Adaptive Spectral Optimization for Communication in the Presence of Crosstalk

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Abstract

Digital Subscriber Line technology is a copper loop transmission technology that solves the bottleneck problem often associated with the last mile between network service providers and user of those services. DSL achieves broadband speeds over ordinary twisted-pair copper wire, an important trait as it eliminates the need for extensive and expensive infrastructure upgrades[4].

The electrical energy transmitted across copper wires as modulated signals, radiate energy into adjacent wires in the same cable binder, an effect known as crosstalk. Recently, optimal signaling techniques have been discovered, giving the best possible power distribution over frequency that minimizes crosstalk interference.

Previous work has been done in a static environment, with a priori knowledge of other the environment. In most cases, this knowledge is not known. This paper investigates methods of adaptive spectral optimization, where the frequency spectra of DSL modems are continually adapting as their knowledge of their environment change. Of special importance is the fact that the adaptive technique does indeed converge to the optimal signal spectra.

1 Introduction

There are many broadband techniques competing for local loop or 'last mile' service. Digital Subscriber Line (DSL) technology is gaining acceptance as a viable way to meet the continually increasing demand for bandwidth.

DSL eliminates the need for expensive infrastructure upgrades, which makes it a very cost effective method of broadband access. It possesses the ability to transform the nearly 700 million installed telephone based on copper wire, the world's most common network medium, into high speed data pipes.

1.1 The Telephone Network

High capacity links exist between telephone company offices due to extensive infrastructure upgrades that have taken place over several decades. Fibre optics exist in nearly every telephone backbone network, while twisted pair copper wire connect most end users to the telephone network. The local access network consists of the local loops and the associated equipment connecting the end user to the central office.

One problem with local loops is that they dissipate electrical energy from signals as they travel along the length of the cable. Signals typically have much lower signal to noise ratios (SNR) the farther along the cable they are.

These same copper wires have been used to provide high speed T1 access. Traditional T1 modulation techniques over copper is only supported over short distances. Longer distances require concatenation of short segments separated by repeaters that regenerate the signal[4].

The existing copper infrastructure was designed to carry voice conversations, which for adequate fidelity, requires lines to handle frequencies up to 3400Hz. Analog modems running over this narrow band of frequencies typically provide data rates ranging from 9.6 to 33.6 Kbps.

1.2 High Speed Access Using DSL

High frequency signals transmitted over metallic loops attenuate energy faster than low frequency signals. One solution to this problem is to use thicker wires, which have lower resistance and attenuate to a lesser degree. However, the infrastructure is already in place and phone companies have designed their systems using the thinnest wire that support the voice service they intended to provide.

Newer, more advanced modulation techniques minimize the effect of signal attenuation. By encoding more bits per cycle of the analog waveform, the range of frequencies required to achieve a desired data rate is reduced. Signaling techniques such as Discrete Multitone (DMT) modulation, 2 Binary, 1 Quaternary (2B1Q) and Carrierless Amplitude and Phase (CAP) modulation are designed to transmit multiple bits per signal cycle.

However, the current T1 carrier technology, does not meet the needs of a mass communications service because of its high cost, special requirements on the subscribers internal wiring, and long, complicated network provisioning process[3].

For low-cost, timely provisioning of high speed service, there is a need for existing telephone lines to transport high speed data, without the costly and complicated methods used for T1 deployment.

DSL is ideal for this situation, as copper-based services will dominate fibre optics for many years. DSL will ease the transition to fibre access by accelerating the use of higher speed services.

2. Background

2.1 Terminology and Notation

To achieve higher transmission speeds, DSL eliminates the 3400Hz upper bound on frequencies and uses a much broader range than the traditional voice channel.

Crosstalk is a significant problem associated with high frequencies. The energy of the modulated signal radiates into adjacent copper wires in the same cable binder. Significant crosstalk is created when systems within the same cable binder, transmit information over the same range of frequencies.

Crosstalk is typically categorized in one of two forms. Near End Crosstalk (NEXT) and Far End Crosstalk (FEXT). NEXT refers to interference between neighboring lines that arise when signals are transmitted in opposite directions. FEXT refers to interference between neighboring lines that arises when signals are transmitted in the same direction. NEXT, typically causes more interference, as FEXT is also attenuated as the signal propagates the length of the line.

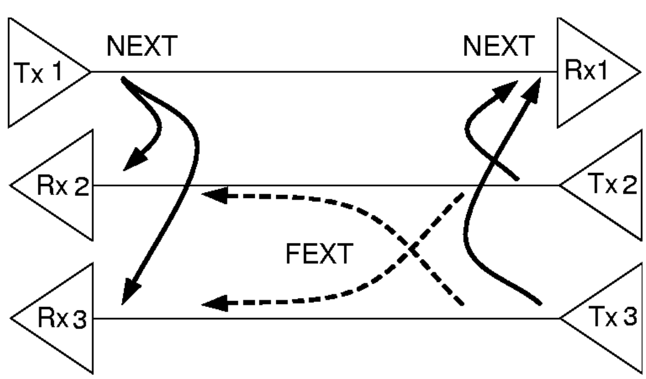

Figure 1: NEXT and FEXT interference between lines

Interference can be further subdivided into self-interference, and interference from other services. Self-interference refers is generated by lines carrying the same service, i.e. other DSL modems carrying the same service. This will be denoted self-NEXT and self-FEXT. Interference from other sources will be denoted DSIN-NEXT and DSIN-FEXT.

Channel capacity is defined as the maximum transmission rate (in bits per second) that can transmitted over a channel with an arbitrarily small bit-error rate (BER).

Performance margin is defined as

$$margin = 10\log_{10}\left(\frac{SNR_{rec}}{SNR_{min}}\right) \tag{1}$$

The argument of the log term is the ratio of the SNRs of the received signal and the minimum SNR required to achieve a given BER. The margin is a measure of degradation that the channel can sustain before being unable to transmit at the given rate for a fixed BER. The higher the performance margin, the more robust the channel is to interference[1].

The distribution of signal energy over frequency is known as the Power Spectral Density (PSD). The maximum allowable PSD for a service in the presence of any interference combination is known as a PSD mask. The transmit spectrum for a service is the PSD of the transmitted signal. The transmit spectrum always lies below the PSD mask, when one is imposed.

2.2 Problem Definition and Signaling Schemes

The general problem statement can be described as follows. Given an arbitrary communications channel with:

1. Self-Interference from other users of service A (self-NEXT,self-FEXT).
2. Interference from users of other services (DSIN-NEXT, DSIN-FEXT).
3. Interference from service A into other services.
4. Other interference including noise.

The goal is to maximize the capacity of users using service A, while minimizing the performance degradation of other services.

