TIME-FREQUENCY ANALYSIS OF SEISMIC REFLECTION SIGNALS

Philippe Steeghs and Guy Drijkoningen

Centre for Technical Geoscience
Delft University of Technology
Mijnbouwstraat 120
2628 RX Delft, the Netherlands

ABSTRACT

In this paper we show the implementation of bilinear time-frequency representations in the interpretation of seismic data. In seismic stratigraphy the reflection patterns in seismic data are interpreted in terms of subsurface stratification and lithological properties. The time-frequency representation clearly elicits seismic reflection patterns and can thus provide useful information on subsurface stratification. A comparison of several $t-f$ representations shows that the best results, from an interpretational point of view, can be obtained with a kernel that is adapted to the characteristics of the signal to be analyzed. We demonstrate that considerable improvement upon an existing method for seismic data analysis, complex-trace attribute analysis, can be achieved. The local frequency that is extracted from the $t-f$ representation provides a better picture of the spectral variation in the data than the instantaneous frequency, that was derived from the analytic signal.

1. INTRODUCTION

In the exploration for oil and gas reservoirs seismic reflection profiling is a widely used method for subsurface imaging. In seismic reflection profiling the response of the subsurface to the excitation by a seismic source is measured with an array of seismometers at the surface (Fig.1). The seismic energy will be reflected at velocity and density contrasts in the subsurface. The recorded signal is a time-series of the ground motion or pressure that is caused by the arrival of the reflected energy at the surface. Stratification of seismic impedance in the subsurface results in specific interference patterns in the seismic image.

The interpretation of seismic images in terms of subsurface stratification and lithological properties is known as seismic stratigraphy. A basic principle of seismic stratigraphy is that seismic reflection patterns contain geologically significant information. Although seismic stratigraphy nowadays is a well-established method for the interpretation of seismic images, only few attempts have been made to come to a more quantitative analysis of seismic reflection patterns. With the increase of the data-volumes as a result of the transition from 2D to 3D subsurface imaging, the demand for quantitative methods for seismic interpretation has grown. Furthermore, a quantitative description of seismic reflection patterns is an important step towards a better understanding of the origins of seismic reflection patterns and their significance in a geological interpretation of seismic reflection data.

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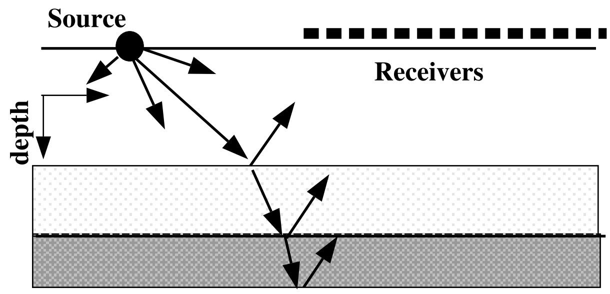

Figure 1: Seismic reflection measurement: seismic energy is reflected at impedance contrasts and the reflected energy is recorded at the surface.

In a signal analysis based approach, the seismic reflection pattern can be characterized by the distribution of spectral properties over time and space. Recently developed techniques in the field of time-frequency analysis have proven to be very well suited for the analysis of seismic reflection sequences [1]. We have applied a $t-f$ representation of the bilinear Cohen's class to the analysis of seismic reflection patterns. We will show how seismic reflection patterns are imaged in the time-frequency domain. We will also give some examples of how we can improve upon classical methods for seismic data analysis using $t-f$ representations.

2. THEORY

A bilinear time-frequency representation from Cohen's class can be given by [1]:

$$P\{u,u\}(t,f) = \int\int W\{u,u\}(t',f')\Psi(t-t',f-f')\,\mathrm{d}t'\,\mathrm{d}f', \tag{1}$$

