IMAGE DATA PROCESSING IN THE COMPRESSED WAVELET DOMAIN

Jun Tian and Raymond O. Wells, Jr.

Computational Mathematics Laboratory, Rice University
Houston, Texas 77251-1892, USA
E-mail: juntian@rice.edu, wells@rice.edu

ABSTRACT

A new embedded image coding method, the *Wavelet-Difference-Reduction* algorithm, was introduced in [8]. It combines the discrete wavelet transform, differential coding, binary reduction, order bit plane transmission, and adaptive arithmetic coding. It is a simple and efficient way to successively approximate the image source and is very suitable for progressive image transmission. In addition, this algorithm provides a clear geometric structure in the compressed wavelet domain, which enables us to process the image data in compressed form. In this paper we will discuss three aspects of this problem, namely denoising, speckle reduction, and zooming. These outline some ideas of how to facilitate image data processing in the compressed wavelet domain.

1. INTRODUCTION

In the last ten years, wavelet analysis (see [1], [2], [6], [7], etc.) has become a cutting-edge technology in signal processing. The wavelet transform provides a compact multiresolution representation of signal data, and it can handle non-stationary signal quite well. In image data processing, the wavelet transform can remove the spatial and spectral redundancies of image sources to a large extend, and thus it is one of the best choices for an image transform coder. Lots of technique have been introduced on the quantization step (usually scalar quantization or vector quantization) of wavelet-based image coding methods. Recently we presented an embedded image coding method which combines the power of the wavelet transform and the efficiency of index coding. This method, the Wavelet-Difference-Reduction algorithm [8], has a very attractive feature in that it provides a clear geometric structure in the compressed wavelet domain. This enables us to perform image data processing in the compressed wavelet domain.

This paper is organized as follows. Section 2 gives a brief overview of the discrete wavelet transform. The Wavelet-Difference-Reduction algorithm is explained in Section 3. In Section 4, we will discuss image data processing in the compressed wavelet domain. And the paper is concluded in Section 5.

in *3rd International Conference on Signal Processing Proc.*, B. Yuan, X. Tang, Editors, 978–981, Beijing, China, 1996.

2. DISCRETE WAVELET TRANSFORM

The discrete wavelet transform (DWT) is a tool that cuts up data, functions, or operators into different frequency components, and then studies each component with a resolution matched to its scale. It gives a compact multiresolution representation of the image data. The DWT can by computed by the Mallat algorithm [5]. Assume that $\{h_k, k \in \mathbf{Z}\}$ and $\{g_k, k \in \mathbf{Z}\}$ are the decomposition scaling vector and wavelet vector, respectively, for a discrete signal $\{a_k, k \in \mathbf{Z}\}$, the low frequency part $\{c_k, k \in \mathbf{Z}\}$ and the high frequency part $\{d_k, k \in \mathbf{Z}\}$ of the DWT of $\{a_k\}$ are

$$c_k = \sum_l h_{l-2k} a_l ,$$

$$d_k = \sum_l g_{l-2k} a_l .$$

And the inverse DWT will give back $\{a_k\}$ from the knowledge of $\{c_k\}$ and $\{d_k\}$,

$$a_k = \sum_l \left(\tilde{h}_{k-2l} c_l + \tilde{g}_{k-2l} d_l \right) ,$$

where $\{\tilde{h}_k, k \in \mathbf{Z}\}$ and $\{\tilde{g}_k, k \in \mathbf{Z}\}$ are the reconstruction scaling vector and wavelet vector, respectively.

Usually the wavelet vectors are obtained from the scaling vectors,

$$g_k = (-1)^k \tilde{h}_{-k+1} ,$$

$$\tilde{g}_k = (-1)^k h_{-k+1} .$$

In this case, the scaling vectors should satisfy the following two conditions to constitute a wavelet system, namely, the linear condition

$$\sum_k h_k = \sum_k \tilde{h}_k = \sqrt{2} ,$$

and the bilinear condition

$$\sum_k h_k \tilde{h}_{k+2l} = \delta_{l,0} = \begin{cases} 1 & \text{if } l = 0 \\ 0 & \text{otherwise} \end{cases} .$$

As can be seen from the above, there are still some freedom of parameters left in designing h_k and $\tilde{h}_k$. One can use such freedom to set h_k and $\tilde{h}_k$ to be symmetric, to have maximal vanishing moments, or have some other nice properties, depending on the specific applications.

