Instantaneous frequency estimation using the reassignment method

Jan E. Odegard, Richard G. Baraniuk and Kurt L. Oehler*
Dept. of ECE, Rice University, Houston, Texas

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SUMMARY

This paper explores the method of reassignment for extracting instantaneous frequency attributes from seismic data. The reassignment method was first applied to the spectrogram by Kodera, Gendrin and de Villedary [5] and later generalized to any bilinear time-frequency or time-scale representation by Auger and Flandrin [1]. Key to the method is a nonlinear convolution where the value of the convolution is not placed at the center of the convolution kernel but rather reassigned to the center of mass of the function within the kernel. The resulting reassigned representation yields significantly improved component localization. In this paper we will study the impact of the reassigned time-frequency representation on our ability to reliably estimate instantaneous frequency for a given seismic signal.

INTRODUCTION

For non-stationary signals found in many important applications, the right tools to apply are time-frequency representations (TFRs), which generalize the concept of the musical score to distributions that measure how the frequency content of a signal changes over time. TFRs like the short-time Fourier transform (STFT), the wavelet transform, and the Wigner distribution figure prominently in a host of different problems, including geophysics; data compression; image coding and analysis; communications; and speech and acoustic signal processing.

Seismic attributes aid the quantitative interpretation of seismic data by extracting information on the nature of its non-stationarity. Attributes such as the instantaneous frequency, bandwidth, and quality absorption parameter Q appear promising. However, these quantities have been defined only in terms of idealized narrowband signals (frequency modulated tones). Since seismic signals generally contain multiple components — and thus have several "instantaneous frequencies" occurring simultaneously — more general seismic attributes must be developed.

The time-frequency plane is a rich feature space for developing new seismic attributes (see [6, 7], for example). A TFR of a multicomponent seismic signal consists of sets of ridges, the orientations and widths of which characterize the signal. For example, once computed, time-frequency images can be processed using edge detection and other image processing algorithms to automatically determine the ridge parameters.

The STFT and continuous wavelet transform have been suggested for the first, image generation step of the feature extraction procedure. In this paper, we will explore the capabilities of seismic attributes based on a new TFR, the reassignment method [5, 1].

TIME-FREQUENCY REPRESENTATIONS

In this Section, we briefly review the elements of the theory of TFRs that we will employ in the sequel. TFRs are two dimensional functions of time t and frequency f that indicate how the frequency content of a signal x changes over time.

The simplest TFR is the *spectrogram,* the squared magnitude of the STFT

$$S_x(t,f) \;\equiv\; \int x(\tau)\, w^*(\tau - t)\, e^{-i2\pi f\tau}\, d\tau. \qquad (1)$$

The classical time-frequency resolution tradeoff of the spectrogram, which is controlled by the analysis window w, has prompted the development of more advanced bilinear TFRs, including the *Wigner distribution* [4]

$$W_x(t,f) \;\equiv\; \int x\Big(t+\frac{\tau}{2}\Big)\, x^*\Big(t-\frac{\tau}{2}\Big)\, e^{-i2\pi f\tau}\, d\tau. \qquad (2)$$

This TFR can be interpreted as a short-time Fourier transform with the window matched to the signal. While the Wigner distribution is highly concentrated, due to its nonlinearity it generates cross-components and is very sensitive to noise.

The spectrogram and Wigner distribution both belong to *Cohen's class* of TFRs. The Wigner distribution can be interpreted as the central, generating member of this class, with each Cohen's class TFR C obtained via the two-dimensional correlation [4]

$$C_x(t,f) \;=\; \iint W_x(\tau,\nu)\, \phi(\tau - t, \nu - f)\, d\tau\, d\nu \qquad (3)$$

with ϕ the *kernel* of C. The spectrogram kernel is the Wigner distribution of the analysis window itself, $\phi_{\text{spec}} = W_w$. Without loss of generality, we will assume that $\phi(t,f)$ is centered at $(0,0)$ in the time-frequency plane.