As previously mentioned NEXT is the dominant interference in DSL service. A simple way to eliminate self-NEXT is to use orthogonal signaling. This could be in the form of Time Division Signaling (TDS), Frequency Division Signaling (FDS), or Code Division Signaling (CDS). In previous work, it has been shown that under a power constraint, that FDS is the optimal signaling technique[1].

To eliminate self-NEXT using FDS, we can force upstream and downstream transmitters to use disjoint frequency bands. The upstream and downstream transmissions are orthogonal and can be easily separated by their respective receivers. Moving to an FDS scheme reduces the bandwidth available to each transmitter by 1/2 of the original bandwidth. There is tradeoff, FDS eliminates self-NEXT, increasing capacity, while also reducing the bandwidth available and decreasing the capacity. Therefore, when self-NEXT is not high enough to warrant FDS, both upstream and downstream transmitters should have the same spectrum, this is referred to and Equal PSD (EQPSD) signaling.

2.3 Signaling Schemes

Previous work[1] found joint signaling schemes for a particular configuration of DSL modems and interferors from different services. These schemes have been proven optimal for several cases.

1. Interference from different services, additive gaussian noise and no self-interference.
   Solution: EQPSD signaling.
2. Interference from different services, additive gaussian noise, self-NEXT and low self-FEXT.
   Solution: EQPSD and FDS signaling.
3. Interference from different services, additive gaussian noise, self-NEXT and high self-FEXT.
   Solution: EQPSD, FDS, and multi-line FDS signaling.

In the first case, when crosstalk and noise are not significant, EQPSD signaling is the scheme that yields maximum capacity. When self-NEXT is high and self-FEXT is low, as in the second case, FDS signaling yields the highest capacity. The available frequency is divided into two halves, with one used for the upstream transmitter and the other for the downstream. This eliminates self-NEXT but does not combat self-FEXT, which is small enough to not be an issue. When both self-NEXT and self-FEXT are high, multi-line FDS signaling is used. Each line receives a separate frequency slot and put both upstream and downstream spectra in the assigned band. This eliminates self-NEXT and self-FEXT as the upstream and downstream bands of each line are orthogonal.

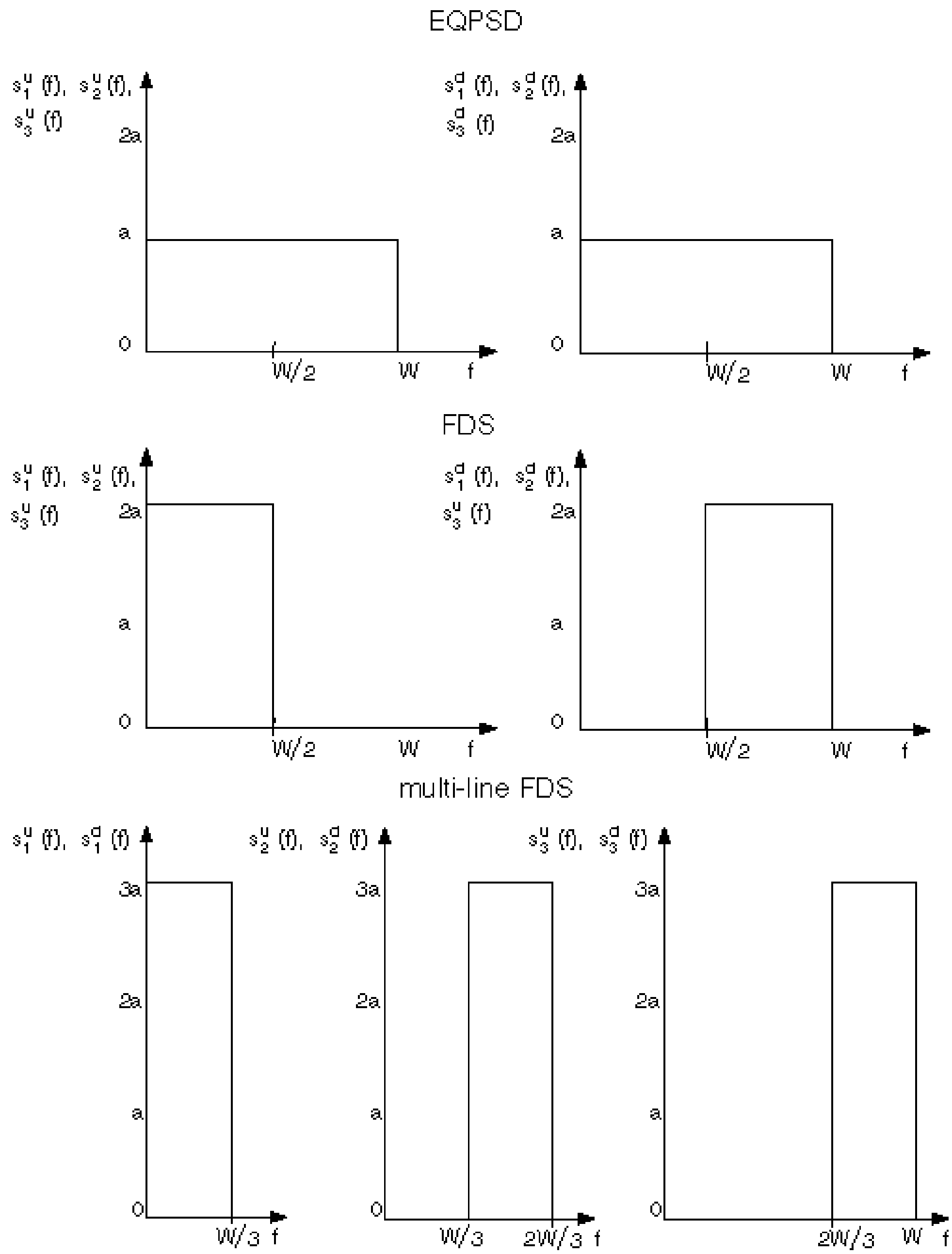

Figure 2: EQPSD, FDS and multiline FDS

3. Spectral Optimization and Joint Signaling Techniques

In a typical telephone channel, the severity of self-NEXT varies with frequency. Switching between FDS and EQPSD as warranted by the severity of the self-NEXT, would yield maximum capacity.

There are 3 cases of interest for which optimal joint signaling schemes have be devised. With multiline FDS we separate each transmitter into a separate frequency band. This greatly reduces the bandwidth available to each transmitter. In practical implementations, its is only worth moving to a multiline FDS solution when self-FEXT is very high and when the number of lines is small. When the number of lines increases, the available bandwidth becomes too small to increase capacity in spite of the fact that self-FEXT is eliminated.

As a simplifcation of the problem only cases 1 and 2 are examined. We will assume that self-FEXT is low enough to warrant not switching to a multiline FDS scheme. Furthermore, case 1 can be thought of as a degenerate example of Case 2, ie. self-NEXT and self-FEXT are so low, that the EQPSD solution is optimal. This leaves one case of interest, the situation where there is self-NEXT present, and a low amount self-FEXT. A more detailed look at joint signaling solution to this problem follows.

