Directional Scale Analysis for Seismic Interpretation

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SUMMARY

A combined space-Fourier representation focused on directional scale analysis is presented. The method leads to a space - log polar frequency distribution. Application to seismic data shows differentiation in scale and orientation. Attributes are extracted to illustrate this differentiation.

INTRODUCTION

Time-frequency distributions have proven to be a powerful tool to get more insights in seismic data (Steeghs, 1997). For seismic characterization a signal can be described in a combined time-frequency domain and as such it can be used for identification and characterization of certain stratigraphic sequences and fault patterns (Steeghs, 1997).
There are many ways to obtain a time-frequency decomposition. In this paper we choose a scale partitioning of the time-frequency plane. The motivation for this is not so much that the earth dilates the source signature (scale), but more that the earth's response (seismic migrated section) appears through different scales. Appropriately this could be called multi-resolution analysis.
For the extension of the wavelet transform to higher dimensions, directional selectivity is required with the eye on laterally varying formations. We do not use the conventional multi-dimensional wavelet transform for this purpose, because these filters have rectangular shapes in the Fourier domain and hence are not designed for directional selectivity. We use a polar scale decomposition or "steerable pyramid" (Freeman and Adelson, 1991; Simoncelli et al., 1992). The operator results in a scale distribution along certain bands of orientations, and can hence be used for *directional scale* analysis.
In this paper a Fourier-based polar scale decomposition is described resulting in a representation in which each spatial coordinate has a distribution in orientation and log polar frequency (scale).
Scale-analysis is discussed, followed by the extension to more dimensions and an angular decomposition. We conclude with some 2-D migrated seismic sections to illustrate some basic attributes that can be extracted from their directional scale representations.

SCALING AND WAVELETS

In order to explain the relevance of using scales in seismic image analysis, we will first briefly address the basic ideas behind wavelet transforms and filterbanks. Wavelet and scale analysis have proven to be very useful in all kinds of signal and image processing applications like analysis, compression and denoising. Wavelet transforms give more insight in the signal shapes and their combined time-frequency behavior without extensive data increase. The wavelet transform is basically a set of bandpass filters applied to the original input signal and downsampled subsequently (Burrus et al., 1998). These bandpass filters can be applied either in the time or in the Fourier domain. Throughout this paper we will work with filterbanks, the Fourier equivalent of wavelets, because we are still investigating convolution of our image with short support 2-D wavelets. Sometimes these have advantages for efficiency and accuracy in edge detection.
The filterbanks are obtained by cascading lowpass and highpass filters. In Fig.(1) an analysis tree and a reconstruction tree are shown for two steps.
The lowpass and highpass filters are indicated by

$$L_j(\omega) = L\left(\frac{\omega}{2^j}\right) \quad \text{and} \quad H_j(\omega) = H\left(\frac{\omega}{2^j}\right), \tag{1}$$

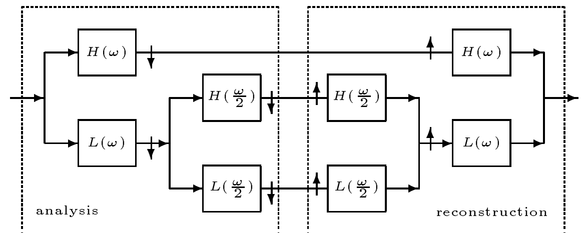

Figure 1: Analysis and reconstruction filterbanks trees obtained by cascading low- and highpass filters.

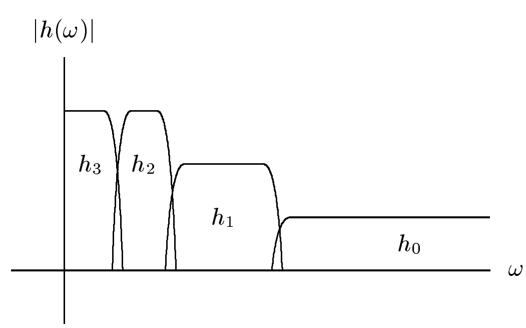

Figure 2: Frequency bands in a wavelet transform. $h_0 - h_3$ are the resultant filterbanks.

