RICE

Wavelet-Based Deconvolution using Optimally Regularized Inversion for Ill-Conditioned Systems

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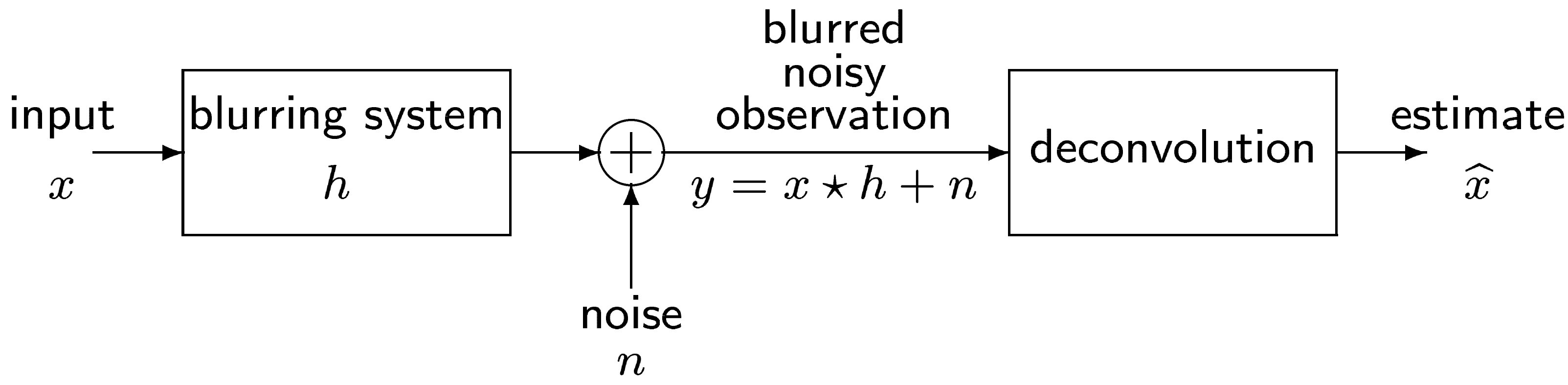
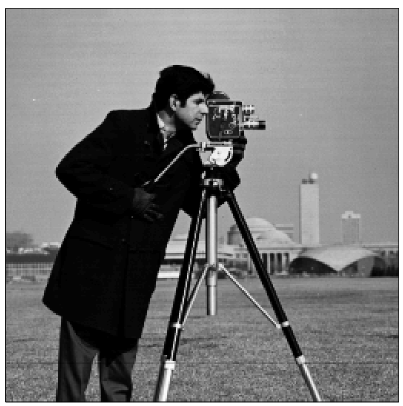
Deconvolution