3.1 Important Assumptions Made in Previous Work

1. Channel noise can be modeled as additive Gaussian noise (AGN).
2. Interference from other services can be modeled as additive colored Gaussian noise.
3. The total transmission bandwidth B (Hz), can be further subdivided into narrow frequency bins of width W (Hz), and the channel, noise, and crosstalk characteristics vary slowly and can be considered constant within the bin. For line i, bin k, we have:

$$|H_C(f)|^2 = \begin{cases} H_{i,k} & |f - f_k| \le \frac{W}{2} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$|H_N(f)|^2 = \begin{cases} X_{i,k} & |f - f_k| \le \frac{W}{2} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$|H_F(f)|^2 = \begin{cases} F_{i,k} & |f - f_k| \le \frac{W}{2} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

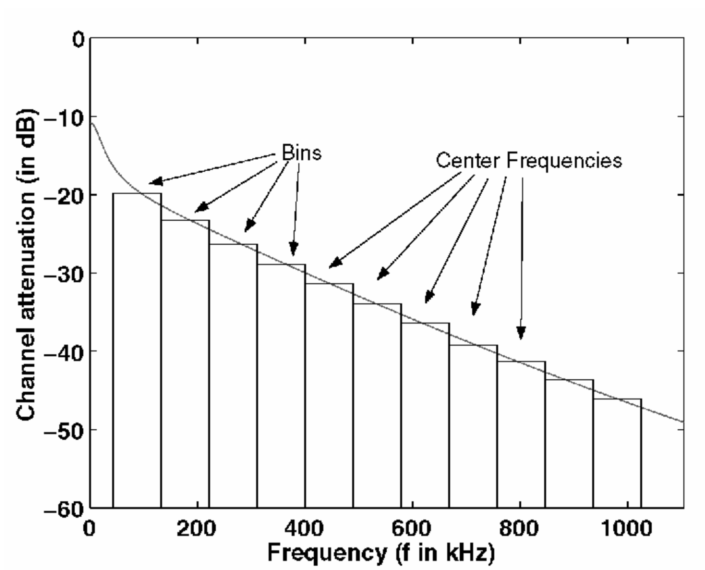

Figure 3: Channel subdivision in to K bins

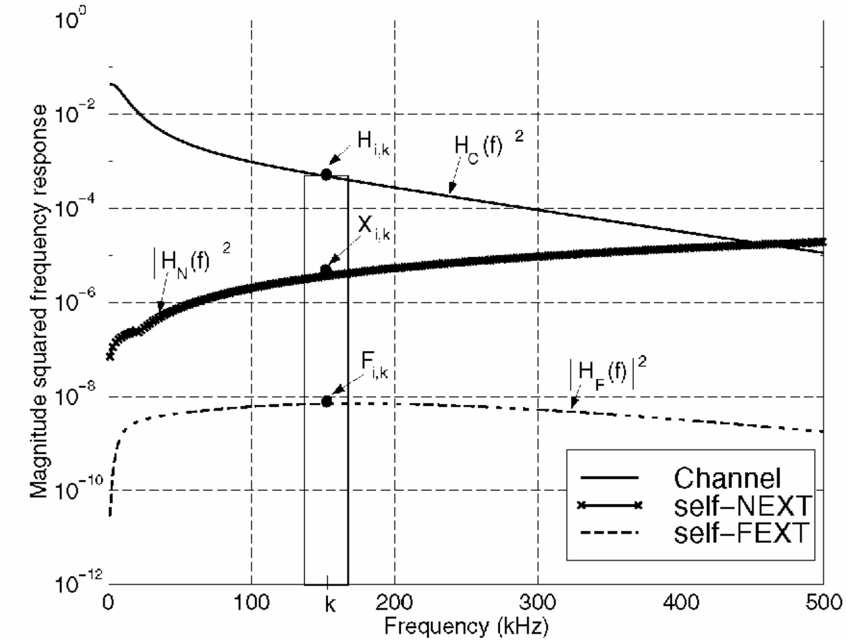

Figure 4: Magnitude squared transfer function of channel, NEXT and FEXT

4. Echo cancellation is being used effectively to cancel crosstalk in a particular line from a transmitter in either the upstream or downstream direction to the receiver in the opposite direction.
5. All sources of DSIN-NEXT can be grouped into one PSD, $DS_N(f)$, and all sources of DSIN-FEXT can be grouped in to another PSD, $DS_F(f)$.
6. All sources of self-NEXT can be added to form one overall self-NEXT source.
7. All sources of self-FEXT can be added to form one overall self-FEXT source.
8. Spectral optimization is done under the average input power constraint.
9. Notation. Let u=upstream, and d=downstream, and denote the PSDs on line i as:

   $S^u_i(f)$: PSD on twisted pair i in upstream direction u.
   $S^d_i(f)$: PSD on twisted pair i in downstream direction d.

   Let the PSDs on line i in frequency bin k, be denoted as:

   $s^u_i(f)$: PSD on twisted pair i in upstream direction u.
   $s^d_i(f)$: PSD on twisted pair i in downstream direction d.

10. Assume a monotonic decreasing channel transfer function.
11. Assume equal channel capacities in the upstream and downstream directions.

3.2 Optimization: DSIN-NEXT, DSIN-FEXT, AGN, High Self-NEXT, Low Self-FEXT

Orthogonal signaling schemes such as FDS can completely eliminate self-NEXT. FDS can give better spectral compatibility than other orthogonal signaling techniques[1]. Additional assumptions that must be used in this derivation are:

12. The self-FEXT level is low enough in all bins that multiline FSD is not required.
13. All M lines have the same channel transfer function, noise characteristics, face the same DSIN-NEXT and DSIN-FEXT in both directions. Therefore, all the upstream PSDs are the same, as are the downstream PSDs.
14. All lines have the same self-NEXT and self-FEXT transfer functions.

3.2.1 Signaling Scheme

As previously mentioned, there is a tradeoff moving to an FDS signaling technique. FDS reduces the available bandwidth and therefore reduces system capacity, while it combats self-NEXT and increases system capacity. In high self-NEXT regions, FDS must be used to reject the effects of self-NEXT. When self-NEXT is not very high, moving to FDS may reduce capacity as the bandwidth available is halved. To maximize capacity we must only use FDS in the regions where the self-FEXT is high enough to warrant it, otherwise use EQPSD.

