Some Exchange Algorithms Complementing the Parks-McClellan Program for Filter Design[1]

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Abstract

In this paper, several modifications of the Parks-McClellan (PM) program are described that treat the band edges differently than does the PM program. The first exchange algorithm we describe allows (1) the explicit specification of δ_p and δ_s and (2) the specification of the half-magnitude frequency, ω_o. The set of lowpass filters obtained with this algorithm is the same as the set of lowpass filters produced by the PM algorithm. We also find that if passband monotonicity is desired in the design of filters having very flat passbands it is also desirable to modify the usual way of treating the band edges. The second multiple exchange algorithm we describe produces filters having a specified δ_p and δ_s but also includes a measure of the integral square error.

1 Introduction

In this paper, several modifications of the Parks-McClellan (PM) program [11, 13, 17] are described. Recall that in their approach to the design of digital filters, the band edges are specified and the weighted Chebyshev error is minimized. The programs described in this paper treat the band edges differently than does the PM program. Moreover, the set of lowpass filters obtained with the program of section 2.1 is the same as the set of lowpass filters produced by the PM algorithm. In section 3, we find that if passband monotonicity is desired in the design of filters having very flat passbands it is also desirable to modify the usual way of treating the band edges. In the last section, we describe a multiple exchange algorithm that produces filters having a specified δ_p and δ_s but also includes a measure of the integral square error.

Consider the usual ideal lowpass filter. Figure 1 shows a typical equiripple frequency response. The passband and stopband edges are denoted by ω_p and ω_s. The Chebyshev errors in the passband and stopband are denoted by δ_p and δ_s. The figure also shows the half-magnitude frequency, ω_o.

We first note, as an aside, that the PM program can be modified so that it achieves a specified Chebyshev error in one band and minimizes the Chebyshev error in the other. This can be achieved by imposing an affine relationship between δ_p and δ_s:

$$\delta_p = K_p\delta + \eta_p, \qquad \delta_s = K_s\delta + \eta_s \tag{1}$$

where K_p, K_s, η_p and η_s are supplied by the user, as are ω_p and ω_s. (At least one of K_p and K_s must be nonzero.) The modified PM program then minimizes δ. When η_p and η_s are both taken to be 0 this becomes the linear relationship permitted by the PM program. However, if $K_s = \eta_p = 0$, then the stopband ripple size δ_s of the resulting equiripple filter has the specified value η_s and the passband ripple size δ_p is minimized. In any case, the linear system of equations to be solved on each iteration is given by eq (1), and by $A(\omega_i) = 1 + (-1)^i\delta_p$ in the passband, and by $A(\omega_i) = (-1)^i\delta_s$ in the stopband. This system of equations is linear in δ, δ_p, δ_s and the filter coefficients and can be solved efficiently as in the PM algorithm. The reference set update procedure is that of the PM algorithm.

2 Specifying δ_p and δ_s

While the PM program allows the user to specify ω_p, ω_s and the ratio δ_p/δ_s, it does not allow the user to specify both δ_p and δ_s directly. Although the user can indirectly control δ_p and δ_s with the PM program by iteratively adjusting the band edges and the ratio δ_p/δ_s [14, 15], the exchange algorithm below produces lowpass linear phase FIR filters having specified δ_p, δ_s, ω_o. It is a hybrid of the algorithm of Hofstetter, Oppenheim and Siegel [7, 8] and the PM algorithm. Like those algorithms, it employs a reference set of frequencies. On each iteration (1) an interpolation problem is solved and (2) the reference set is updated.

Although the algorithm of Hofstetter et al predates the PM program and produces equiripple filters with specified δ_p and δ_s, it is not widely used because it produces only extra-ripple filters and because it permits

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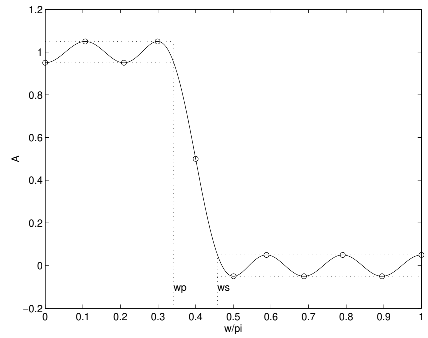


Figure 1: $N = 21$, $\delta_p = \delta_s = 0.05$, $\omega_o = 0.4\pi$

limited control over the location of the band edges. The design formulation leading to these extra-ripple filters was originally described by Hermann and Schüßler [5, 6] and is also described in [16, 17].