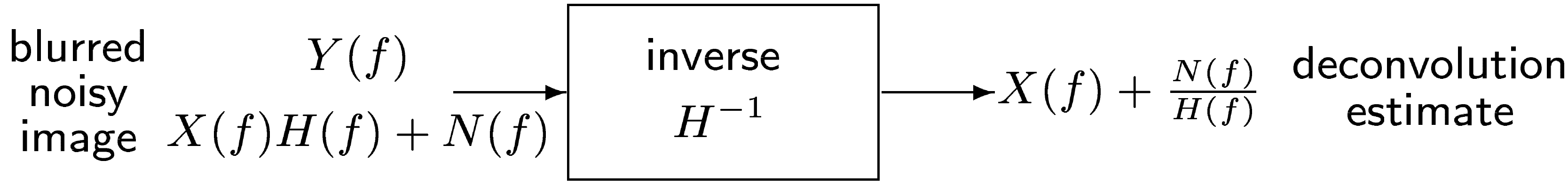
input

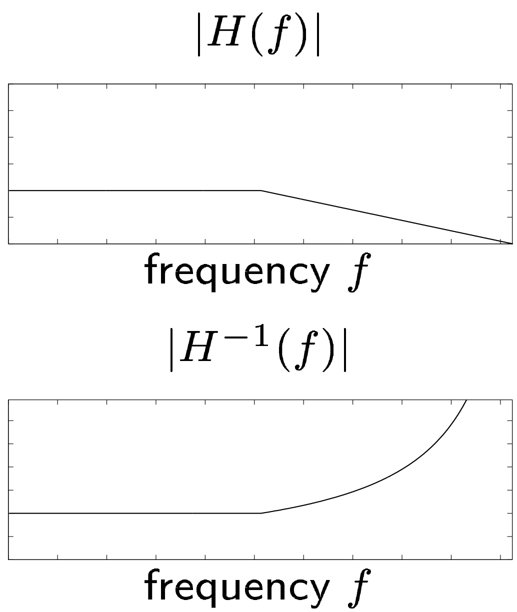
observed

estimate

- Problem: given y, h, find x
- Applications: satellite imagery, seismic exploration, ...

Deconvolution is Ill-Posed

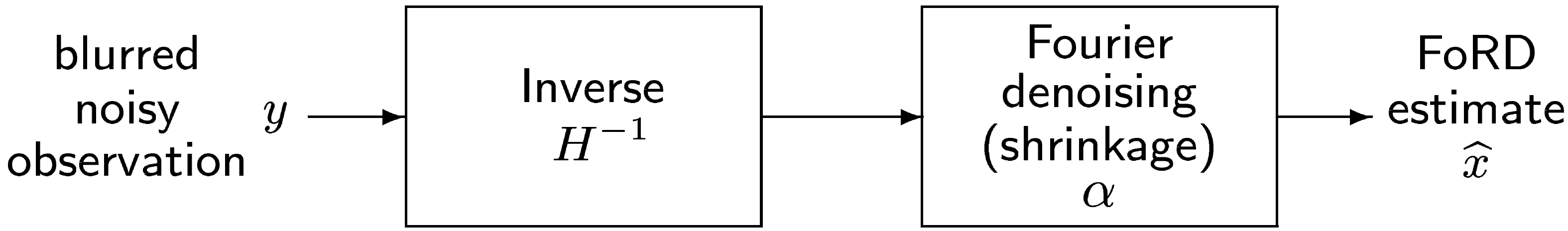




after pure inversion

- $|H(f)| \approx 0 \;\Rightarrow\;$ noise $\frac{N(f)}{H(f)}$ explodes!
- Solution: *regularization* (approximate inversion)

Fourier-domain Regularized Deconvolution (FoRD)



- Fourier transform diagonalizes H

  $\Rightarrow$ identifies and attenuates amplified noise frequency components

- Ex: Wiener filter (MSE-optimal LTI estimator)

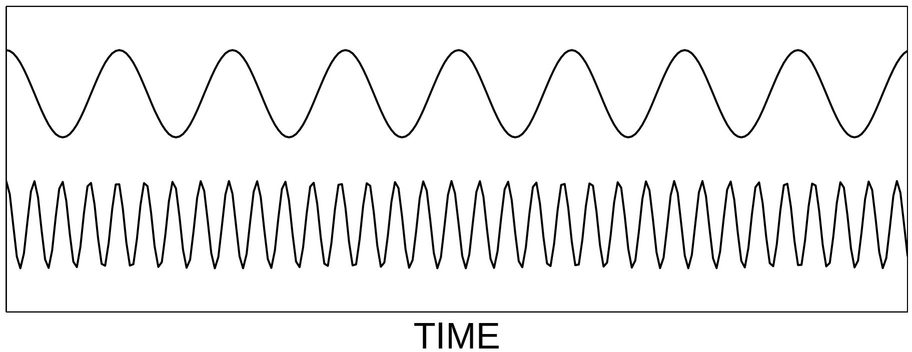
$$\widehat{X}(f) = Y(f) \frac{1}{H(f)} \frac{|H(f)|^2 \, \mathsf{SNR}(f)}{|H(f)|^2 \, \mathsf{SNR}(f) + \alpha}$$