In the case of 2 lines, the upstream and downstream capacities are as follows:

$$C^u = \sup_{S^u(f),S^d(f)} \int_0^\infty \log_2\left[1+\frac{|H_C(f)|^2 S^u(f)}{N_o(f)+DS_N(f)+DS_F(f)+|H_N(f)|^2 S^d(f)+|H_F(f)|^2 S^u(f)}\right] df, \tag{5}$$

and

$$C^d = \sup_{S^u(f),S^d(f)} \int_0^\infty \log_2\left[1+\frac{|H_C(f)|^2 S^d(f)}{N_o(f)+DS_N(f)+DS_F(f)+|H_N(f)|^2 S^u(f)+|H_F(f)|^2 S^d(f)}\right] df. \tag{6}$$

To simplify analysis, we subdivide the channel into bins of width W, and design each bin individually. The achievable rate for each bin then becomes:

$$R_A(s^u(f),s^d(f)) = \int_0^W \log_2\left[1+\frac{s^u(f)H}{N+s^d(f)X+s^u(f)F}\right] df, \tag{7}$$

Where

$$H = H_{i,k}, \quad X = X_{i,k}, \quad F = F_{i,k} \tag{8}$$

and

$$N = N_o(f_k) + DS_N(f_k) + DS_F(f_k), \tag{9}$$

and $s_u(f)$ and $s_d(f)$ are the upstream and downstream regions of the PSD are as follows:

$$s^u(f) = \begin{cases} \alpha \frac{2P_m}{W} & \text{if } |f| \le \frac{W}{2}, \\ (1-\alpha)\frac{2P_m}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$s^d(f) = \begin{cases} (1-\alpha)\frac{2P_m}{W} & \text{if } |f| \le \frac{W}{2}, \\ \alpha \frac{2P_m}{W} & \text{if } \frac{W}{2} < |f| \le W, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Here α is a parameter that determines if we use EQPSD, FDS or any other signaling scheme in that particular bin and P_m is the average power in the frequency range [0,W]. It has been shown[1] that to maximize capacity that either EQPSD or FDS signaling has to be used in every bin. The test condition for which method to use is:

$$\text{If } X^2 - F^2 - HF < 0 \text{ then}$$

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{EQPSD}}{\gtrless}} \frac{H - 2(X-F)}{X^2 - F^2 - HF}. \tag{12}$$

$$\text{If } X^2 - F^2 - HF > 0 \text{ then}$$

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{EQPSD}}{\lessgtr}} \frac{H - 2(X-F)}{X^2 - F^2 - HF}. \tag{13}$$

Therefore in each bin the upstream and downstream capacities are

$$c^u = \begin{cases} W \log_2 \left[1 + \frac{P_m H}{NW + P_m(X+F)}\right], & \text{if } \alpha = 0.5, \\ \\ \frac{W}{2} \log_2 \left[1 + \frac{P_m H}{N\frac{W}{2} + P_m F}\right], & \text{if } \alpha = 1. \end{cases} \tag{14}$$

Due to the channel monotonicity assumptions that were made, we will assume all bins in $[1,M_E]$ satisfy the above conditions such that EQPSD signaling is used. Bins from $[M_F,K]$ where K is the last bin, use FDS. There is an optimal switchover point, M_{E2F}, in the interval (M_E,M_F) where EQPSD will change to FDS.

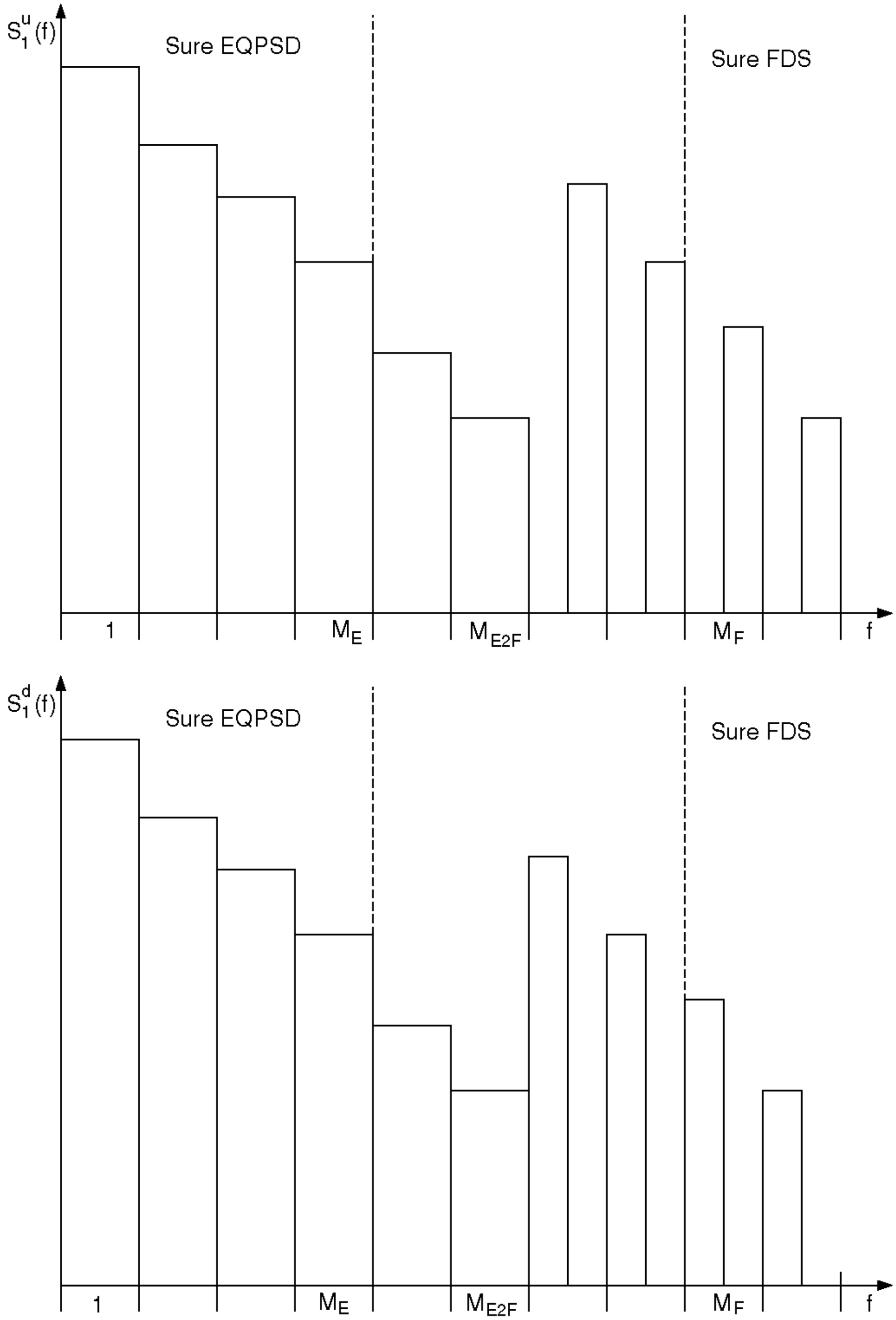

Figure 5: EQPSD and FDS regions in Upstream and Downstream Spectra

3.2.2 Overall Algorithm

1. Set up K equispaced bins of width W, over the transmission bandwidth B.
2. Estimate DSIN-NEXT, DSIN-FEXT, self-NEXT, self-FEXT, and AGN.
3. Compute M_E and M_F using formulae.
4. Estimate M_{E2F}.
5. Estimate how much power, P_E, of P_{MAX} in EQPSD region and how much, P_F, in FDS region.
6. Use water-filling technique to distribute P_E and P_F in the appropriate regions, compute each subchannel capacity and then sum to find total channel capacity.
7. Re-estimate P_E and P_F.
8. Repeat 6 and 7 until channel capacity is maximized.
9. Re-estimate M_{E2F} crossover bin.
10. Repeat 5 to 9 until a maximum total capacity is found.
11. Set M_{E2F} as the final crossover bin and find the final channel capacity.