In [23] Shpak and Antoniou present an interesting and distinct modification to the PM program in which the Chebyshev error in respective bands are not constrained by ratios. Although they obtain extra-ripple filters without sacrificing the ability to specify the band edges, control over δ_p and δ_s is not explicit.

The frequency response of a linear phase FIR filter is given by the discrete-time Fourier transform of its impulse response and can be written as $H(\omega) = A(\omega)e^{-jM\omega}$ where $A(\omega)$ is the amplitude and $M = (N-1)/2$ for length-N filters [12]. Let $E(\omega) = (A(\omega) - D(\omega))\,/\delta(\omega)$ where $D(\omega)$ is the desired amplitude and where $\delta(\omega)$ equals δ_p in the passband and δ_s in the stopband.

2.1 A New Equiripple Lowpass Filter Design Algorithm: Specified δ_p, δ_s and ω_o

We propose an exchange algorithm that allows (1) the explicit specification of δ_p and δ_s and (2) the specification of the half-magnitude frequency, ω_o. The reference set here does not contain two band edges as in the PM program, instead, it contains ω_o. The circular marks in fig 1 indicate the reference frequencies upon convergence. The filter satisfies the alternation property for the correct choice of band edges, so it could have been designed using the PM program if the band edges had been known in advance.

We find the filter that alternately interpolates $1 + \delta_p$, $1 - \delta_p$ in the passband, alternately interpolates δ_s, $-\delta_s$ in the stopband, and interpolates 0.5 at ω_o. Interpolation formulas can be used. At each iteration, the local extrema of $A(\omega)$ are found. If there are M extrema, then ω_o is appended to these frequencies to obtain a new reference set. If, however, there are $M+1$ local extrema of $A(\omega)$, then one of these frequencies must be excluded. This is similar to the PM program, and likewise, the frequency to exclude is either 0 or π. To decide which, we inspect the difference between $E(\omega)$ at that extremal frequency and $E(\omega)$ at the neighboring extremal frequency. We exclude the frequency for which this difference is smaller.

When the specified δ_p, δ_s are achievable, this algorithm produces exactly the same lowpass filters as does the PM program, however, it allows one to specify a different set of parameters in the design process. The corresponding design of multiband filters requires more care however [22].

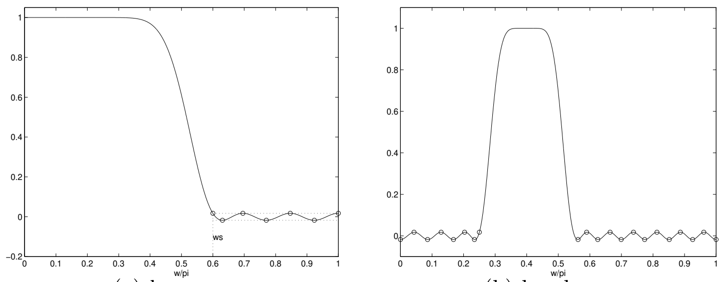
Table 1 classifies four approaches. We note, however, that the Shpak-Antoniou algorithm, when applied to multiband filter design, achieves a number of extra ripples specified by the user. This table clarifies the relationship among previously reported exchange algorithms for equiripple linear phase filter design and the way in which the algorithm described in this section relates to them.

Table 1: Exchange Algorithms for Equiripple Filters

	Nonextra-ripple	Extra-ripple
Band edges ω_p,ω_s specified	PM [13]	SA [23]
Chebyshev error δ_p,δ_s specified	New	HOS [7], HS [5, 6]

3 Filter Design with Flatness Constraints

The design of linear phase FIR filters having very flat passbands and equiripple stopbands has been studied by several authors [4, 9, 25, 26, 27]. Vaidyanathan [27] presents a method based upon the PM algorithm [12] and a special filter structure. This structure, also used by Schüßler and Steffen [19, 20, 24], enforces a specified degree of flatness at $\omega = 0$. However, the filters obtained by the method of [27] do not necessarily have monotonic passbands, which is sometimes a requirement. This section modifies the method of [27] to produce filters with equiripple stopbands and flat *monotonic* passbands having a specified degree of flatness at $\omega = 0$. Figure 2 shows the amplitude of such filters. Lowpass digital differentiators can also be designed using this approach [21].



