same optimal "finishing" path. At each non-leaf node of the tree, there are two contenders for the "surviving path", the parent and its children, with the winner having the lower Lagrangian cost. According to this sub-optimality, we can apply a dynamic programming to construct the optimal subtree starting from the leaf nodes upwards. When we reach the root node, the best basis with the minimum Lagrangian cost is known.

We first perform a full WP decomposition and soft-thresholding. Then, we initialize a slope value λ and compute the rate-distortion relations and the minimum Lagrangian costs of all subbands with the admissible quantizer sets. We then start from the leave nodes of the entire WP tree and compare each node with its four child nodes. For each node, if its Lagrangian cost is less than the sum of those costs of its four child nodes, we mark this node as a "merge" node; otherwise, we mark this node as a "split" node and update its information (Lagrangian cost and the corresponding rate and distortion) with those of its child nodes. We recursively do the comparisons until we reach the root node. Then, we can curve out the optimal subtree for this λ by pruning all the sub-trees rooted at those "merge" nodes and the corresponding rates and distortions. We can repeat the above process using a bisection search to find the proper λ so that the resulting rate is identical to the desired bit budget.

The computational complexity of computing the subband distortion-rate curves counts for a significant percentage of the total cost of our algorithm. We propose a pruning method based on the fact that the distortion-rate function is monotonically decreasing and convex, to fast search for the minimum Lagrangian cost in each subband image. We ignore the straightforward proof.

Assume that $\{R_i\}$ is a set of admissible bit-rates for a certain subband image, satisfying $R_i < R_2 < \cdots < R_{k-1} < R_k < R_{k+1} < \cdots$. Define the Lagrangian cost $J(R_i) = D(R_i) + \lambda R_i$, where $D(\cdot)$ is the distortion-rate function and λ is the Lagrangian multiplier. The goal is to find

$$R^* = \arg\min_{R_i} J(R_i) = \arg\min_{R_i} [D(R_i) + \lambda R_i].$$

We claim the following two pruning conditions to accelerate the search:

Condition 1: $J(R_{k-1}) < J(R_k) \Rightarrow J(R_{k-1}) < J(R_{k+1})$;

Condition 2: $J(R_k) > J(R_{k+1}) \Rightarrow J(R_{k-1}) > J(R_{k+1})$.

Since we include the de-noising procedure in our coding algorithm, which obviously causes some error between the de-noised image and the original image, we use the MSE between the coefficients of the de-noised image and those of the decompressed image as the distortion.

In the above algorithm, the computational complexity is mainly determined by the size of the image. In most applications, we only decompose the image up to $4 \sim 6$ levels. The number of admissible quantizers for each subband is also a small constant compared to the size of image. Also, the number of iterations for bisection search of a convex curve is relatively small. Therefore, the total computational complexity of our algorithm is $O(MN)$ for an image of size $M \times N$.

5. EXPERIMENTAL RESULTS

We apply our algorithm to a SAR image provided by the Lincoln Laboratory, which is an 800×800, 256-gray-scale image. The de-noised image is compressed at 0.126 bits-per-pixel with both the JPEG algorithm and our WP algorithm for the purpose of comparison. The speckle phenomena is clearly visible in the original image (Figure 1) and greatly removed in the de-noised image without visual loss of detail features (Figure 2). The severe blocking effect can be clearly viewed in the JPEG-compressed image with a PSNR $34.52dB$ (Figure 3). The WP-compressed image is free of any artifact and with a much higher PSNR, $38.18dB$ (Figure 4).

6. SUMMARY

An efficient algorithm is proposed for representing the SAR image with the optimal WP basis in the rate-distortion framework, and simultaneously suppress the speckle noise and compress the image data, while maintaining the resolution and sharp features of the original image. The reconstructed image will significantly improve the performances of detection, classification and recognition, and require much fewer number of bits for transmission and storage.

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Figure 1. Original SAR image

Figure 2. De-noised image

Figure 3. Compressed image using JPEG

Figure 4. Compressed image using WP