Optimal Transmit Spectra for Communication in the Presence of Crosstalk

Rohit V. Gaikwad and Richard G. Baraniuk

Department of Electrical and Computer Engineering
6100 Main Street, Rice University
Houston, Texas 77005

Abstract— **We present a general framework for designing optimal transmit spectra for symmetric bit-rate communication services dominated by crosstalk, in particular Digital Subscriber Line (DSL) services such as the proposed HDSL2. Using the channel, noise, and interference transfer functions, we set up and solve an optimization problem to maximize the joint capacity of neighboring lines. Joint signaling techniques and optimal power distribution yield significant gains in bit rates (or performance margins) over current schemes. Furthermore, by design, the spectra are spectrally compatible with existing neighboring services. The framework is quite general — it does not depend on the exact choice of modulation scheme, for example. It is also extremely simple and of low computational complexity.**

Keywords— **Digital Subscriber Line (xDSL) systems, capacity, multiuser interference.**

I. Introduction

Digital subscriber line (DSL) modems, the next generation of high-speed telephone line modems, exploit large bandwidths (> 1 MHz) to yield high bit rates (> 1 Mbps). The various DSL services (xDSL in general) are categorized according to the bit rates they deliver:

ADSL — Asymmetric DSL — provides a high-speed (on the order of 6 Mbps) downstream (from central office to subscriber) channel and a low-speed (on the order of 640 kbps) upstream (from subscriber to the central office) channel over each twisted pair.

VDSL — Very high bit-rate DSL — will provide a symmetric or asymmetric high-bit-rate (on the order of 50 Mbps) channel over a single twisted pair less than 3 to 6 kft long.

HDSL2 — High bit-rate DSL 2 — will provide a symmetric bit–rate of 1.544 Mbps over a *single twisted pair* (< 18 kft long) without repeaters.

Telephone lines are packed closely together into binders in a cable. Crosstalk (near-end (NEXT) and far-end (FEXT)) results due to the proximity of the lines (see Figure 1) and significantly limits achievable bit-rates [1].

In this paper, we employ *crosstalk avoidance* between same-service lines in a binder, using orthogonal signaling techniques to design optimal transmit spectra. We solve an optimization problem to maximize the channel capacity given the channel, noise, and crosstalk characteristics. By design, we maintain spectral compatibility with existing neighboring services. This problem was first solved in [2] for symmetric-bit-rate services facing self-NEXT (NEXT from same-service lines) and white additive Gaussian noise (AGN). Here, we solve the problem in presence of self-NEXT, self-FEXT, AGN, and interference from other services. Optimization can also be done under an additional peak frequency-domain power constraint [3]. The techniques developed here are general and can be applied to any symmetric-bit-rate communication channel with appropriate crosstalk characteristics. In this paper, we target symmetric-bit-rate DSL services, e.g., HDSL2 [4].

This work was supported by the National Science Foundation, grant no. MIP–9457438, Nortel Networks, and SBC Communications.
Email: {rohitg, richb}@rice.edu Web: www.dsp.rice.edu

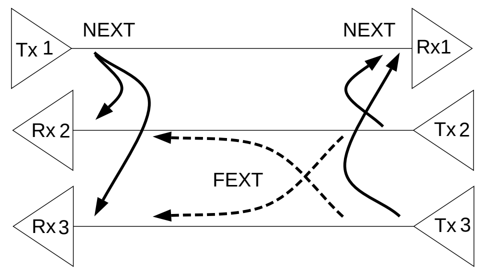

Fig. 1. *NEXT and FEXT between neighboring lines in a binder. Tx's are transmitters and Rx's are receivers.*

Section II outlines the definitions and notation used. Details on obtaining optimal transmit spectra are presented in Section III. We discuss simulation results in Section IV and present conclusions in Section V.

II. Definitions and Notation

There are two types of crosstalk (see Figure 1):

Near-end crosstalk (NEXT): Interference between neighboring lines that arises when signals are transmitted in opposite directions. If the neighboring lines carry the same type of service, then the interference is called self-NEXT; otherwise, it is called as different-service (DS) NEXT.