[1]all integrals run from $-\infty$ to $+\infty$

where $\Psi(t,f)$ is a kernel function and $W\{u,u\}(t,f)$ is the Wigner distribution. The Wigner distribution is given by

$$W\{u,u\}(t,f) = \int \exp(-j2\pi f\tau) R\{u,u\}(t,\tau)\,\mathrm{d}\tau \qquad (2)$$

where $R\{u,u\}(t,\tau)$ is the instantaneous auto-correlation function:

$$R\{u,u\}(t,\tau) = u(t+\frac{1}{2}\tau)u^*(t-\frac{1}{2}\tau). \qquad (3)$$

Seismic reflection signals are multi-component signals. The bilinearity of the time-frequency representation introduces cross-terms between the components. These cross-terms make an interpretation of the Wigner distribution of a seismic reflection signal impracticable. For that reason we choose the kernel function $\Psi(t,f)$ such that it suppresses the cross-terms. The convolution with the kernel $\Psi(t,f)$ of the Wigner distribution transforms into a window operation on the two-dimensional Fourier transformation of Eq.(1):

$$\begin{aligned} P\{u,u\}(t,f) = \int\int &\exp(j2\pi(\nu t - \tau f) \\ &\Psi(\nu,\tau)A\{u,u\}(\nu,\tau)\mathrm{d}\nu\mathrm{d}\tau, \end{aligned} \qquad (4)$$

where $A(\nu,\tau)$ is the symmetric Ambiguity function given by

$$A\{u,u\}(\nu,\tau\} = \int \exp(-j2\pi\nu t)R\{u,u\}(t,\tau)\mathrm{d}t. \qquad (5)$$

The auto-terms of the Wigner distribution are mapped to region around the origin in the Ambiguity domain. The cross-terms between signal components occur at increasing time lags τ and frequency lags ν, and are thus located away from the origin. The kernel function $\Psi(\nu,\tau)$ with good cross-term suppression properties will therefore be a two-dimensional window around the origin.

The problem of finding a window that optimally separates cross-terms from auto-terms while retaining the highest possible resolution, can be formulated as an optimization problem [2]. This optimization problem can be solved by adapting the shape and size of the window to the auto-terms in Ambiguity function of the signal that is analyzed. The window is chosen in such a way that (1) it is located around the origin of the Ambiguity function, (2) has a fixed size. The optimization is then performed by maximizing the energy output of the window, given the constraints (1) and (2). The size of the window determines the resolution of the representation. We determine the optimal window with a linear optimization method; the Condensing Sort and Select Algorithm (CSSA) of Baraniuk and Jones [2].

Additionally, a running time version of this algorithm, using a recursive computation of the short-time Ambiguity function, has been implemented [3]. In the running time version the kernel is adapted as a function of time. This results in a better match of the kernel to the auto-terms where the characteristics of the signal vary with time. The short-time Ambiguity function of a signal $u(t)$ is given by

$$\begin{aligned} A_s\{u,u\}(t;\nu,\tau) = \int &\exp(-j2\pi\nu t')R\{u,u\}(t',\tau) \\ &w(t'-t-\frac{\tau}{2})w^*(t'-t+\frac{\tau}{2})\mathrm{d}t', \end{aligned} \qquad (6)$$