  – SNR high $\Rightarrow$ less shrinkage SNR low $\Rightarrow$ more shrinkage

- Inversion and shrinkage done together in practice

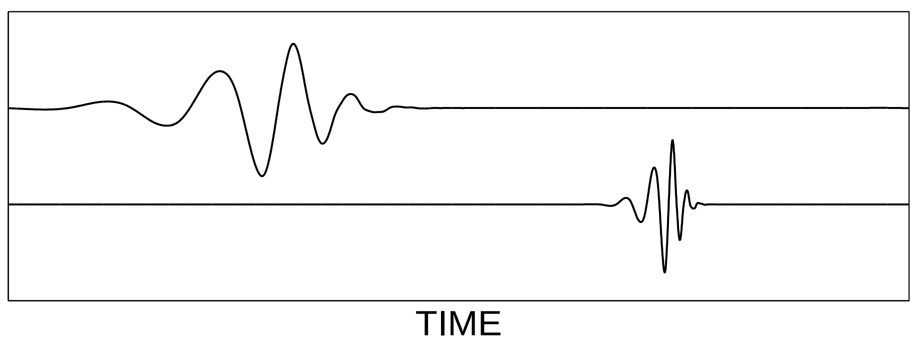
Matchmaking

- Fourier basis: not suited for images with edges

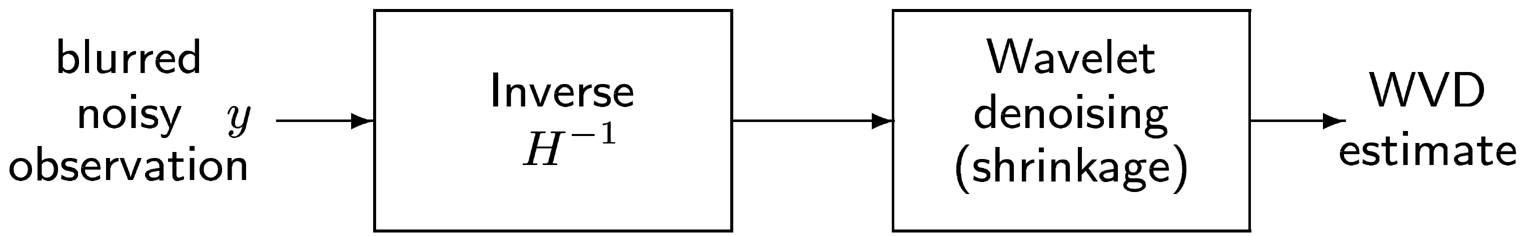


Wiener estimate

- Fourier basis: *matched* to operator but *unmatched* to signal
- Wavelets: *matched* to signal (economical representation)

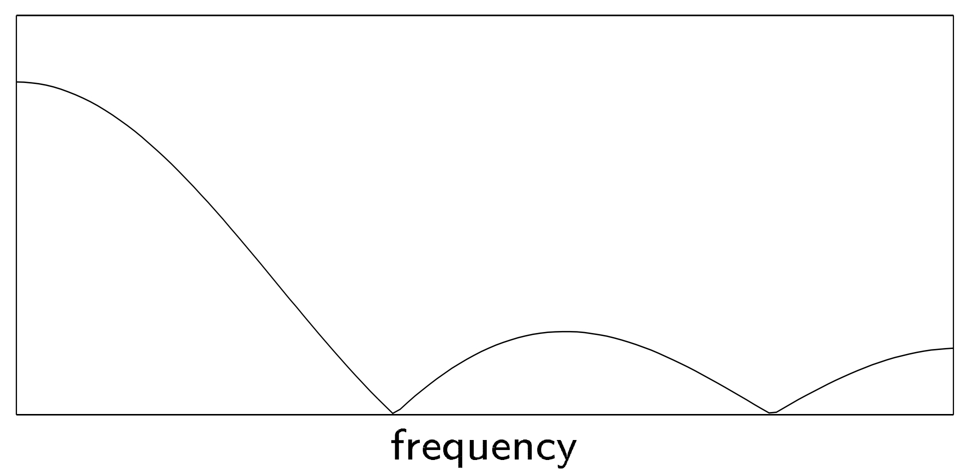


Wavelet Vaguellete Deconvolution (WVD)