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where j indicates the scaling space in which the filter is located and $L(\omega)$ and $H(\omega)$ are lowpass and highpass filters. The resulting filterbanks $h_0 - h_3$ that are convolved with the original signal in order to obtain the wavelet coefficients are shown in Fig.(2).

The bandwidths of the filters have a logarithmic spacing, which is representative for scaling. For perfect reconstruction we need conservation of energy, so the filters $L(\omega)$ and $H(\omega)$ are constrained by the following condition:

$$\left|L\left(\frac{\omega}{2^j}\right)\right|^2 + \left|H\left(\frac{\omega}{2^j}\right)\right|^2 = 1, \quad \text{for} \quad j = 0, 1, .., N \qquad (2)$$

and the condition that the filters are real.

Each filtered portion of the signal can be proportionally downsampled in order to prevent redundancy of information, because they are bandlimited (Oppenheim and Schafer, 1989) . This results in a critically sampled wavelet transform in which the aliasing in one frequency band is undone by the other frequency bands.

ORIENTATION AND SCALE

In order to analyse images, we need a way to describe multi-dimensional scaling. A lot of work has been done in this direction, especially for compression and image coding. Usually for these kind of purposes a straight forward two-dimensional wavelet transform is used.

Conventional 2-D wavelet transforms have the great advantage of having separable basis functions spanning the subspaces so they are quite easily implemented in the spatial domain. The problem however is that the filters have rectangular shapes in the Fourier domain and thus do not describe orientation optimally.

For directional analysis, the Fourier domain may better be parameterized in terms of polar coordinates. Intuitively, this is a more natural coordinate system for directional analysis than the Cartesian system, because the angular coordinate corresponds to orientation and the (logarithmic) radial component corresponds to polar frequency (scale) at that specific orientation.

The decomposition can be used for analysis in a combined space - log polar Fourier domain as opposed to the conventional space-Fourier domain. Translation of a signal in this two-dimensional log polar Fourier domain corresponds to dilation and rotation of the signal in the spatial domain. The transform with this characteristic partitioning of the Fourier plane is called a steerable pyramid (Freeman and Adelson, 1991), for the filters are steerable in orientation and have a logarithmic pyramid structure as in the wavelet transform. In this paper we have implemented the steerable pyramids in the Fourier domain. The subbands in this decomposition are polar separable. The partitioning of the Fourier domain for a decomposition into two scales ($j = 2$) and four orientation bands ($\alpha = 4$) is shown in Fig.(3). The filterbanks in this particular partitioning are obtained by cascading three basis filters, a polar lowpass filter, a polar highpass filter and an orientational bandpass filter. The low and highpass filters are indicated by $L(\boldsymbol{\omega})$ and $H(\boldsymbol{\omega})$, the bandpass filters by $B^{\alpha}(\boldsymbol{\omega})$, in which the angleband is indicated by α, and $\boldsymbol{\omega} = (k_x, f)$. The filterbanks $H_j^{\alpha}(\boldsymbol{\omega})$ in Fig.(3) are then given by

$$H_j^{\alpha}(\boldsymbol{\omega}) = L\left(\frac{\boldsymbol{\omega}}{2^{j-1}}\right) H\left(\frac{\boldsymbol{\omega}}{2^j}\right) B^{\alpha}(\boldsymbol{\omega}) \qquad (3)$$

For perfect reconstruction we constrain the filters by

$$\left|H\left(\frac{\boldsymbol{\omega}}{2^j}\right)\right|^2 + \left|L\left(\frac{\boldsymbol{\omega}}{2^j}\right)\right|^2 = 1, \quad \text{for } j = 1, 2, .., N \qquad (4)$$

$$\sum_{\alpha=\alpha_1}^{\alpha_M} |B^{\alpha}(\boldsymbol{\omega})|^2 = 1, \qquad (5)$$

in which N is the number of scales and α_M gives the number of orientation bands. Furthermore we constrain the filter to be real.