3.2.3 Fast Suboptimal Solution

In practice, it is often found that the optimal M_{E2F} crossover bin is close to the initial estimate of M_E, usually within several bins. Thus to save a significant amount of computation, we simply set M_{E2F} as M_E, and do steps 5 through 8, and then step 11, of the above algorithm.

3.2.4 Grouping of Bins

It is often simpler to regroup the bins in the FDS region in an alternate configuration, such that we form wider subchannels. This is often useful when using modulation schemes such as PAM or QAM, rather than DMT.

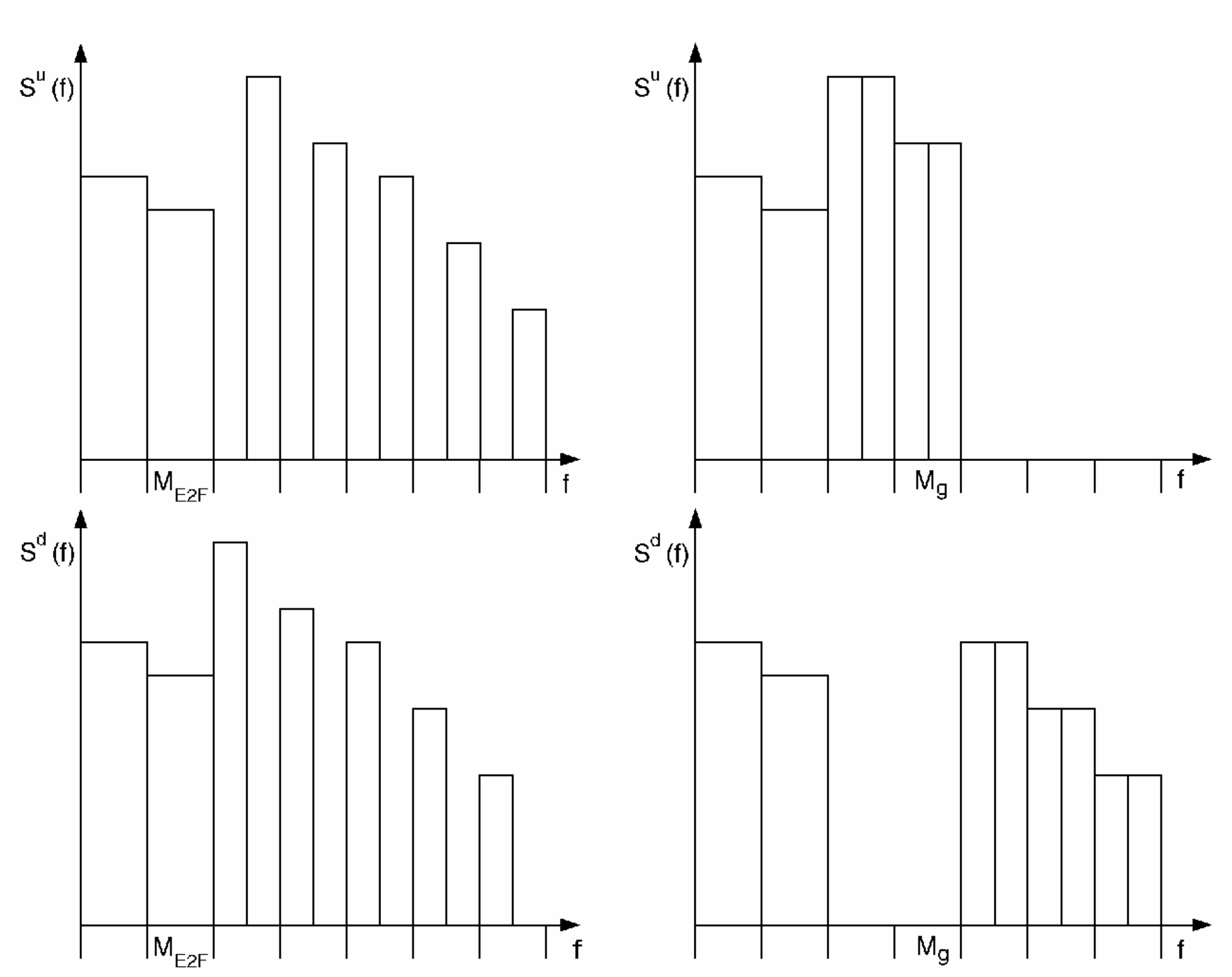

Figure 6: 'Discrete' spectra grouped into 'contiguous' spectra

When regrouping these bins, careful attention must be paid so that the condition of orthogonality of the upstream and downstream spectra in the FDS region is not violated. There are many ways in which the bins can be grouped, however, only one possible configuration creates the largest possible contiguous regions under the constraints that upstream and downstream spectra have equal power and equal performance margins (or data rates).

3.2.5 Algorithm: Equal Power and Margin Constrained Grouping

1. Solve for optimal transmit spectra.
2. Estimate bins M_C in $(M_{E2F},K]$ and M_G in $(M_C,K]$, and group the bins of the optimal spectrum such that:

$$S^u_{\mathrm{opt}}(f) = \begin{cases} S(f) & \forall f \text{ in } E_{\mathrm{EQPSD}}, \text{ and} \\ & \forall f \text{ in bins } (M_C, M_G], \\ 0 & \text{otherwise}, \end{cases} \tag{15}$$

$$S_{\mathrm{opt}}^{d}(f) = \begin{cases} S(f) & \forall f \text{ in } E_{\mathrm{EQPSD}}, \text{ and} \\ & \forall f \text{ in bins } (M_{\mathrm{E2F}}, M_C], \text{ and} \\ & \forall f \text{ in bins } (M_G, K], \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

3. M_C is chosen such that equal performance margins (equal data rates) result in both the upstream and downstream portions of the FDS region, while M_G is chosen such that both upstream and downstream directions have equal average powers. Iterate for different values of M_C (and calculating the corresponding value of M_G) until both constraints are satisfied.