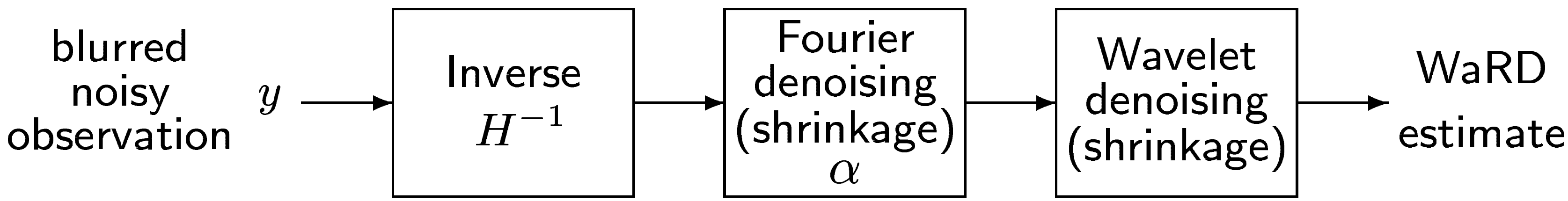


- Donoho '95: near optimal performance for *certain* H
- Kalifa el al. '98: extended class of applicable H
- Bottom line: do not apply to arbitrary H

Ex: ⊓ blur



Wavelet-domain Regularized Deconvolution (WaRD)



- Fourier denoising: exploits convolution operator structure

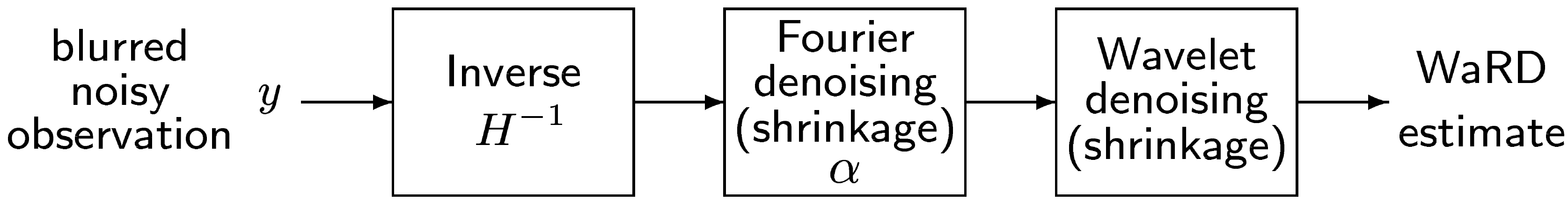
  Wavelet denoising: exploits input signal structure

- Choice of α: balance Fourier and wavelet denoising

- Applicable to all convolution operators

- Simple and fast algorithm: $O(M \log_2^2 M)$ for M pixels

How Much Fourier Regularization?



- Tradeoff:

$$\alpha \text{ large } \Rightarrow \begin{cases} \text{noise in wavelet domain reduced} \\ \text{edges smeared} \end{cases}$$

$$\alpha \text{ small } \Rightarrow \begin{cases} \text{noise in wavelet domain increased} \\ \text{edges preserved} \end{cases}$$

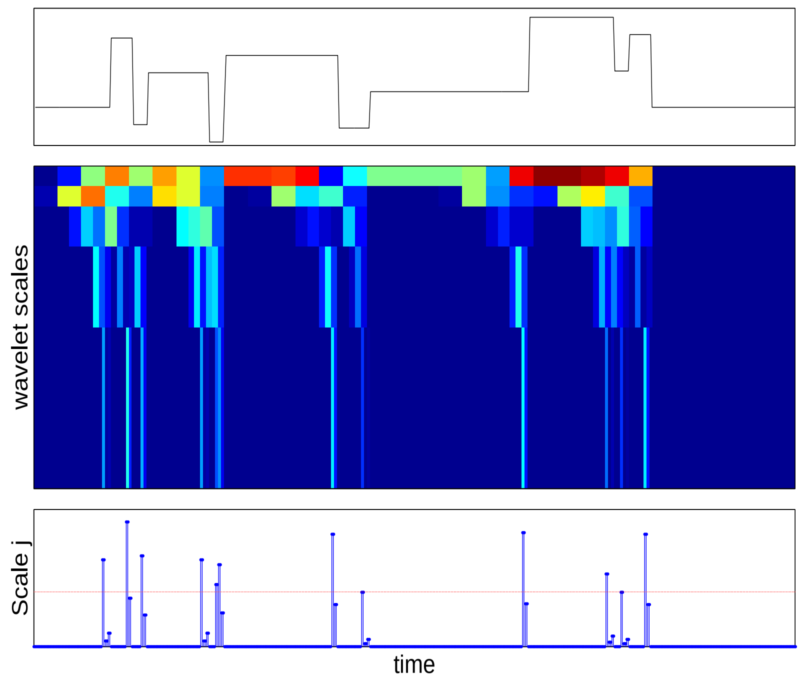
- Goal: balance Fourier and wavelet denoising