3. EMBEDDED IMAGE CODING

Embedded image coding is a way to successively approximate the original image source. As the coding continues, more and more fine detail will be obtained, in the order of importance. So we can reach the finest resolution we are looking for in an embedded fashion. And the embedded image coding is especially suited for progressive image data transmission.

The Wavelet-Difference-Reduction algorithm [8] is a new simple and efficient embedded image coding algorithm. It is based on the DWT and index coding. The basic structure is described in Figure 1. First we take the DWT of the image data. The DWT coefficients (which is a matrix being the same size of the original image data) will be ordered from coarser scales to finer scales. Then we get a one-dimensional ordered list, which consists of DWT coefficients. Next, the magnitudes of all DWT coefficients will be compared with a threshold T. For those significant coefficients (which means their magnitudes are equal to or greater than T), their index set (a set of all indices of significant coefficients) will be coded by differential coding and binary reduction. Differential coding takes the difference of successive values, and binary reduction removes the most significant binary digit in the binary representation. The combination of differential coding and binary reduction, which is called *index coding* here, gives a very compact representation of the index set. After index coding, the refinement values (either "1" or "0") of significant coefficients with respect to T will be output in the next step. When one cycle (consisting of an index coding and a refinement pass) is complete, T will be divided by 2, and we move to the next cycle. The resulting symbol stream (which contains four symbols, "1", "0", "+", and "−") will be further coded by adaptive arithmetic coding [9]. When some target rate or distortion metric is met, the encoding stops.

Experimental results have shown that this new algorithm is very competitive with other leading image coding algorithms on still images, medical images, seismic waves, and synthetic aperture radar images, etc. In addition, since we didn't assume any statistic model

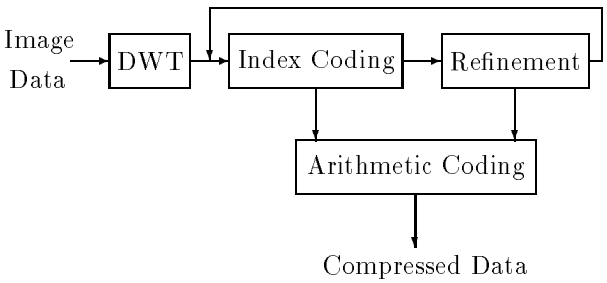

Figure 1: The Wavelet-Difference-Reduction Algorithm

on the positions of significant coefficients, it is a direct method to locate these significant coefficients. In the next section, we will discuss how to take this advantage to process image data in the compressed domain.

4. PROCESSING IN THE COMPRESSED DOMAIN

The Wavelet-Difference-Reduction algorithm locates the significant wavelet transform coefficients directly and has a clear geometric structure. In the compressed wavelet domain, we know the exact locations of significant coefficients, along with their proceeding significant bits values. With such a property, we may process the compressed data directly without applying the decompression first.

4.1. Denoising

Wavelet analysis has shown big success in both image compression and signal denoising. Denoising describes in an informal way various schemes which attempt to reject noise by damping or thresholding in the wavelet domain.

In his celebrated paper [3], D. L. Donoho showed that the soft thresholding method is an optimal procedure to recover data from additive Gaussian noise. His method consists of three steps. First take the DWT of the noisy data. Then apply the soft thresholding on the DWT coefficients. The soft thresholding acts like a shrinkage on real numbers (see Figure 2),

$$\mathcal{ST}_t(x) = \begin{cases} x - t & \text{if } x \geq t \\ 0 & \text{if } -t < x < t \\ x + t & \text{if } x \leq -t \end{cases} ,$$

where $t > 0$ is the chosen threshold. The last step is to take the inverse DWT on the resulting numbers. Then we get the "denoised" data which optimally (in