(a) lowpass (b) bandpass

Figure 2: (a) $N = 33$, $L = 22$, $\omega_s = 0.6\pi$, $\delta_s = 0.0175$ (b) $N = 55$, $L = 16$, $\omega_p = 0.4\pi$, $\omega_t = 0.15\pi$

While algorithms for general linear programs can be used to produce such filters [9, 25, 26] here we are interested in the type of filters that can be obtained using the Remez and Remez-like exchange algorithms. To obtain filters having flat monotonic passbands using Remez-like algorithms, it is necessary to omit the passband from the exchange procedure. Therefore the passband edge does not fit naturally into the corresponding algorithm and the passband is shaped by the flatness constraints.

The transfer function of the lowpass filter structure is $H(z) = z^{-(N-1)/2} + \left(\frac{1-z^{-1}}{2}\right)^L H_2(z)$. Taking H_2 to be a highpass filter whose impulse response is symmetric and of length $N-L$, $H_2(e^{j\omega})$ can be written [12] as $H_2(e^{j\omega}) = e^{-j\frac{N-L-1}{2}\omega} A_2(\omega)$. When L is even, $H(e^{j\omega})$ can be written as $H(e^{j\omega}) = e^{-j\frac{N-1}{2}\omega} A(\omega)$ where

$$A(\omega) = 1 + A_2(\omega)(-1)^{L/2} \left(\sin \frac{\omega}{2}\right)^L. \tag{2}$$

Two problem formulations for which exchange algorithms can be used are: (1) Specify N, L, ω_s; minimize δ_s. (2) Specify N, L, δ_s; maximize the stopband width, (minimize ω_s). Option(1) is the traditional approach in which the bands of interest are well defined. Option(2) is a variation in which δ_s is specified but the band edge ω_s, however, is not fixed. In this case, no band edge is actually used during the course of the design algorithm. The band edge that results is the one that corresponds to the specified Chebyshev error δ_s and the specified degree of flatness L. Option(1) is solved by applying the Remez exchange algorithm [12] over just the stopband, yielding the coefficients of H_2 minimizing $||A(\omega)||_\infty$ over the stopband. To specify δ_s and leave ω_s variable in option(2), we use an approach similar to that described in the previous section [21].

The passband can be shown to be monotonic by the following reasoning. Recall that when no degree of flatness is imposed upon $A(\omega)$ the maximum number of frequencies in $[0, \pi]$ at which the derivative of $A(\omega)$ equals zero is $(N + 1)/2$ [16]. Note also that additional degrees of flatness imposed at $\omega = 0$ reduces the number of frequencies at which $A'(\omega)$ can be equal to zero. Because the filters above have the property that $A'(\omega)$ equals zero at $(N + 1 - L)/2$ frequencies in the stopband, it appears that there can be no passband frequencies (other than $\omega = 0$) at which $A'(\omega)$ equals zero. Therefore, the passband will be monotonic.

This method can also be applied to the design of bandpass filters having very flat passbands. In this case, we specify a passband frequency, ω_p, where we wish to impose flatness constraints. The deviations from 0 in the stopbands are denoted by δ_1 and δ_2. The appropriate structure has the transfer function $H(z) = z^{-(N-1)/2} + H_1(z)H_2(z)$ with $H_1(z) = \left((1 - 2(\cos\omega_p)z^{-1} + z^{-2})/4\right)^{L/2}$ where L is even, N is odd, and H_2 is a filter whose impulse response is symmetric and of length $N - L$. The overall frequency response

$H(e^{j\omega})$ can then be written as $H(e^{j\omega}) = e^{-j\frac{N-1}{2}\omega}A(\omega)$ where

$$A(\omega) = 1 + (-1)^{L/2}\left(\frac{\cos\omega_p - \cos\omega}{2}\right)^{L/2} A_2(\omega). \tag{3}$$

To ensure that the passband is monotonic on both sides of ω_p, it is necessary that L be a multiple of 4. When 4 divides L, $A_1(\omega)$ is a nonnegative function. Then $A(\omega)$ is concave over the passband with appropriately chosen H_2. As above, there are two approaches for which exchange algorithms are well suited: (1) Specify N, L, ω_p, stopband edges, $K = \delta_2/\delta_1$; minimize δ_1. (2) Specify N, L, ω_p, δ_1, δ_2; maximize stopband widths.

4 A Multiple Exchange Algorithm for Constrained Least Square Filter Design

Adams has suggested that it is desirable to consider both the integral square error and the Chebyshev error in filter design [1, 2], but most multiple exchange algorithms do not allow this. Adams et al have developed programs for constrained least square filter design that minimize the weighted square error subject to a constraint on the Chebyshev error. Programs to solve general constrained least square problems require more than simple multiple exchange algorithms, however. Here we modify the approach taken by Adams and in doing so, we find that a simple multiple exchange algorithm is very effective for lowpass filter design. Like the algorithms above, this approach treats the band edges differently than do traditional methods. The user specifies δ_p, δ_s, and the cut-off frequency of the ideal lowpass filter.