THE REASSIGNMENT METHOD

Cohen's class TFRs have a simple interpretation as a *smoothed Wigner distributions*: To compute a TFR at the point (t,f) in time-frequency using (3), we translate the kernel ϕ to (t,f), multiply with the

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Wigner distribution W_x, integrate the product, and then place the result at the point (t, f) (see Figure 1(a)). For lowpass kernels ϕ, smoothing suppresses cross-components and reduces noise sensitivity in the Wigner distribution, but simultaneously "smears" the time-frequency representation.

The idea behind reassignment is simple and ingenious [5, 1]: Rather than placing the convolution value (the integral of the $W_x\,\phi$ product) at the center point (t, f) of the kernel ϕ, we place the value at the *center of mass* of the $W_x\,\phi$ product (see Figure 1(b)). Reassignment smoothes the Wigner distribution, but then refocuses the TFR back to the true regions of support of the signal components. For many "real world" signals, a highly concentrated TFR results. Note that a reassigned distribution is highly nonlinear, and no longer merely quadratic as in (3).

An elegant algorithm for computing reassigned TFRs is given in [1]. In particular, the reassigned spectrogram can be computed from three different spectrograms (using windows $w(t)$, $tw(t)$ and $\frac{d}{dt}w(t)$).

In Figure 2, we demonstrate the power of reassignment on a multicomponent synthetic signal. Notice the sharp frequency focusing effect of the reassignment method. For additional examples, see [5, 1].

INSTANTANEOUS FREQUENCIES

Traditional instantaneous seismic attributes of a seismic trace $x(t)$ are based on the amplitude and phase representation [4, 8]. Given a real signal $s_r(t)$ we seek to define a complimentary imaginary signal component $s_i(t)$ such that the interpretation of the complex signal achieves a sensible physical and mathematical description,

$$x(t) \;=\; s_r(t) + j s_i(t) \;=\; A(t)\, e^{j\theta(t)}, \tag{4}$$

where $A(t)$ represents the instantaneous amplitude and $\theta(t)$ the instantaneous phase. Hence, notice that $x(t)$ is non-unique and the challenge is to define $s_i(t)$ such that $A(t)$ and $\theta(t)$ is robust to signal noise and gives raise to a meaningful interpretation of signal properties of $s_r(t)$. However, assuming that $s_i(t)$ can be defined the instantaneous frequency is unambiguously defined by

$$\gamma_1(t) = \frac{d}{dt}\theta(t) \tag{5}$$

[4, 8]. While noise sensitivity is a real issue a second consequence of the above analysis for extracting instantaneous frequency and amplitude is that it only gives a single instant frequency for each time and hence can not deal with multicomponent signals. For example, instant frequency of two sinusoidal tones at frequencies f_1 and f_2 is at the average frequency $\frac{1}{2}(f_1 + f_2)$ [2].

An alternative approach to instantaneous frequency applicable to complicated multicomponent signals is to track ridges in time-frequency domain. Using this method multiple strong frequency components can be extracted for each time t giving raise to a vector $\underline{\gamma}_2(t)$ of instantaneous frequencies for a given time. An algorithm for extracting multicomponent frequency vectors is numerically quite fast given the desired TFR. Alternatively, if we view a TFR as an energy density function one can look at statistical-type averages such as

$$\gamma_3(t) \;=\; \frac{\int f\, C(t,f)\, df}{\int C(t,f)\, df}, \tag{6}$$

which can also be generalized to a "local mean" for multicomponent signals.

SEISMIC DATA EXAMPLE

In Figure 3 we compare the spectrogram with the reassigned spectrogram and the corresponding multicomponent estimates $\underline{\gamma}_2(t)$ for a particular seismic trace.

CONCLUSIONS

The reassignment method shows great promise for analysis of seismic signals with the goal of extracting instantaneous attributes. Further analysis is still required and the computational speed will be a real issue given that one needs to first compute the equivalent of multiple time-frequency representation before instantaneous attributes can be extracted based on a reassigned TFR. In fact, how to reassign in discrete grid of TF points is nontrivial. Furthermore, while some preliminary results can be found in Chassande-Mottin, Auger and Flandrin [3] additional analysis of noise sensitivity is required.