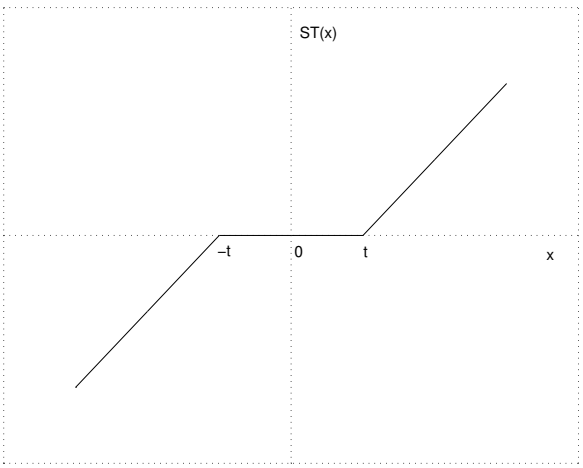

Figure 2: Soft Thresholding

the mean square error sense) approximates the original data, subject to the condition that with high probability the "denoised" data is at least as smooth as the original data. Thus the big improvement of his method over other denoising methods lies in the fact that Donoho's method keeps the smoothness of the data (at least with high probability which tends to 1).

It is clear that the soft thresholding method only involves the magnitudes of the wavelet transform coefficients. Thus we may denoise the image data in the compressed wavelet domain directly. In the Wavelet-Difference-Reduction algorithm, the magnitudes of the DWT coefficients are compared with a threshold T and the indices of those significant coefficients will be coded by differential coding and binary reduction. The magnitudes of those insignificant coefficients will be compared with the next threshold $T/2$ in the next cycle. And so on. Assume a target rate or distortion metric is met at some point, so the encoding job is terminated at this point, and assume the final threshold is T_l. There are two possible cases. First, if $T_l \geq t$, where t is the threshold chosen for denoising. In this case, the magnitudes of all insignificant coefficients are set to zero, by the virtue of the Wavelet-Difference-Reduction algorithm. Then we may simply subtract the magnitudes of significant coefficients by t, and the denoising is done. The second case is that if $T_l < t$. Now since in the Wavelet-Difference-Reduction algorithm, the consecutive thresholds in the consecutive cycles are different by a factor of 2, there must exist some cycle with the threshold T_c, such that $T_c < t \leq 2T_c$. To carry out the soft thresholding method in the compressed domain, we just set all magnitudes of those coefficients which are found significant at this cycle (with the threshold T_c) or after this cycle (with an even small threshold $T_c/2$, or $T_c/4$, etc) to zero, and subtract by t all magnitudes of those coefficients which are found significant before the T_c cycle. This completes the denoising. Because the magnitudes of those significant coefficients found at or after the T_c cycle are set to zero, the "denoised" data will has a smaller data size than the compressed data, which means we have compressed the data further simultaneously.

4.2. Speckle Reduction

Speckle results from the need to create the image with coherent radiation. Speckle phenomena can be found in synthetic aperture radar (SAR) images, acoustic imagery, and laser range data. A fully developed speckle pattern appears chaotic and unordered. Therefore, it becomes very important to reduce speckle in several applications of coherent imaging.

H. Guo et al. [4] found out that both the soft thresholding method and the hard thresholding method are computational efficient and can significantly reduce the speckle while preserving the resolution of SAR images. The hard thresholding method is almost identical to Donoho's soft thresholding method, except that replacing soft thresholding with hard thresholding. The hard thresholding behaves like an ideal stop band (see Figure 3),

$$\mathcal{HT}_t(x) = \begin{cases} x & \text{if } x \geq t \text{ or } x \leq -t \\ 0 & \text{if } -t < x < t \end{cases},$$

where $t > 0$ is the chosen threshold. Compared with the soft thresholding method, the hard thresholding method doesn't require a subtraction operation. Thus the hard thresholding method is much easier than the other one to be implemented in the compressed wavelet domain. Using the same argument as for the denoising, we may reduce the speckle on the compressed SAR data directly. An interesting observation of the hard thresholding method is that if the denoising threshold t is less than the final threshold T_l in the Wavelet-Difference-Reduction algorithm, then the "denoised" data will be exactly the same as the compressed data. Or in other words, the compressed data is already "denoised", with respect to the threshold t for the hard thresholding method.