SEISMIC DATA EXAMPLE

From seismic migrated images we try to extract relevant information about geological structures. Horizons are tracked with correlation algorithms, faults are often tracked by hand and other stratigraphic information about rock properties and sedimentary environments are interpreted by geologists with the bare eye and with the assistance of attributes. The question is if there is a mathematical formulation that describes what is observed now with the bare eye. It is important to study the character of seismic signals in a multi-dimensional way. For instance, faults can be interpreted ideally as step functions in the image. These step functions are mainly oriented in a near vertical direction. Horizons are continuous amplitudes that mainly extend in a near horizontal direction.

The steerable pyramid describes the image locally in orientation and scale. This seems an appropriate domain to characterize seismic images and the geological features they describe. In our implementation we used analytic

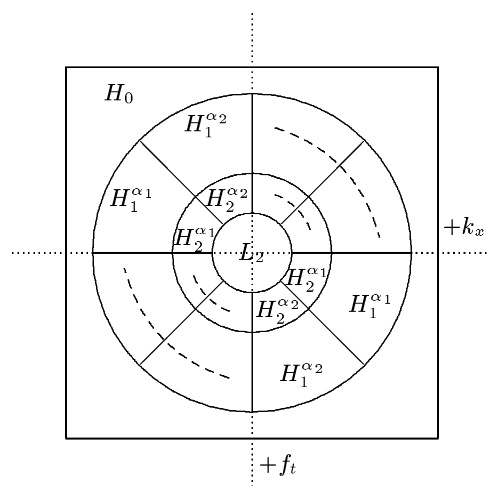

Figure 3: Spectral decomposition performed by the steerable pyramid. L_2 is a lowpass filter, H_j^{α} are the oriented wavelet subbands at scale j and angle α, H_0 indicates the residual band.

filters, in which the imaginary part is obtained by taking the temporal Hilbert transform of the filter. In the Fourier domain this basically means that we have taken the inner product of the filters with a step function $\mathcal{H}(\omega)$, defined by

$$\mathcal{H}(\omega) = \{0, 1, 2\} \text{ for } \{\omega < 0, \omega = 0, \omega > 0\} \qquad (6)$$

and hence mute the negative frequencies. This prevents aliasing and zero-crossing problems in the space domain for analysis and attribute calculation.

In Fig.(5) a representation is shown of the absolute value of a complex steerable pyramid up to five scales and eight orientation bands of the seismic image in Fig.(4 a.). The center angles are noted along the sides at the highest scales. Decreasing framesize indicates increasing scale.

This Figure clearly illustrates the difference of the pyramid coefficients of the faults and of the stratigraphy. The first remarkable observation is that the stratigraphy lives mainly in a very narrow set of near horizontal orientation bands and the faults tend to live in more vertical orientation bands. The difference not only profilates in the angles, but also in scales. Although the faults lack high energies, they can be observed at the highest scales. The layering mainly manifests itself in the four lower scales. The stratigraphy shows differentiation through scales.

We have performed an inverse complex steerable pyramid transform only including bands containing fault information in order to get a feeling for the selectivity of the transform and the ability to cut out certain parts that appear to have different coefficients. In order to get a clear picture we have put a threshold on the included bands. The resulting image is plotted in Fig.(4 b.). Although the faults don't show perfect continuous lines, they indicate the location with a sufficient precision. Hardly any information of the stratigraphy is visible in the image. This is a very rough way of choosing an attribute, but it shows the ability to separate information.

For three-dimensional seismic data the dip attribute can be very useful and gives much insight in the structures. In order to calculate a local dip, the seismic section in Fig.(6) has been transformed to the directional scale domain consisting of sixteen angles and four scales. After inverse transformation of the wedges separately, we picked the maximum value from the resulting sixteen dip representations at each space location. By noting in which of the orientation bands the maximum was located we found the local dip at each location. Results of this analysis can be found in Fig.(6) (right) in which the grayvalue is representative for the dip.