Far-end crosstalk (FEXT): Interference between neighboring lines that arises when signals are transmitted in the same direction. If the neighboring lines carry the same type of service, then the interference is called self-FEXT;

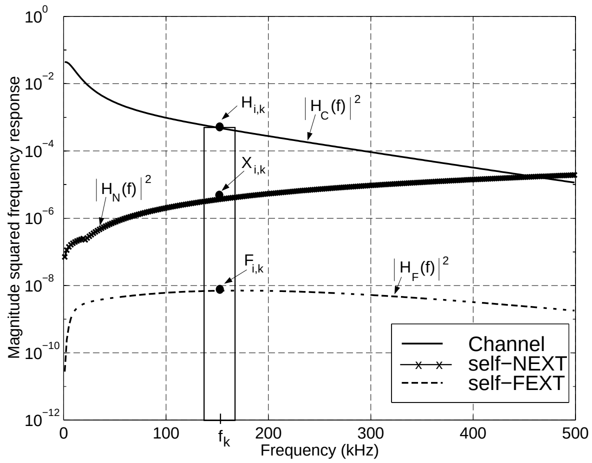

Fig. 2. *Magnitude-squared transfer function of the channel (CSA loop 6),* 39 *self-NEXT interferers, and* 39 *self-FEXT interferers.*

otherwise, it is called as different-service (DS) FEXT.

The term *self-interference* refers to the combined self-NEXT and self-FEXT. Channel noise is modeled as AGN.

Figure 2 illustrates the channel, self-NEXT, and self-FEXT transfer functions, denoted by $H_C(f)$, $H_N(f)$, and $H_F(f)$, respectively. We assume that the channel can be characterized as a linear time-invariant system. We divide the *transmission bandwidth* B of the channel into K narrow frequency bins; each of width W Hz and assume that the channel, noise and the crosstalk characteristics vary slowly enough with frequency that they can be approximated as constant over each bin.

We use the following notation for the channel transfer function of line i [5]

$$|H_C(f)|^2 = \begin{cases} H_{i,k} & \text{if } |f - f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

self-NEXT transfer function [6]

$$|H_N(f)|^2 = \begin{cases} X_{i,k} & \text{if } |f - f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

and self-FEXT transfer function [6]

$$|H_F(f)|^2 = \begin{cases} F_{i,k} & \text{if } |f - f_k| \le \frac{W}{2}, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Here f_k are the center frequencies (see Figure 2) of the K bins with index $k \in \{1, \ldots, K\}$. We consider real signals with symmetric frequency responses. Thus, we denote quantities only over the non-negative frequency region.

Similarly, DS-NEXT is denoted by $DS_N(f)$, DS-FEXT by $DS_F(f)$, and AGN by $N_o(f)$. We sum the DS-NEXT, DS-FEXT, and AGN to get the total Gaussian noise as

$$N(f) := N_o(f) + DS_N(f) + DS_F(f). \quad (4)$$

xDSL modems transmit in two directions on the same line via a 4–2 line hybrid circuit. We denote the upstream and downstream transmit power spectral densities (PSDs) by $S^u(f)$ and $S^d(f)$, respectively. Similarly, $s^u(f)$ and $s^d(f)$ denote the PSDs in frequency bin k. An *Equal PSD* (EQPSD) signaling scheme in frequency bin k is one for which $s^u(f) = s^d(f) \neq 0$ for all f in the bin. (that is, both upstream and downstream transmissions occupy the band $|f - f_k| \le \frac{W}{2}$ in the same way). A *Frequency Division Signaling* (FDS) scheme in frequency bin k is one for which $s^u(f) = 0$ when $s^d(f) \neq 0$ for all f in the bin and vice versa (that is, both transmissions occupy orthogonal frequency bands within $|f - f_k| \le \frac{W}{2}$.)