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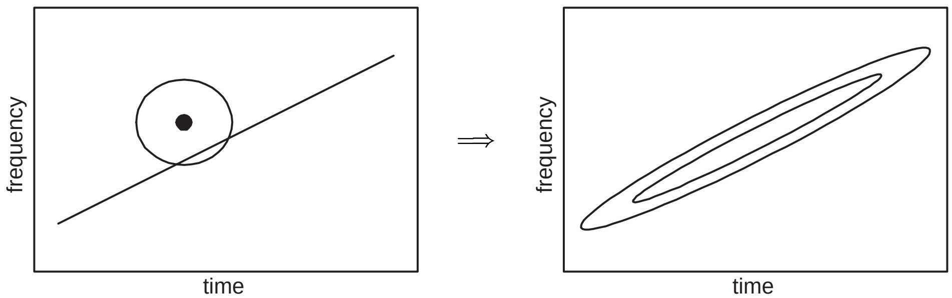
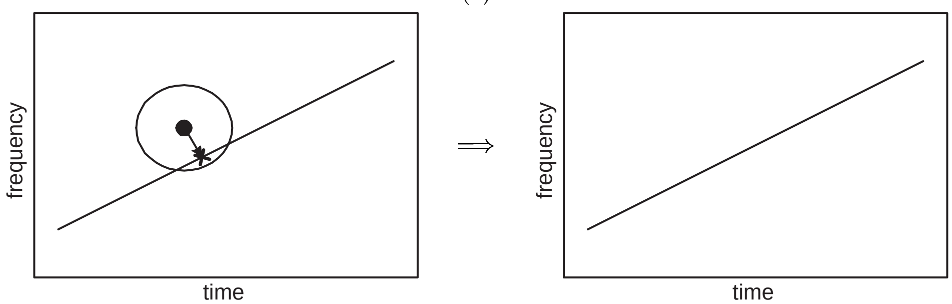
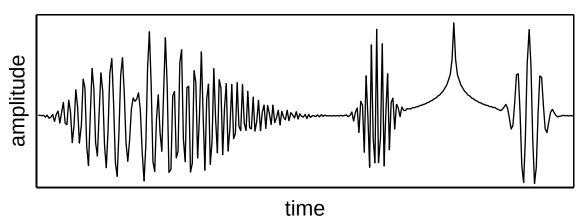


(a)

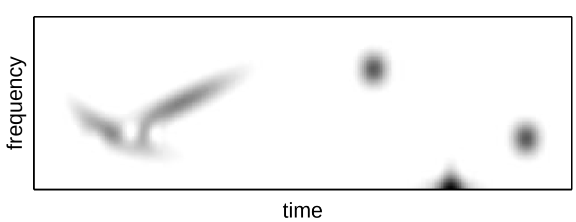


(b)

Figure 1: A pictorial comparison of "normal convolution" versus the "reassignment convolution" using a simple linear chirp test signal. (a) Normal convolution and the resulting time-frequency plan after performing a normal convolution giving raise to a classical TFR with significant smearing of the frequency. (b) Illustrates the reassignment type convolution. Notice how the result of convolving the kernel with the signal is reassigned to the center of mass within the kernel support giving raise to the reassigned TFR.



(a)



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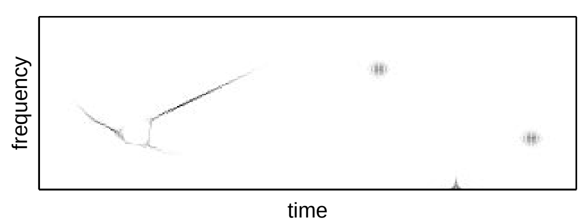

(c)

Figure 2: (a) Synthetic test signal. (b) Spectrogram. (c) Reassigned spectrogram. Notice the sharp focusing of the frequency achieved by applying the reassignment method to the spectrogram.