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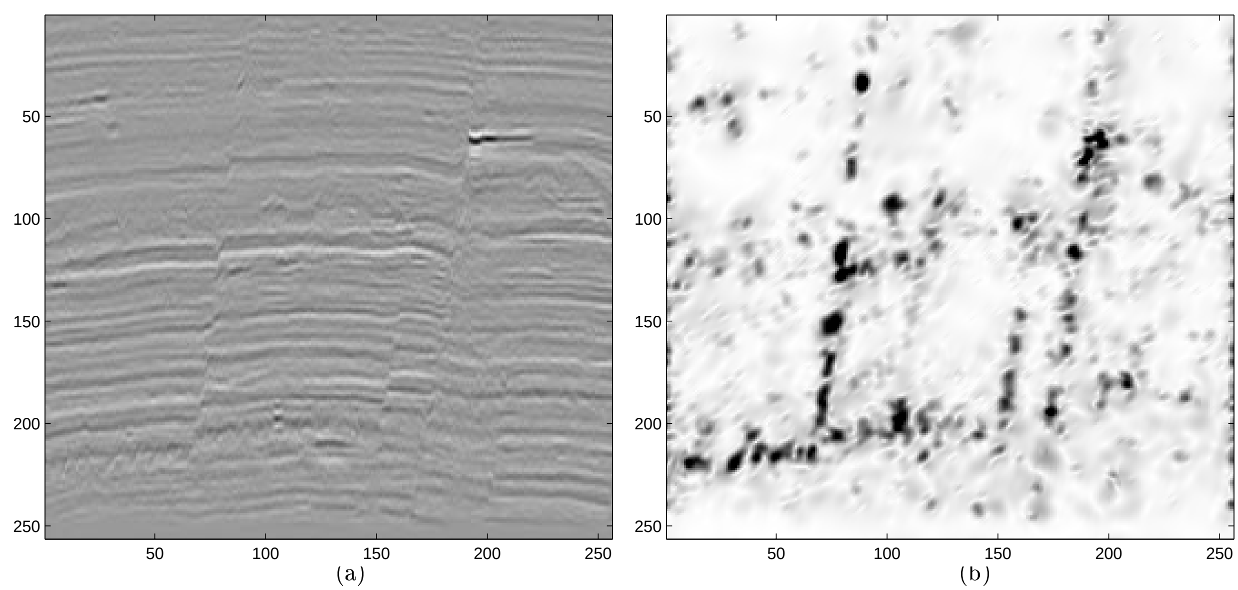