III. Optimal transmit spectra

A. Absence of self-interference

In the absence of self-NEXT and self-FEXT, the interference combination consists exclusively of different service interferers (such as HDSL, T1, ADSL, etc.) and AGN. This interference can be lumped together with the AGN [7] as in 4 to obtain the optimal power distribution in each direction of transmission by the classical water-filling solution [8].

B. Presence of self-interference

When present, self-NEXT and self-FEXT severely limit the achievable bit rates in symmetric-bit-rate xDSL services. In this scenario, we assume self-NEXT dominates self-FEXT and self-FEXT is small (see Figure 2). This is the case of interest for HDSL2. However, self-FEXT still factors into our design in a significant way. This is a new, non-trivial extension of the work of [2].

Our goal is to maximize the upstream capacity (C^u) and the downstream capacity (C^d) given an average total power constraint of $P_{\max}$ and the equal capacity constraint $C^u = C^d$.

Consider the case of two neighboring lines carrying the same service. Line 1 upstream capacity is C^u and line 2 downstream capacity is C^d. Under the Gaussian channel assumption, we can write these capacities (in bps) as

$$C^u = \sup_{S^u(f), S^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S^u(f)}{N(f) + |H_N(f)|^2 S^d(f) + |H_F(f)|^2 S^u(f)}\right] df, \quad (5)$$

and

$$C^d = \sup_{S^u(f), S^d(f)} \int_0^\infty \log_2 \left[1 + \frac{|H_C(f)|^2 S^d(f)}{N(f) + |H_N(f)|^2 S^u(f) + |H_F(f)|^2 S^d(f)}\right] df. \quad (6)$$

The supremum is taken over all possible $S^u(f)$ and $S^d(f)$ satisfying

$$S^u(f) \geq 0, \quad S^d(f) \geq 0 \quad \forall f,$$

and the average power constraints for the two directions

$$2\int_0^\infty S^u(f)df \leq P_{\max}, \;\; 2\int_0^\infty S^d(f)df \leq P_{\max}. \quad (7)$$

We can solve for the capacities C^u and C^d using "water–filling" if we impose the restriction of EQPSD, that is $S^u(f) = S^d(f)\ \forall f$. However, this gives low capacities. Therefore, we employ FDS ($S^u(f)$ orthogonal to $S^d(f)$) in spectral regions where self-NEXT is large enough to limit our capacity and EQPSD in the remaining spectrum. This gives much improved performance.

To ease our analysis, we divide the channel into K bins of equal bandwidth W (see Figure 2) and continue our design and analysis on the single frequency bin k assuming the subchannel frequency responses (1)–(3). For ease of notation, in this section set

$$H := H_{i,k}, \quad X := X_{i,k}, \quad F := F_{i,k} \quad \text{in (1)–(3)}, \quad (8)$$

and let $N := N(f_k)$ denote the total noise PSD in bin k. Let $s^u(f)$ denote the PSD in bin k of line 1 upstream direction and $s^d(f)$ denote the PSD in bin k of line 2 downstream direction (for capacity purposes we will consider the bin k demodulated to baseband). Denote the corresponding capacities of bin k by c^u and c^d.

We desire a signaling scheme that includes FDS, EQPSD, and all combinations in between in each bin. Therefore, we divide bin k in half and set

$$s^u(f) = \begin{cases} \alpha\frac{2P_m}{W} & \text{if } 0 \leq f \leq \frac{W}{2}, \\ (1-\alpha)\frac{2P_m}{W} & \text{if } \frac{W}{2} < f \leq W, \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

$$s^d(f) = \begin{cases} (1-\alpha)\frac{2P_m}{W} & \text{if } 0 \leq f \leq \frac{W}{2}, \\ \alpha\frac{2P_m}{W} & \text{if } \frac{W}{2} < f \leq W, \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

Here P_m is the average power over the bandwidth W in bin k and $0.5 \leq \alpha \leq 1$. The factor α controls the power distribution in the bin. When $\alpha = 0.5$, $s^u(f) = s^d(f)$ $\forall f \in [0, W]$ (EQPSD signaling); when $\alpha = 1$, $s^u(f)$ and $s^d(f)$ are disjoint (FDS signaling). These two extreme transmit spectra along with other possible spectra (for different values of α) are illustrated in Figure 3.