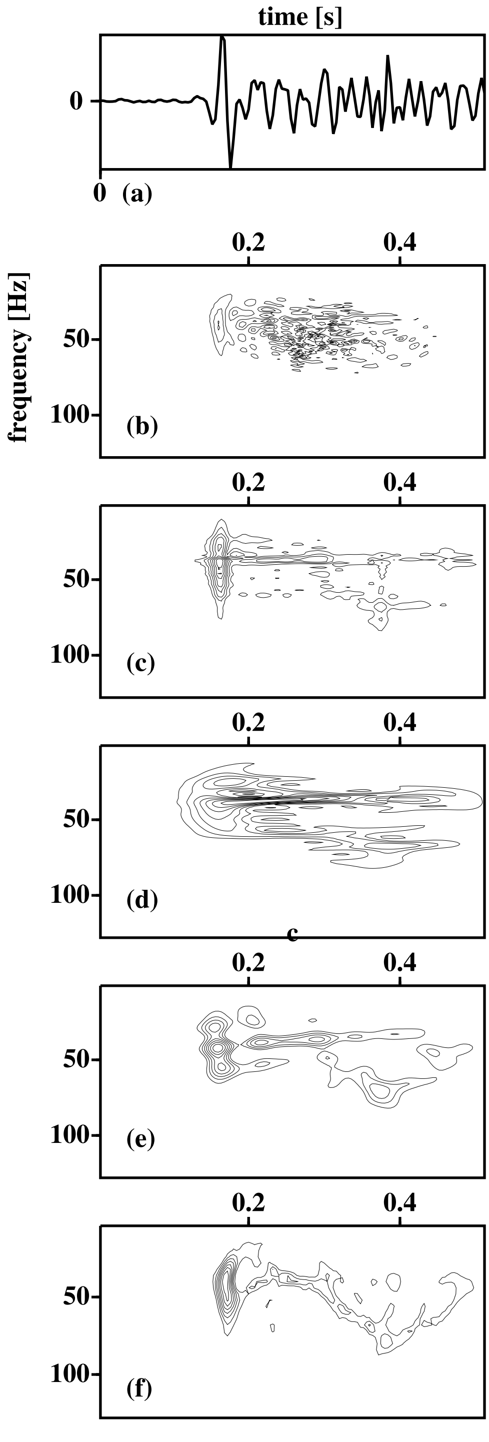

Figure 2: Comparison of bilinear TFR's. (a) Seismic recording, (b) Wigner distribution, (c) Choi-Williams distribution [4], (d) Spectrogram, (e) Optimal kernel TFR [2], (f) time-adaptive optimal kernel TFR.

where $w(t')$ is a symmetrical window function. For every time sample the short-time Ambiguity function is calculated and for each $A_s\{u,u\}(t;\nu,\tau)$ the optimal kernel is determined with the CSSA algorithm. The time-frequency representation is then computed as a slice of the two-dimensional Fourier transformation of the product of the time-adaptive optimal kernel $\Psi_s(t;\nu,\tau)$ and the short-time Ambiguity function:

$$P\{u,u\}(t,f) = \int\int \exp(j2\pi(\nu t - \tau f) \\ A_s\{u,u\}(t;\nu,\tau)\Psi_s(t;\nu,\tau)\mathrm{d}\nu\mathrm{d}\tau. \quad (7)$$

3. RESULTS

Example 1: bilinear $t-f$ representations of a seismic signal

Figure 2 shows the first 128 samples of a seismic recording together with several time-frequency representations. The first high-amplitude peak is the reflection from the sea-bottom. It is followed by the (interfering) reflections from the sediment layers below. In the Wigner distribution (Fig.2b) the time-frequency structure of the signal is obscured by the cross-terms, making an interpretation in terms of time-varying spectral content impossible. The Choi-Williams distribution (Fig.2c) gives a clearer picture [4]. However, the cross-shaped Choi-Williams kernel introduces a bias in the representation, because it tends to localize the energy at single times and frequencies. Figure 2d is a spectrogram with a 32 point Gaussian window. It shows the general time-frequency structure, but the time resolution is not sufficient to separate the different components. The optimal kernel representation (Fig.2e) gives the clearest time-frequency picture and for interpretational purposes it appears to be superior to a fixed kernel representation. The time-adaptive optimal kernel (Fig.2f) achieves a slightly sharper localization of energy in the $t-f$ plane. For large data sets it has the additional advantage of less computational costs compared to the non-time adaptive optimal kernel.

Example 2: $t-f$ patterns in seismic sequences

In Figure 3 we show a part of a seismic section from the Alboran Sea (SE Spain) together with the $t-f$ representation of the leftmost trace of this section. The time-frequency representation clearly shows how the frequency of the signal increases with time. This was interpreted to be the result of an increased thinning of the sediment layers with depth. The interpretation was used for the creation of an input model for the calculation of synthetic seismic data. [4]. The depth-velocity model and its seismic response is shown in the lower half of the figure, together with the time-frequency representation of the modelled data. The reflection patterns in the real and synthetic data are in excellent agreement with each other.

Example 3: Instantaneous frequency

The classic method for the description of waveforms in seismic data is complex-trace attribute analysis [5]. The most widely used attributes are the instantaneous amplitude and intantaneous frequency of the analytic signal. The Wigner distribution is related to the instantaneous amplitude and instantaneous frequency through its zero and first order moments. With the Wigner distribution we find for the instantaneous energy and instantaneous frequency the same values as from the analytic signal. An advantage when using the Wigner distribution is that we now have the possibility of manipulating the time-frequency representation before extracting its attributes. This results in a robust estimation of the attributes in the presence of noise and more control over the averaging window in time and frequency.