Figure 4: Seismic section and extracted faults

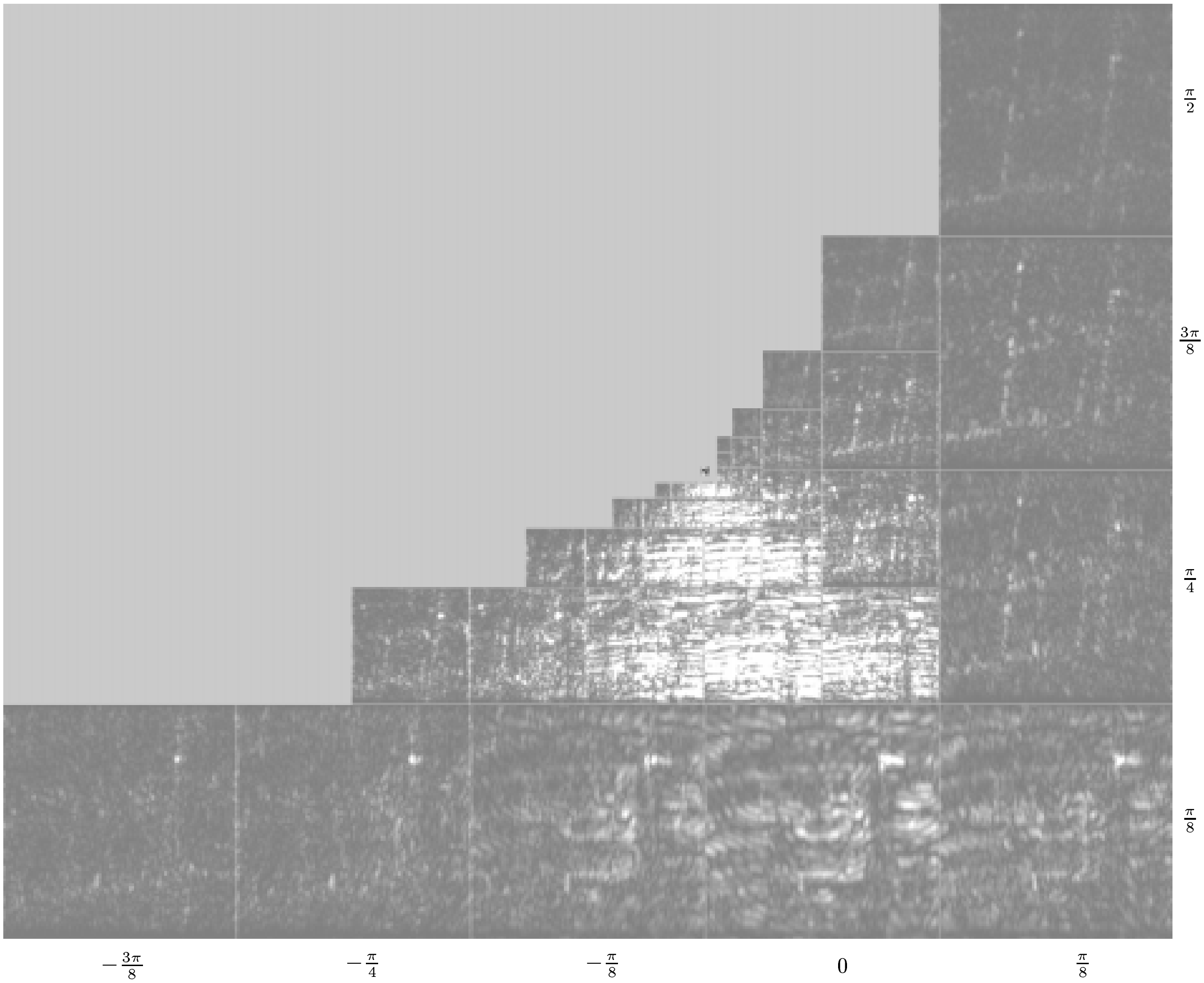

Figure 5: Steerable pyramid representation of the section in Fig(4 a.)

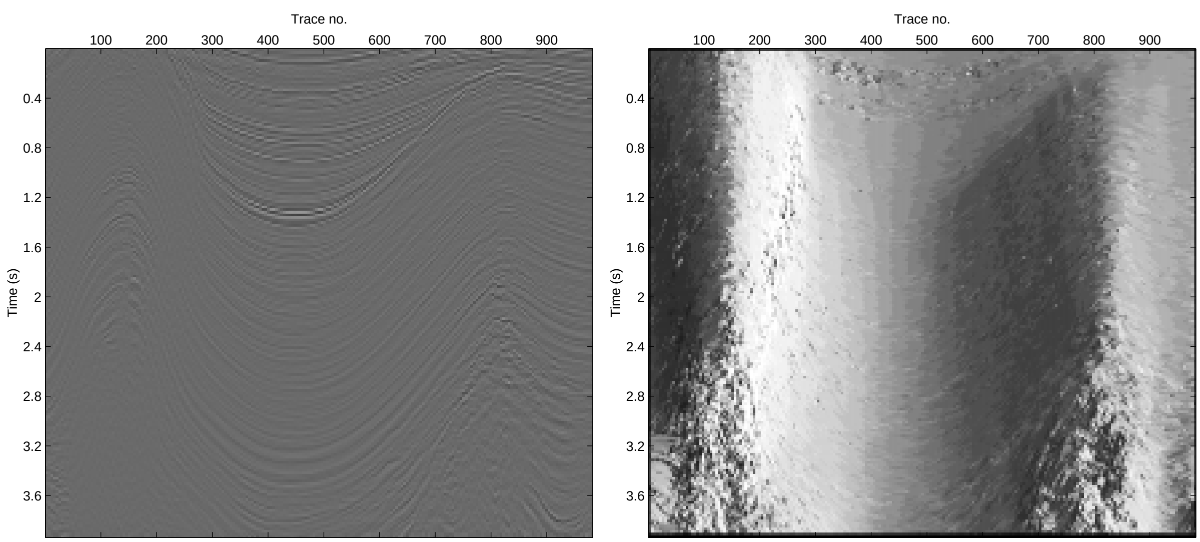

Figure 6: Seismic dataset and its dip representation. White=high positive dips, black=high negative dips.