B.1 Optimal Spectrum: One frequency bin

If we define the achievable rate as

$$R_A(s^u(f), s^d(f)) = \int_0^W \log_2\left[1 + \frac{s^u(f)H}{N + s^d(f)X + s^u(f)F}\right] df, \quad (11)$$

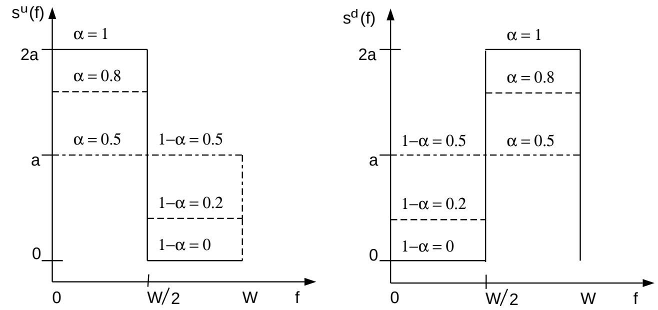

Fig. 3. *Upstream and downstream transmit spectra in a single frequency bin ($\alpha = 0.5 \;\Rightarrow$ EQPSD signaling and $\alpha = 1 \;\Rightarrow$ FDS signaling).*

then

$$c^u = \max_{0.5 \leq \alpha \leq 1} R_A(s^u(f), s^d(f)) \quad \text{and}$$
$$c^d = \max_{0.5 \leq \alpha \leq 1} R_A(s^d(f), s^u(f)). \quad (12)$$

Due to the power complementarity of $s^u(f)$ and $s^d(f)$, the channel capacities c^u and c^d are equal. Therefore, we will only consider the upstream capacity c^u expression. Further, we will use the shorthand R_A for $R_A(s^u(f), s^d(f))$ in the remainder of this section. Let $G = \frac{2P_m}{WN}$ denote the signal-to-noise ratio (SNR) in the bin.

Substituting the PSDs (9) and (10) into (11) and using (12), we obtain

$$c^u = \max_{0.5 \leq \alpha \leq 1} \frac{W}{2}\left\{\log_2\left[1 + \frac{\alpha GH}{1 + (1-\alpha)GX + \alpha GF}\right] + \log_2\left[1 + \frac{(1-\alpha)GH}{1 + \alpha GX + (1-\alpha)GF}\right]\right\}. \quad (13)$$

Note from (12) and (13) that the expression after the max in (13) is the achievable rate R_A. Differentiating the R_A expression in (13) with respect to α yields

$$\frac{\partial R_A}{\partial \alpha} = G(2\alpha - 1)\left[2(X - F) + G(X^2 - F^2) - H(1 + GF)\right] L, \quad (14)$$

with $L > 0\ \forall \alpha \in (0, 1]$.

Setting this derivative to zero gives us the single stationary point $\alpha = 0.5$. The achievable rate R_A is monotonic in the interval $\alpha \in (0.5, 1]$. If the value $\alpha = 0.5$ corresponds to a maximum, then it is optimal to perform EQPSD signaling in this bin. If the value $\alpha = 0.5$ corresponds to a minimum, then the maximum is achieved by the value $\alpha = 1$, meaning it is optimal to perform FDS signaling in this bin. *No other values of α are an optimal option*. We can write test conditions to determine the signaling nature (FDS or EQPSD) in a given bin by solving (14):

$$\text{If } X^2 - F^2 - HF < 0, \text{ then}$$

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{EQPSD}}{\gtrless}} \frac{H - 2(X-F)}{X^2 - F^2 - HF}, \qquad (15)$$

If $X^2 - F^2 - HF > 0$, then

$$G = \frac{2P_m}{NW} \underset{\text{FDS}}{\overset{\text{EQPSD}}{\lessgtr}} \frac{H - 2(X-F)}{X^2 - F^2 - HF}. \qquad (16)$$

Note: In each bin the optimal spectra exclusively employ EQPSD or FDS signaling; that is, $\alpha = 0.5$ or 1 only. FDS scheme is a special case of the more general orthogonal signaling concept. However, of all orthogonal signaling schemes, *FDS signaling gives the best results in terms of spectral compatibility under an average power constraint* and hence is used here (see proof in [9]).