- Criterion: minimize overall MSE

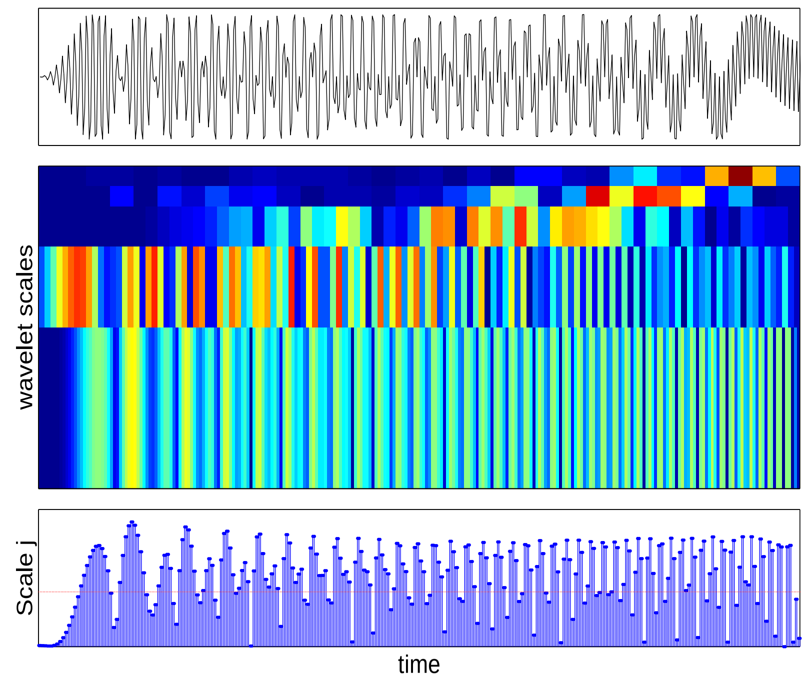
$$\text{MSE} \approx \begin{array}{c}\text{Fourier}\\ \text{distortion}\\ \text{error}\end{array} + \begin{array}{c}\text{wavelet}\\ \text{thresholding}\\ \text{error}\end{array}$$

Optimal regularization α

Optimal α for wavelet scale j = % of wavelet coeffs > noise variance in scale j



small α (rely on wavelet denoising)



large α (rely on Fourier denoising)

Optimal balance controlled by economics of wavelet representation

Optimality of WaRD

- $\alpha = 0 \Rightarrow$ WaRD = WVD
  - WaRD inherits asymptotic optimality from WVD
  - Optimal $\alpha \neq 0 \Rightarrow$ WaRD outperforms WVD (small samples)
- Optimal $\alpha \ll 1$ (for most real world signals)
- WaRD applies for all convolution operators

Original

Observed

Wiener (SNR = 20.6 dB)

WaRD (SNR = 22.4 dB)