Let $D(\omega)$ be the desired amplitude, see for example fig 3(a). The algorithm below minimizes the square error ($||E||_2 = \int (A(\omega) - D(\omega))^2 d\omega$) such that (1) the local extrema of $A(\omega)$ in the passband lie between $1 + \delta_p$ and $1 - \delta_p$ and (2) the local extrema of $A(\omega)$ in the stopband lie between δ_s and $-\delta_s$.

Notice that this approach employs the unweighted square error: there is no transition region that is excluded from the square error criterion. This is attractive because, if a zero-weighted transition band is used, then the best Chebyshev and L_2 filters are optimal in the meaningful way described in [28] only if the signals in the input class have no frequency content in the transition band, which is not necessarily true.

We present a rapidly converging, multiple exchange algorithm for the design of optimal peak constrained least square lowpass FIR filters. Because it does not require the use of explicitly specified transition bands, this approach does not ignore the square error around the cut-off frequency, and thereby does not implicitly assume that input signals have no frequency content there. The algorithm uses Lagrange multipliers and the Kuhn-Tucker (KT) conditions, as suggested by Adams [1] and further developed in [2, 10], to guarantee optimality upon convergence. It gives the least integral square error filter and a continuum of Chebyshev filters as special cases.

4.1 The Algorithm

The algorithm we use solves a succession of *equality* constrained square error minimization problems, where the constraints are on $A(\omega)$ for the frequency points in a constraint set. The constraint set is updated so that at convergence the only frequency points at which constraints are imposed are those where $A(\omega)$ touches the constraint. The equality constrained problem is solved with Lagrange multipliers. According to the KT conditions, the *equality* constrained problem solves the corresponding *inequality* constrained problem if all the multipliers are non-negative.

Let $T(\omega)$ equal δ_p and δ_s in the passband and stopband. Let the constraint set be $\{\omega_1, \ldots, \omega_L\}$. If ω_i is a candidate local maximum (minimum) of $A(\omega)$, then it is necessary to impose the constraint $A(\omega_i) \leq D(\omega_i) + T(\omega_i)$ ($A(\omega_i) \geq D(\omega_i) - T(\omega_i)$). Together, these can be written as $s_i A(\omega_i) \leq s_i D(\omega_i) + T(\omega_i)$ where s_i is 1 and -1 respectively. For an even symmetric odd length filter, $A(\omega)$ can be written as $\frac{a_0}{\sqrt{2}} + \sum_{k=1}^{M} a_k \cos 2\pi k\omega$ [12] so, to minimize $||E||_2$ subject to $s_i A(\omega_i) = s_i D(\omega_i) + T(\omega_i)$, the use of Lagrange multipliers yields the equations

$$\begin{bmatrix} \mathbf{I}_{m+1} & \mathbf{G}^t \\ \mathbf{G} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \end{bmatrix} \tag{4}$$

where the unknowns are the coefficients, $\mathbf{a} = (a_0, \ldots, a_m)^t$, and the multipliers, $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_L)^t$. In eq (4) $\mathbf{c}$ is given by the Fourier series of $D(\omega)$, $\mathbf{G}$ is a matrix of cosine terms, and $\mathbf{d}_i = s_i D(\omega_i) + T(\omega_i)$. Note that $\boldsymbol{\mu} = (\mathbf{G}\mathbf{G}^t)^{-1}(\mathbf{G}\mathbf{c} - \mathbf{d})$ and $\mathbf{a} = \mathbf{c} - \mathbf{G}^t\boldsymbol{\mu}$ give the solution to eq (4).

The algorithm begins with the best unconstrained L_2 filter obtained by truncating the Fourier coefficients. Then constraints are iteratively imposed upon $A(\omega)$ at selected frequencies until the best constrained L_2 filter is obtained. The algorithm can be summarized as follows:

1. Initialize the constraint set to the empty set.

2. Calculate the multipliers and the filter that minimizes $||E||_2$ subject to the equality constraints $s_i A(\omega_i) = s_i D(\omega_i) + T(\omega_i)$ for all ω_i in the constraint set.

3. If there is a constraint set frequency ω_i for which the Lagrange multiplier μ_i is negative, then remove from the constraint set the frequency corresponding to the most negative multiplier and go back to step 2. Otherwise, go on to step 4.