4.3. Zooming

When the image data size is very large, such as the medical images, seismic waves, image processing will be remarkable slow. Very oftenly we are interested in only a small region of the whole image. Thus it is desirable that we can zoom in the interested region as quick as possible. Here we propose a novel method to

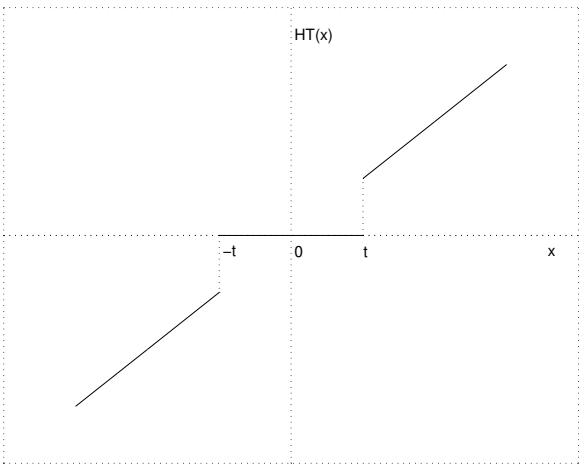

Figure 3: Hard Thresholding

zoom image data in the compressed domain. Assume the image data is stored in a compressed form by the Wavelet-Difference-Reduction algorithm. Since it is an embedded coding algorithm, at the beginning of the bit stream, the embedded code contains all lower rate (here the rate is the bit rate, or the number of bits per pixel) codes. Decompressing these beginning bit stream will give us a reconstruction image at a very low resolution with a relatively small data size. Now it becomes much easier to pick up our interested region in this decompressed image, which has a small data size. After that, as the decoding continues, more and more fine detail in the order of importance will be sent out to produce reconstruction images with higher and higher resolutions. Since we already know the location of our interested region, we also know the locations of significant wavelet transform coefficients, by the nature of the Wavelet-Difference-Reduction algorithm, we can simply ignore those parts of the bit stream not related to our interested region and get the reconstruction image of the interested region in a fast fashion.

5. CONCLUSIONS

In this paper we discuss how to process image data in the compressed wavelet domain. Due to the clear geometric structure of the Wavelet-Difference-Reduction algorithm, we are able to perform some image processing tasks directly in the compressed domain. Considering the large data sets in use today, developing tools for image processing in the compressed domain will become increasingly more important.

6. ACKNOWLEDGMENT

We would like to thank C. Sidney Burrus and Haitao Guo of the Computational Mathematics Laboratory, Rice University, and Hartmut Juergens and Markus Lang of the Center for Medical Visualization and Diagnostic Systems, University of Bremen, for fruitful discussion and valuable help. This work was supported in part by ARPA and Texas ATP.

7. REFERENCES

[1] Charles K. Chui. *An Introduction to Wavelets.* Wavelet Analysis and Its Applications. Academic Press, Boston, 1992.

[2] Ingrid Daubechies. *Ten Lectures on Wavelets.* CBMS-NSF regional conference series in applied mathematics. Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania, 1992.

[3] David L. Donoho. De-noising by soft-thresholding. *IEEE Trans. on Information Theory*, 41(3):613–627, May 1995.

[4] Haitao Guo, Jan E. Odegard, Markus Lang, Ramesh A. Gopinath, Ivan W. Selesnick, and C. Sidney Burrus. Wavelet based speckle reduction with application to SAR based ATD/R. In *Proceedings of the International Conference on Image Processing*, Austin, TX, November 1994.

[5] Stephane G. Mallat. Multiresolution approximation and wavelet orthonormal bases of $L^2(\mathbf{R})$. *Transactions of the American Mathematical Society*, 315(1):69–87, September 1989.

[6] Yver Meyer. *Wavelets and Operators.* Cambridge University Press, 1992.

[7] Howand L. Resnikoff and Raymond O. Wells, Jr. *Wavelet Analysis and the Scalable Structure of Information.* Springer-Verlag, New York, 1996. To appear.

[8] Jun Tian and Raymond O. Wells, Jr. A lossy image codec based on index coding. In *Proceedings of the IEEE Data Compression Conference*, Snowbird, Utah, 1996. IEEE Computer Society Press.

[9] Ian H. Witten, Radford M. Neal, and John G. Cleary. Arithmetic coding for data compression. *Communications of the ACM*, 30:520–540, June 1987.