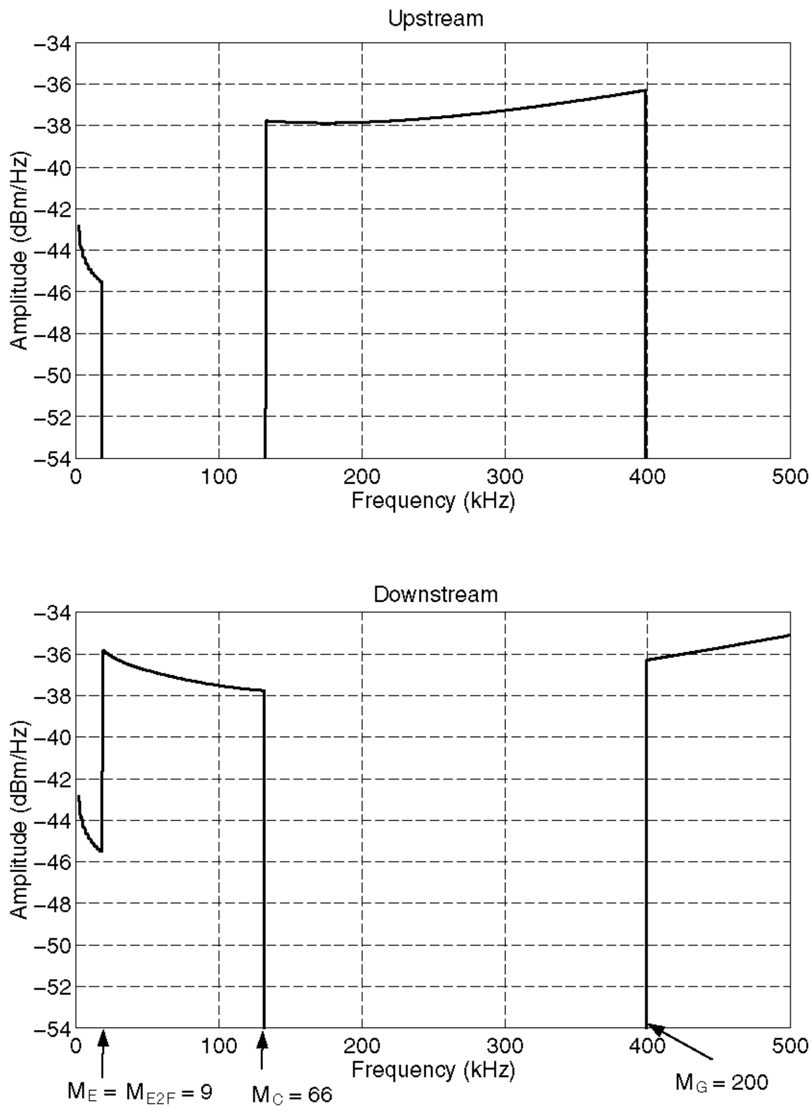

Figure 7: 'Contiguous' spectra under equal power and margin constraint

4 Adaptive Spectral Optimization

The previous work summarized is done under the assumption of a static environment, where the number of users of a service and the configuration of the interferors do not change over time. Each modem has a priori knowledge of the entire system; how many users of the same service there are, and numbers of the various interferors. This knowledge is used to calculate the total interference and compute the optimal PSD for the modems.

In practice, modems do not have this knowledge. In an adaptive system, the spectra of DSL modems continually adapt as the modems' knowledge of the environment changes. In an ideal situation, the adaptive solution will converge to the same solution that was obtained with joint optimization.

There are several advantages of a system that can converge to an optimal signaling scheme adaptively, by measuring line characteristics and taking steps to maximize capacity.

1. Modems do not need knowledge of the final configuration of users of the same service and of other interferors, they simply adapt to the environment as it changes. This is key, as the environment is continually changing as users turn the DSL service on and off. The same is true for configuration of interferors, which also changes with time.}
2. There is verification of the optimal solution based on measured line characteristics. That is, modems can change their spectra and verify which PSD results in maximum capacity, rather than on computation from formulae.
3. It will be key in future work when the assumptions that all DSL lines in a cable binder share the same channel transfer function, self-NEXT and self-FEXT transfer functions, interference from other sources (DSIN-NEXT and DSIN-FEXT), are relaxed. Analytical solutions in this case can become difficult to solve and a solution that adaptively converges to the optimal PSD is preferred.}

4.1 Algorithms

There are several steps required to converge to the optimal PSD as defined by the joint signaling method. The first is to properly define the EQPSD and FDS regions, and then ensure orthogonality of the upstream and downstream transmission in the FDS portion of the spectra.

4.1.1 Finding Optimal EQPSD and FDS Regions

Each time a change in the DSL environment occurs, we change the amount of interference, and the EQPSD to FDS crossover point may change. To keep the optimality of the signaling, we should recompute this point when environmental changes occur.

Finding M_{E2F} is an N-dimensional (where N represents the number of DSL modems) search for the EQPSD to FDS crossover point. There is a single bin that results in a maximum achievable rate.

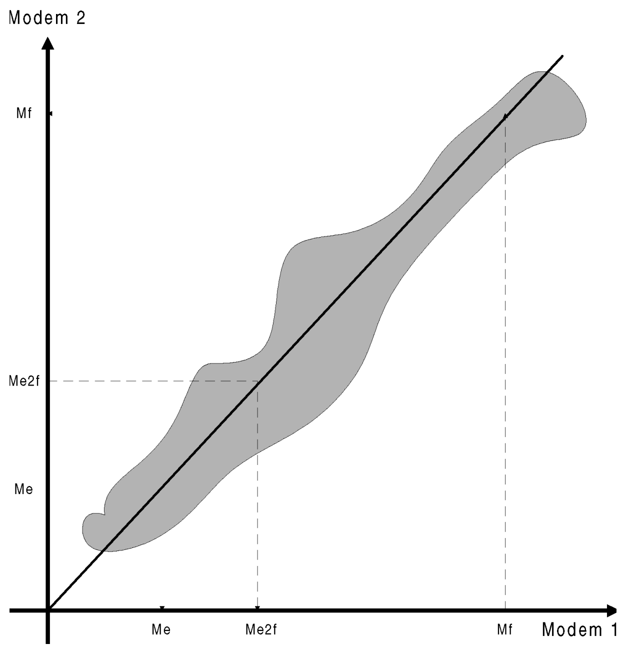

Figure 8: Search space for M_{E2F}

The algorithm attempts to find this maximum point with each modem functioning independently. Modems typically put less power per bin in the EQPSD region than in the FDS region. All upstream and downstream transmitters are sharing this region, and to minimize the effect of self-NEXT, a smaller amount of power is placed in the region. When modems do not share the same crossover point, the FDS regions of some modems will overlap with the EQPSD regions of others. The net effect is that the capacity of all modems will drop as there is too much power in the EQPSD region of some modems, resulting in higher self-NEXT terms, while at the same time the orthogonality of the FDS region of other modems is violated. Thus, it is key that all modems share the same EQPSD to FDS crossover bin.