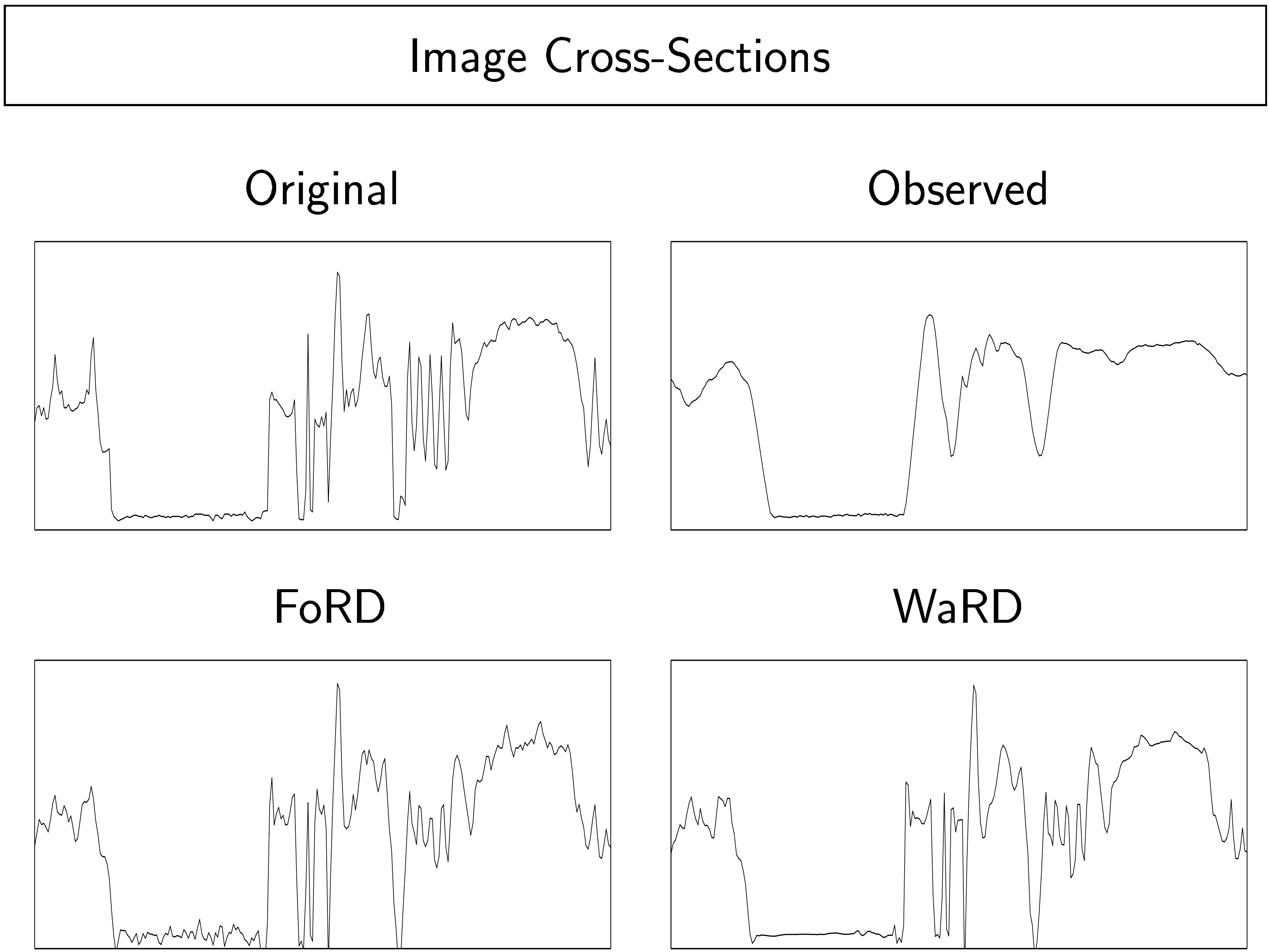


Image Cross-Sections
Original
Observed
FoRD
WaRD

Conclusions

- WaRD: Balances Fourier-domain and wavelet-domain denoising
- WaRD simultaneously preserves critical edges and smooth regions
- Simple and fast algorithm: $O(M \log_2^2 M)$ for M pixels
- Near-optimal asymptotic performance
  Good small sample performance
- Applicable even when H not invertible
- Outperforms conventional WVD

Web: `http://www.dsp.rice.edu`