B.2 Optimal Spectra: All frequency bins

The above analysis dealt with only a single frequency bin centered around frequency f_k (see Figure 2). To obtain the complete optimal spectra, we apply the test conditions in (15) and (16) to each frequency bin in $[0, B]$. A simple iterative algorithm yields the complete optimal transmit spectra [9]:

1. Estimate which bins employ EQPSD signaling and FDS using (15) and (16). We have shown that in our case (low self-FEXT) we get an EQPSD region to the left of a switch-over bin M_{E2F} and FDS region to the right of it [9]. Estimate the switch-over bin M_{E2F}.
2. Perform optimal power distribution within the EQPSD region using water-filling [8] and in the FDS region using another optimization technique (optimization in the presence of self-interference) [10].
3. Loop between 1 and 2 until convergence is reached.

We have found a simple test condition that closely approximates the optimal switch-over bin M_{E2F} [9]. This approximation reduces the above algorithm to a single, computationally simple step of "water-filling".

The resulting spectra may not have contiguous power allocation over frequency. However, we present optimal ways of grouping bins in [9] to yield contiguous upstream and downstream spectra.

It is key to note that the optimal transmit spectra do not dictate any specific modulation scheme, but rather simply describe how a modulation scheme should optimally distribute its power over frequency. Thus, optimal spectra can be used with a number of different modulation schemes, including, but not limited to, DMT, CAP, QAM, PAM, etc.

IV. Simulation Results and Discussion

A. Examples

Figures 4, 5, and 6 illustrate the optimal transmit spectra on CSA loop 6 for HDSL2 in the presence of 49 HDSL, 25 T1, and 39 HDSL2 interferers, respectively.[1]

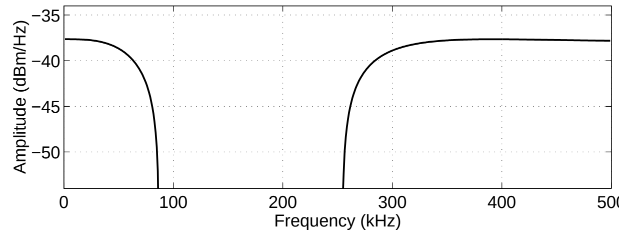

Fig. 4. *Optimal transmit spectra for HDSL2 on CSA loop 6 with 49 HDSL NEXT interferers and AGN of -140 dBm/Hz. Since there is no self-interference, FDS is not required. The upstream and downstream transmissions employ the same spectrum.*

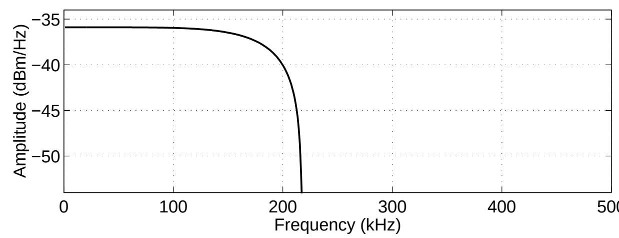

Fig. 5. *Optimal transmit spectra for HDSL2 on CSA loop 6 with 25 T1 NEXT interferers and AGN of -140 dBm/Hz. Since there is no self-interference, FDS is not required. The upstream and downstream transmissions employ the same spectrum.*

In the case of different service interferers (HDSL and T1 in Figures 4 and 5), the optimal upstream and downstream spectra are the same (EQPSD throughout). In the case of HDSL2 interferers (Figure 6), self-NEXT at high frequencies forces the optimal upstream and downstream spectra to separate in frequency giving rise to an FDS region. As an added bonus, no echo cancellation will be required in the large FDS region.