For the attribute extraction the optimal kernel $t-f$ representation of every time series of seismic data is calculated. The result is a data volume representing the energy distribution over space, time and frequency. From the $t-f$ representations the mean frequency for every time sample is extracted and plotted for every sample of the seismic image. The mean frequency derived from $t-f$ representation clearly brings out features from the data that are not easily recognized in the original data, as they are masked by strong amplitudes (Fig.(4)). If we compare this with the instantaneous frequency, we see that the mean frequency that was extracted from the $t-f$ representation is less sensitive to noise than the complex-trace attribute and provides a better picture of the variation of spectral content in the data.

4. CONCLUSIONS

The $t-f$ analysis of real and modelled seismic sequences shows that the $t-f$ representation elicits seismic reflection patterns and can provide meaningful information about lithologic stratification. As such time-frequency analysis is a valuable addition to existing methods for seismic data analysis. The best results, from an interpretational point of view, were obtained with a signal adaptive kernel.

Another application of $t-f$ analysis is found in complex-trace attribute analysis, where the $t-f$ distribution enables robust extraction of local frequency. The data example shows that the the $t-f$ representation yields attributes that are more representative of the information content of the data, than the equivalent attribute extracted from the analytic signal.

5. REFERENCES

[1] T.P.H. Steeghs and G.G. Drijkoningen, "Time-frequency analysis of seismic sequences", Expanded Abstracts , 65th Annual Meeting of the Society of Exploration Geophysicists, Houston, 1995.

[2] R.G. Baraniuk and D.L. Jones, "A signal dependent time-frequency representation: Optimal kernel design", IEEE Transactions on Signal Processing, vol. 41, No. 4, 1993.

[3] D.L. Jones and R.G. Baraniuk, "An adaptive optimal-kernel time-frequency representation", IEEE Transactions on Signal Processing, October 1995.

[4] H.I. Choi and W.J. Williams, "Improved time-frequency representation of multicomponent signals using exponential kernels", IEEE Trans. on Acoust. , Speech, Signal Processing, vol. 37, pp.862-871, 1989

[5] T.H. Taner, F. Koehler and R.E. Sheriff, "Complex seismic trace analysis", Geophysics, vol. 44, 1979.

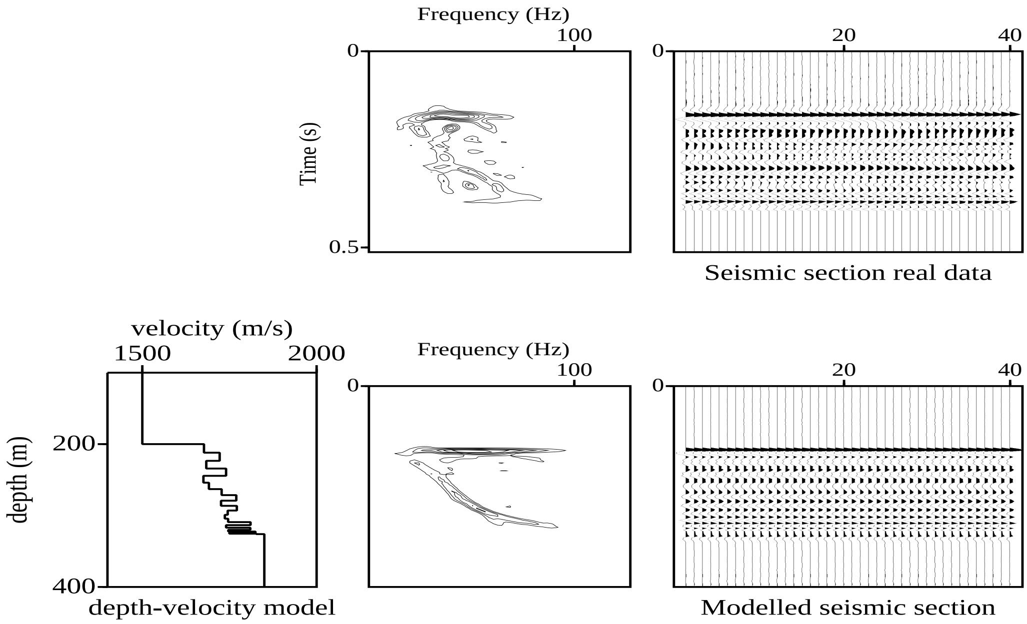

Figure 3: Time-frequency representation of a seismic sequence.

Figure 4: (a) seismic section, (b) mean frequency derived from the $t - f$ representation and (c) instantaneous frequency.