Knowing that the capacity is maximum when the all the modems share the same crossover bin, greatly simplifies the search procedure. We do not need to search the entire N dimensional space, but rather only the region near the line that is traced out when all modems share the same crossover bin. We also know that M_{E2F} is fairly close to M_E, thus using M_E as an initial value is an intelligent starting point.

The search procedure involves searching the various combinations of crossover bins near the line defined above. By moving along this line, we will find a single value for M_{E2F} that will maximize the capacity of all modems.

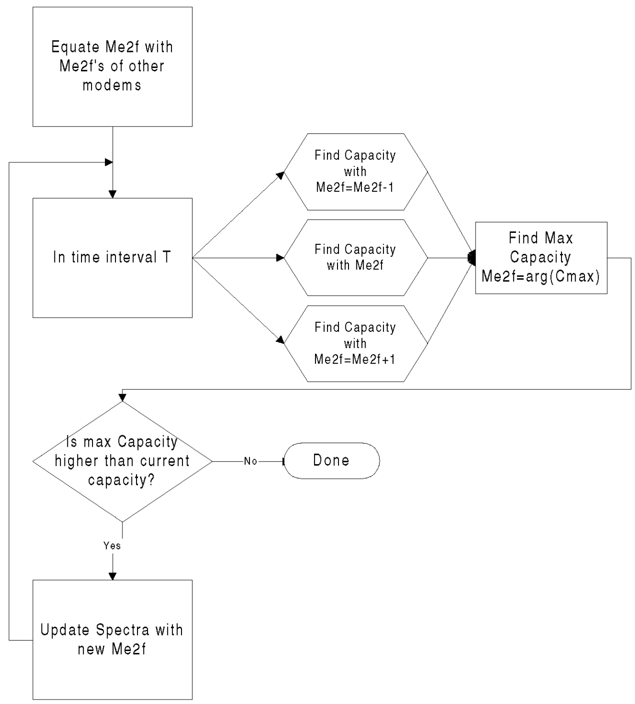

Figure 9: Algorithm for finding M_{E2F}

4.1.2 Ensuring orthogonality in FDS region

From the joint signaling solution, we know the FDS region is to have the upstream and downstream spectra orthogonal. That is, the upstream transmitters should not be signaling in the regions where the downstream transmitters are signaling. Once we place the constraints that both the upstream and downstream spectra should have equal power and equal performance margins, and that the grouping of bins should be as large as possible, there is only one optimal solution.

The algorithm starts by choosing a value for M_C and then computing M_G such that the equal power constraint is met. In the joint signaling technique, All possible values for M_C, and the corresponding M_G, are checked. Since the equal power constraint is met with the computation of M_G, we choose the M_C that gives us equal capacities in the upstream and downstream direction in the FDS region.

In the adaptive solution, each modem computes overall capacity in both upstream and downstream directions for values of M_C both above and below the current value. If a capacity is found to be higher, M_C is updated and the spectrum changed accordingly. This alone causes the spectrum to converge to a solution such that the upstream and downstream transmit spectra are orthogonal. In the FDS region, if there is an overlap in the upstream and downstream transmit spectra of some modems, then some modems are putting power in a region where self-NEXT is high. The interference due to self-NEXT becomes significant and thus reduces overall capacity, hence modems move away from this state.

Once the upstream and downstream portions of the spectra are orthogonal, the next step is to equate the upstream and downstream capacities. Spectra for values of M_C both above and below the current value are computed, and the upstream and downstream capacities are calculated. The value of M_C which minimizes the difference between upstream and downstream capacities is chosen. Once one modem makes this change, the upstream and downstream spectra between modems will no longer be orthogonal, and the other modems update their spectra accordingly. In this manner the modems 'chase' the first modem which tries to equate upstream and downstream capacities.

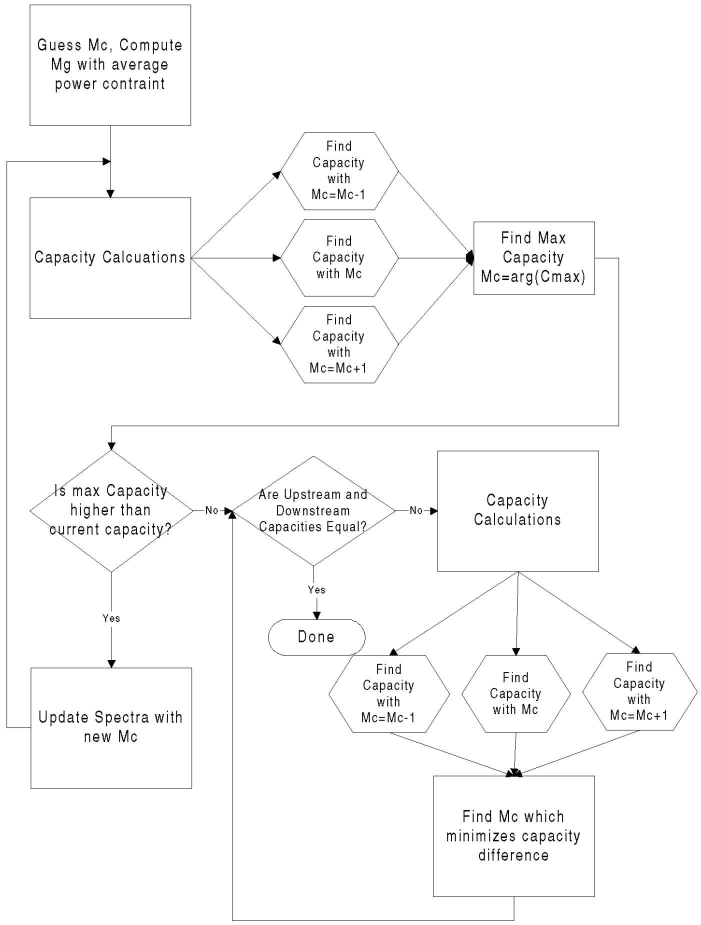

Figure 10: Algorithm for finding M_C

5 Simulations

The simulations for the adaptive algorithms were straight forward. The HDSL2 service was chosen, as it is a symmetric data service having a rate of 1.544 Mbps. The number of HDSL2 modems was set at two for simplicity, however, it can easily be changed to a different value.

Power spectral densities of the two HDSL2 modems were initially computed, and the crossover bins for each modem were offset by random amounts such that the EQPSD and FDS regions of both modems were different. The algorithm was applied and as expected the spectra converged such that both modems had the same crossover bin, which was varied until a maximum capacity was found.