Note that the optimal transmit spectra vary significantly with the interference combination.

B. Performance margins

The amount of noise (in dB) a channel can sustain while maintaining a fixed bit rate and bit error rate is known as the *noise margin* or *performance margin* [14].

[1] Simulation Details:
Bit rate fixed at 1.552 Mbps. Total average input power (one-sided) in each direction $P_{\max} = 16.78$ dBm.
Different service interference models obtained from Annex B of T1.413-1995 (from [5], the ADSL standard), with exceptions as in [11]. Self-NEXT interference modeled as a 2-piece Unger model [6].
Margins calculated according to [12].
OPTIS transmit spectra obtained by tracking 1 dBm/Hz below the OPTIS PSD masks [13]. OPTIS performance margin figures from [13].
AGN of -140 dBm/Hz added to the interference.
DMT modulation scheme:
Sampling frequency $f_s = 1000$ kHz.
Bin width $W = 2$ kHz. Number of bins $K = 250$.
Start frequency = 1 kHz. Bit error rate = 10^{-7}.
SNR gap = 9.8 dB. No cyclic prefix. No limitation on maximum number of bits per tone.

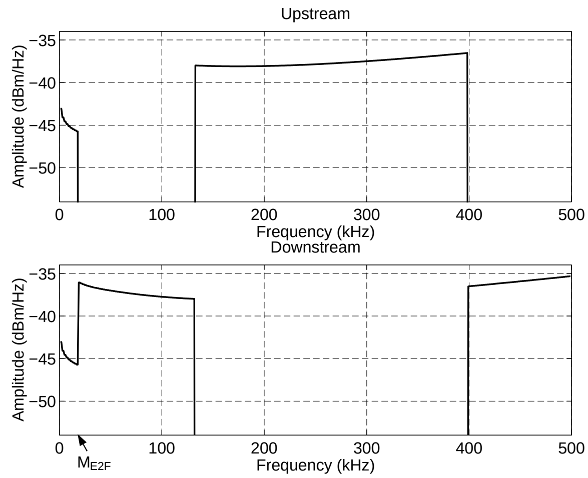

Fig. 6. *Optimal upstream and downstream transmit spectra for HDSL2 on CSA loop 6 with 39 self-NEXT and 39 self-FEXT interferers. EQPSD signaling takes place to the left of switch-over frequency M_{E2F} and FDS to the right. (Note that the placement of the FDS regions is nonunique. Here we choose a grouping such that upstream and downstream transmissions use equal transmit powers and result in equal performance margins. Other groupings are possible; see [9] for details.)*

TABLE I

Uncoded performance margins (in dB) for HDSL2 on CSA loop 6: OPTIS vs. Optimal. OPTIS figures were obtained from [13]. Diff = Difference between worst-case optimal and worst-case OPTIS.

Crosstalk source	OPTIS		**Optimal**		Diff
	Up	Dn	Up	Dn	
49 HDSL	2.7	12.2	**18.5**	**18.5**	**15.8**
25 T1	19.9	17.5	**21.3**	**21.3**	**3.8**
39 self	2.1	9.0	**18.3**	**18.3**	**16.2**
24 self+24 T1	4.3	1.7	**5.4**	**5.4**	**3.7**

Table I lists the performance margins of the optimal transmit spectra vs. those obtained using the OPTIS fixed transmit spectra (ANSI T1E1.4 committee's standard for the HDSL2 service [13]) for CSA loop 6. For different service interferers (HDSL and T1), only the NEXT powers were considered; for HDSL2, "self" comprises both self-NEXT and self-FEXT. Equal performance margins were obtained for upstream and downstream transmissions. We can clearly see that the optimal scheme outperforms OPTIS with large gains in all the cases.