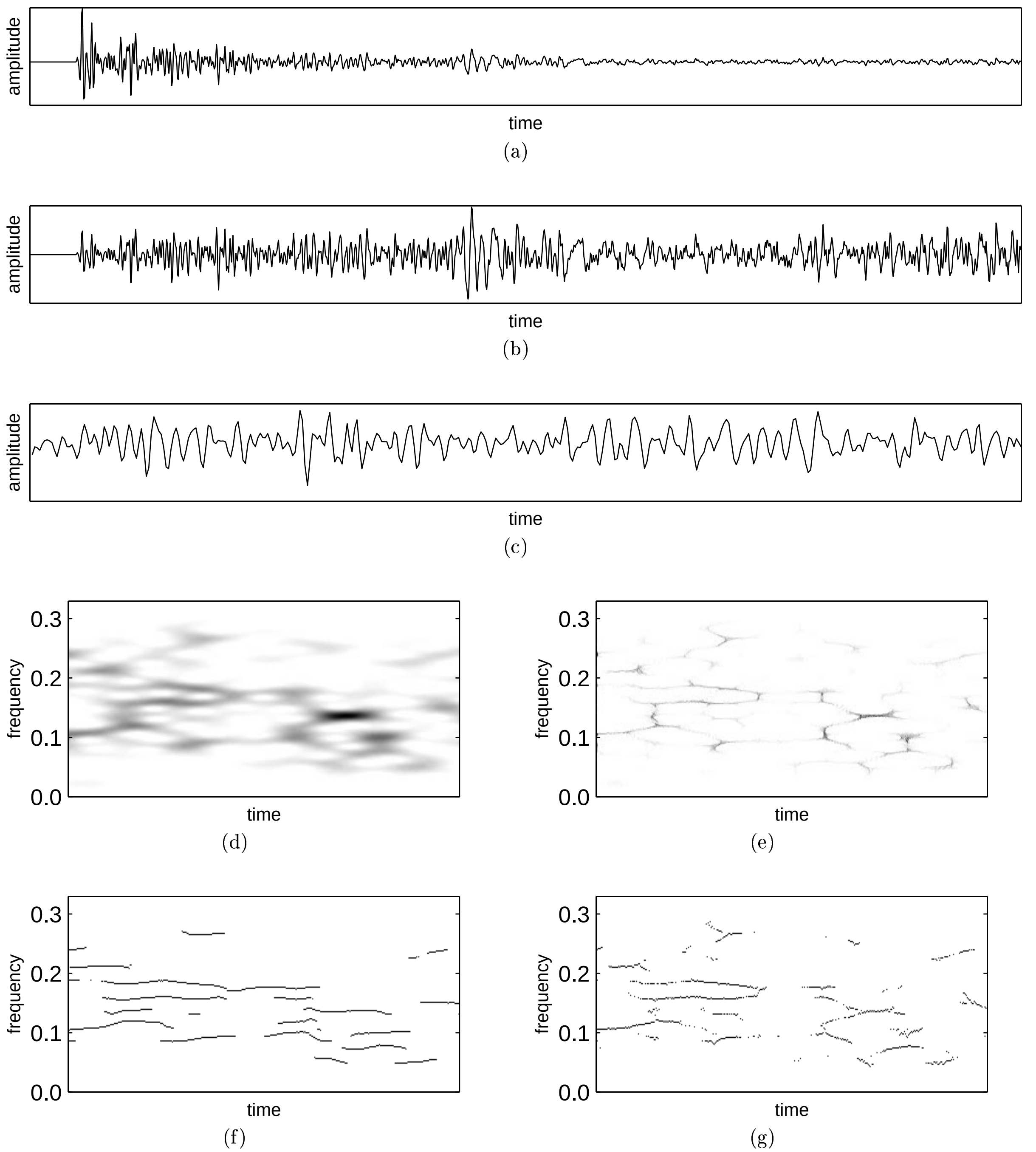

Figure 3: (a) Original seismic trace courtesy Caspian Geophysical and the Mobil Corporation. (b) Seismic trace gain equalized. (c) 400 point subset of trace to be analyzed. (d) Spectrogram (Nyquist=0.5). (e) Reassigned spectrogram (Nyquist=0.5). (f) $\underline{\gamma}_2(t)$ extracted based on the spectrogram. (g) $\underline{\gamma}_2(t)$ extracted based on the reassigned spectrogram.