Shown in the table below are stages of the simulation. Initially M_{E2F} of the two modems are bin 5 and bin 13 respectively. The modems independently change the value of the bin until they are equal, in the case below, they iterate until M_{E2F} became 9 in both cases. Once this occurred they independently maximized capacity and both settled on a final value 10, which is exactly the solution that results by using the joint optimization technique.

Modem\Iteration	Time 1	Time 2	Time 3	Time 4	Time 5	Time 6
Modem 1	5	6	7	8	9	10
Modem 2	13	12	11	10	9	10

Table 1: Example of 2 HDSL2 modems iteratively finding M_{E2F}

In adaptively finding the proper grouping of bins in the FDS region, random values of M_C and M_G were chosen for each modem as initial starting points. When the algorithm was applied, the spectra independently shifted until the upstream and downstream portions were orthogonal, and then shifted to equate capacity and converged to a final solution.

An example in is illustrated in figure 11. Colors moving from light to dark indicate direction of motion. Modem 1's spectra are on the top of the figure and on the bottom for modem 2. The figures on the left illustrate the modems attempting to make the FDS region orthogonal, and the right illustrates the modems equating upstream and downstream capacities. The modems start with values of M_C of 114 and 264 respectively. They adapt such that the upstream and downstream spectra between modems are orthogonal. In this case, they change such that M_C is set to 189 for both modems. At this point the upstream and downstream spectra between modems are orthogonal, however, the modems no longer have equal upstream and downstream capacities in the FDS region, and they begin to equate them. The finally settle on the value of 114 for M_C, and a corresponding value of 398 for M_G. The final state is illustrated in black. This corresponds exactly with joint signaling solution.

The algorithm is quite robust, and every test condition attempted under a variety of environmental configurations led to the optimal solution.

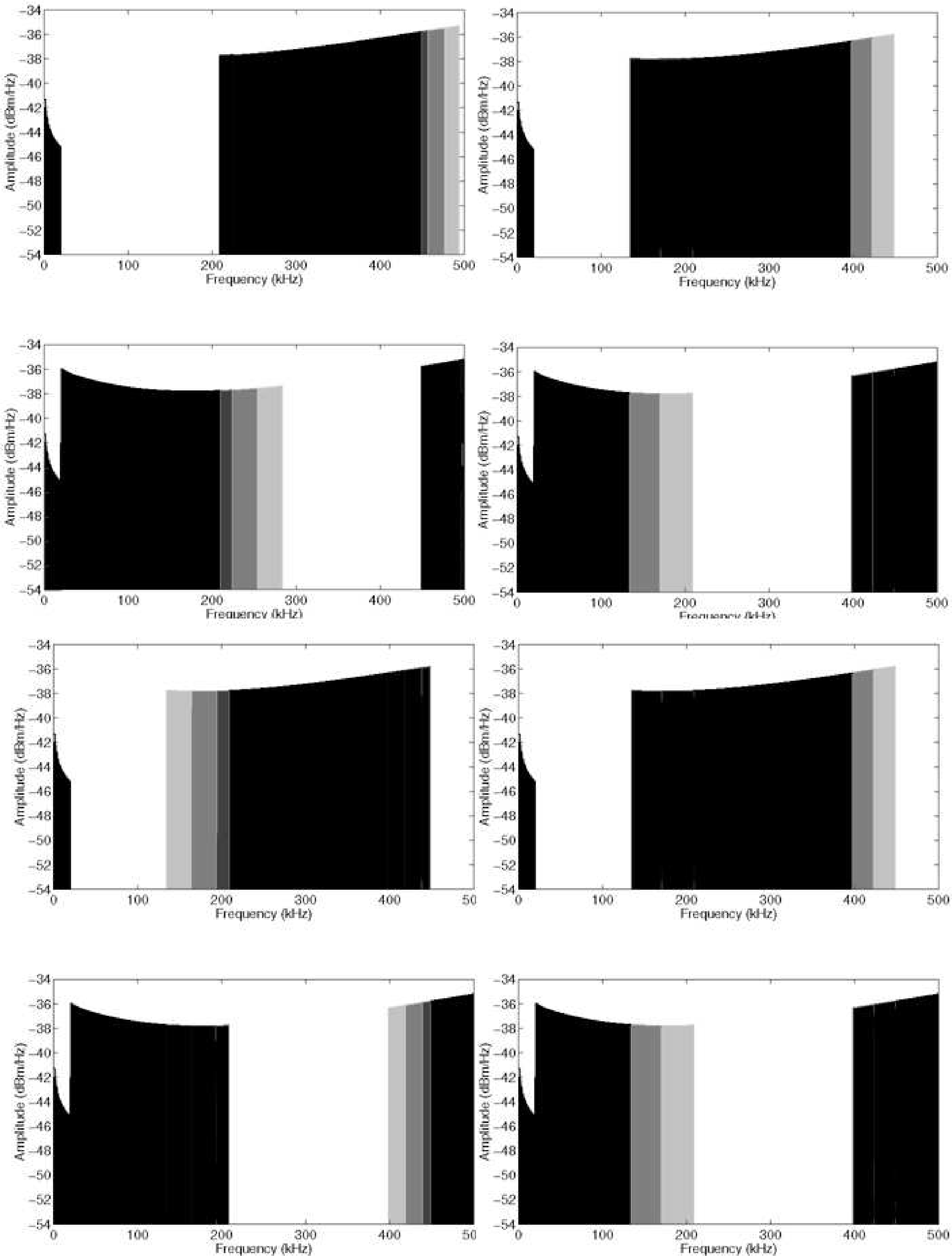

Figure 11: Left (top and bottom): modem1 and modem2 orthogonal (in FDS). Right(top and bottom): modem1 and modem2 equating up and downstream capacities. Color change from light to dark indicates direction of motion

6 Conclusion

Previous work in spectral optimization has been done in a static environment where the configuration of users of a particular DSL service and interferors do not change over time. Each modem is assumed to have a priori knowledge of its environment. In practice, this is not generally the case, and adaptive solution to the signaling problem is preferred. The key contribution of this project illustrates that when using an iterative optimization scheme, the spectra of the DSL users converge to a solution. It is of special importance to note that the solution achieved adaptively is precisely the solution obtained by using a joint-signaling technique.

7 Future work

Future work in the area involves loosening some of the assumptions that were made. In previous work, it was assumed that all modems in a cable binder have the same channel transfer function, which may not be true. There may be individual lines which are longer than others, some may have a different guage wires, etc. These differences will certainly lead to different channel models. For these same reasons, there will be different self-NEXT and self-FEXT transfer functions, and DSIN-NEXT and DSIN-FEXT interferences may be different for different lines. In other cases the channel transfer function may or may not be a monotonic decreasing function as it is currently assumed. In conditions such as these an analytical solution may be very difficult to achieve, and it would be advantageous to have an iterative solution which will converge to an optimal PSD for each modem.

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