V. Conclusions

In this paper, we have derived optimal transmit spectra for symmetric bit-rate communication channels dominated by crosstalk, in particular for DSLs. We solved an optimization problem to jointly maximize the capacity of each DSL line in a binder given the channel, noise, and crosstalk characteristics. The key advantages are:

1. Optimal transmit spectra yield large gains in performance margins compared to fixed-mask schemes. These gains can also be traded for increased bit rates or decreased average transmission power.
2. Optimal spectra are not bound to any particular modulation scheme.
3. Near-optimal transmit spectra are easy to compute.
4. Equal performance margins can be obtained for upstream and downstream directions.
5. FDS regions require no echo cancellation.
6. Transmit spectra can be adapted on-line to changes in line conditions (due to temperature variations, etc.).
7. Optimal spectra yield bounds on maximum achievable bit rates.

Our scheme requires a priori knowledge of the characteristics of neighboring interfering services. These can either be estimated at start-up or analyzed in a worst-case manner for a particular line under consideration. This information could also be obtained from a central office database that specifies the type of services in each binder group in the telephone cable.

References

[1] I. Kalet, "The Multitone Channel," *IEEE Trans. Commun., vol. 37, no. 2, Feb. 1989.*

[2] A. Sendonaris, V. Veeravalli and B. Aazhang, "Joint Signaling Strategies for Approaching the Capacity of Twisted Pair Channels," *IEEE Trans. Commun., vol. 46, no. 5, May 1998.*

[3] R.V. Gaikwad and R.G. Baraniuk, "Optimal Transmit Spectra for HDSL2 under a Peak Frequency-Domain Power Constraint," *ANSI T1E1 contribution T1E1.4/98-188R2.*

[4] R.V. Gaikwad and R.G. Baraniuk, "Optimal Transmit Spectra for HDSL2," *ANSI T1E1 contribution T1E1.4/98-162R1.*

[5] American National Standard for Telecommunications, "Network and Customer Installation Interfaces–Asymmetric Digital Subscriber Line (ADSL) Metallic Interface," *T1.413-1995.*

[6] K.J. Kerpez, "Full-duplex 2B1Q Single-pair HDSL Performance and Spectral Compatibility," *ANSI T1E1 contribution T1E1.4/95-127.*

[7] K.J. Kerpez, "Near-End Crosstalk is almost Gaussian," *IEEE Trans. Commun., vol. 41, no. 1, Jan. 1993.*

[8] R.G. Gallager, "Information Theory and Reliable Communication," *New York: Wiley, 1968.*

[9] R.V. Gaikwad and R.G. Baraniuk, "Spectral Optimization and Joint Signaling Techniques for Communication in the Presence of Crosstalk," Technical report #9806, Department of Electrical and Computer Engineering, Rice University, July 1998. On Web: `www.dsp.rice.edu/~rohitg/Papers/tr9806.ps`

[10] J.T. Aslanis and J.M. Cioffi, "Achievable Information Rates on Digital Subscriber Loops: Limiting Information Rates with Crosstalk Noise," *IEEE Trans. Commun., vol. 40, no. 2, Feb. 1992.*

[11] G. Zimmerman, "Performance and Spectral Compatibility of OPTIS HDSL2," *ANSI T1E1 contribution T1E1.4/97-237.*

[12] G. Zimmerman, "Normative text for Spectral Compatibility Evaluations," *ANSI T1E1 contribution T1E1.4/97-180R1.*

[13] J. Girardeau, M. Rude, H. Takatori and G. Zimmerman, "Updated OPTIS PSD Mask and Power Specification for HDSL2," *ANSI T1E1 contribution T1E1.4/97-435.*

[14] M. Barton and M.L. Honig, "Optimization of Discrete Multitone to Maintain Spectrum Compatibility with Other Transmission Systems on Twisted Copper Pairs," *IEEE J. Select. Areas Commun., vol. 13, no. 9, pp. 1558-1563, Dec. 1995.*