4. Set the constraint set equal to the set of frequency points satisfying both (i) $A'(\omega) = 0$ and (ii) $|A(\omega) - D(\omega)| \geq T(\omega)$. If $A(\omega_i)$ is a local maximum (minimum), then set $s_i = 1$ ($s_i = -1$). If $|A(\omega) - D(\omega)| \leq T(\omega) + \epsilon$ for all frequency points in the new constraint set, then convergence has been achieved. Otherwise, go back to step 2.

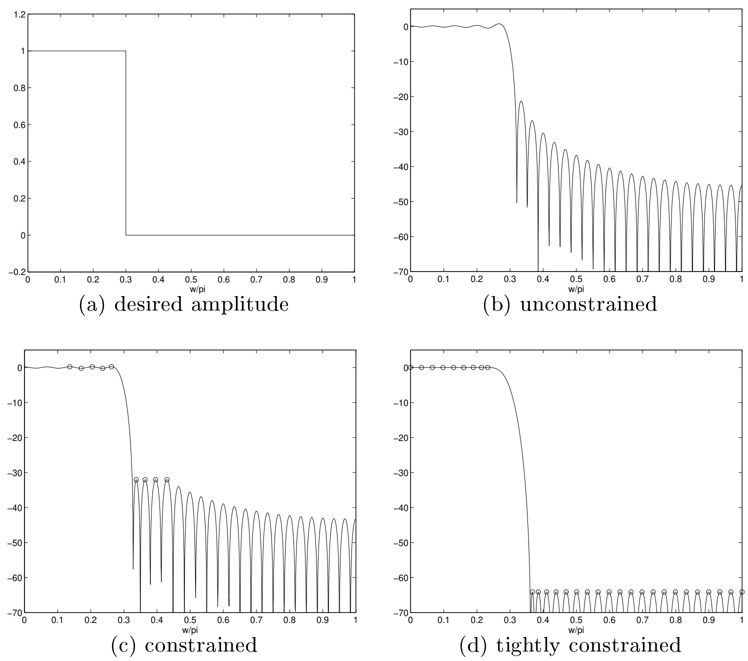


Figure 3: $N = 61$, $\omega_o = 0.3\pi$, (b) Fourier Method (c) $\delta_p = \delta_s = 0.025$ (d) $\delta_p = \delta_s = (0.025)^2$

For example, when $D(\omega)$ is the ideal lowpass filter with the cut-off frequency at 0.3π and $T(\omega) = 0.025$ the proposed algorithm converges to the length 61 filter in fig 3(c) in 3 iterations.

The proposed algorithm gives as special cases a continuum of best Chebyshev filters. First, observe that if, for a fixed filter length, the constraint on $||E||_\infty$ in [1, 2, 10] is chosen too small, then no filter satisfies the constraint and the algorithms of [1, 2, 10] can not converge. However, there is no minimum $T(\omega)$ below which the proposed approach fails to converge. If $T(\omega)$ is taken to be small, then the transition between the bands adjusts itself in the course of the algorithm: when $T(\omega) = (0.025)^2$, the resulting response in fig 3(d) is obtained in 6 iterations.

The constrained minimization problem described above is not, strictly speaking, a quadratic program and can not be posed as one because the resulting band edges are not known in advance. However, it can be posed as a sequence of similar quadratic programs. Even so, the convergence of this algorithm is not supported by the theory of quadratic programming. That it does in fact converge in practice is due to properties of the particular problem of lowpass filter design. When used for bandpass filter design, for example, this algorithm does not converge in general. For multiband filter design we have found as in [2] that it is necessary to use a single point update procedure for some iterations to obtain robust convergence.

4 Conclusion

By making minor changes in the way in which band edges are treated, a variety of new algorithms emerge. The first of these algorithms discussed above produce the same set of lowpass filters as does the PM program, but gives the user a different set of parameters to specify in the design process. In section 3, it was found that passband monotonicity can be ensured by omitting the passband from the exchange algorithm. Previously, passband monotonicity had been obtained by general linear programming methods, which require a dense grid and are more computationally intensive. The algorithm of section 4 is the most interesting because it is a multiple exchange algorithm that includes the integral square error as well as δ_p and δ_s. It is distinct from many other filter design methods because it does not exclude from the integral square error a region around the cut-off frequency and yet, it overcomes Gibbs phenomenon without resorting to windowing or 'smoothing out' the discontinuity of the ideal lowpass filter.

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