WIGNER-RADON REPRESENTATIONS FOR 3-D SEISMIC DATA ANALYSIS

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ABSTRACT

In this paper a local Radon representation is proposed and applied to 3-D seismic data analysis. The derivation of the local Radon power spectrum is based on an extension of the relation between the global Radon transform and multi-dimensional Fourier transform to the non-stationary case. This Wigner-Radon power spectrum is closely related to the Cohen's class of quadratic time-frequency representations. The local Radon power spectrum is very well suited to characterize the geometry of seismic reflection surfaces. The normalized first moment of the Wigner-Radon representation can be used as a measure for the dip angle of the 3-D signal. Analysis of this geometrical information greatly facilitates the geological interpretation of the data.

1. INTRODUCTION

Three-dimensional seismic imaging plays a crucial role in the exploration for oil and gas reservoirs. The growing demand for more detailed, but less time-consuming, interpretation of large 3-D seismic data volumes has initiated an increased effort to develop more effective methods for seismic data analysis. In three dimensions, seismic reflections are surfaces that delineate a transition in the physical properties of the earth. Geological interpretation of seismic data is based on an assessment of the geometrical properties of these reflective surfaces. In this paper we will describe a technique for extracting geometrical information from 3-D seismic data.

The Radon transform, or slant stack, is a widely used data representation in exploration seismology. The Radon transformation plays a prominent role in the analysis of acoustic wave -fields, because it essentially performs a decomposition of the data into its plane-wave components. However, in certain seismic applications the global Radon transform is not the ideal analysis tool. Sometimes it is not sufficient to know which plane wave components or dip angles are present in the data, but also where they occur. For instance, using a Radon transformation for the geological interpretation of a seismic image, only makes sense if the locations of the dipping reflectors are known. In this type of applications a localized Radon transformation would be required. Localized Radon transformations have been defined and applied for different purposes [2], [3], [4]. These local Radon representations are all based on the application of the classical slant stack on windowed portions of the data. In the same way as the sliding-window power spectrum or spectrogram is a member of a very large class of possible local power spectra, the sliding-window Radon transformation is just one choice from an infinite number of possible localized Radon transformations. This new class of localized Radon representations has many properties in common with Cohen's class of time-frequency representations.

Instantaneous temporal frequency and bandwidth are widely used to characterize seismic signals in one dimension. They can be extracted as the normalized first and second moments of a time-frequency frequency representation of the signal. In order to characterize the geometry of a seismic reflection surface in three dimensions we propose the normalized moments of the local Radon representation in cylindrical coordinates. The volume dip and azimuth attributes effectively reveal the discontinuities in the signal. These signal discontinuities in many cases delineate significant geological features.

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2. THE LOCAL RADON POWER SPECTRUM

The global Radon transformation of the 3-D seismic signal $u(\boldsymbol{x}, t)$ is defined as

$$\breve{u}(\boldsymbol{p}, \tau) = \int_{\boldsymbol{x} \in I\!R^2} u(\boldsymbol{x}, \tau + \boldsymbol{p} \cdot \boldsymbol{x}) \mathrm{d}\boldsymbol{x}, \tag{1}$$

where $\boldsymbol{x} = \{x_1, x_2\}$ is the spatial coordinate vector, and t is the time coordinate. The vector $\boldsymbol{p} = \{p_1, p_2\}$ represents the angles with respect to the (x_1, x_2)-plane in the x_1 and x_2 directions, and τ is the intercept with the time axis. The Fourier transformation of Eq. (1) with respect to intercept time τ of Eq. (1) is given by

$$\breve{u}(\boldsymbol{p}, f) = \int_{\boldsymbol{x} \in I\!R^2} \exp(\mathrm{j} 2\pi f \boldsymbol{p} \cdot \boldsymbol{x}) \hat{u}(\boldsymbol{x}, f) \mathrm{d}\boldsymbol{x}, \quad f \in I\!R^+, \tag{2}$$

where $\hat{u}(\boldsymbol{x}, f)$ denotes the temporal Fourier transform of $u(\boldsymbol{x}, t)$. In all practical cases we consider either that $u(t)$ is real-valued or its associated complex-valued analytic signal. In both cases we can restrict the analysis to positive frequency f. The domain occupied by positive frequencies is introduced as $I\!R^+ = \{f \in I\!R; f \geq 0\}$. The characteristic function of this domain $\chi_{I\!R^+}(f)$ is given by $\chi_{I\!R^+}(f) = \left\{0, \frac{1}{2}, 1\right\}$ for $\{f < 0, f = 0, f > 0\}$. Equation 2 is a 3-D Fourier transformation of $u(\boldsymbol{x}, t)$. Consequently, we can obtain the temporal frequency domain representation of the Radon transform from the 3-D Fourier transform $\tilde{u}(\boldsymbol{k}, f)$ of the signal $u(\boldsymbol{x}, t)$. i.e.

$$\breve{u}(\boldsymbol{p}, f) = \tilde{u}(f\boldsymbol{p}, f) = \tilde{u}(\boldsymbol{k}, f), \quad f \in I\!R^+, \tag{3}$$

where $\boldsymbol{k} = \{k_1, k_2\} = f\boldsymbol{p}$ is the spatial frequency vector. We shall exploit this relation in order to define a local Radon power spectrum, based on the multi-dimensional Wigner distribution of the signal u.

The Wigner distribution of the 3-D seismic signal will be defined as the Fourier transform of a local auto-correlation function. The 3-D local autocorrelation function of $u(\boldsymbol{x}, t)$ is defined as

$$R(\boldsymbol{x}, t; \boldsymbol{\xi}, \tau) = u(\boldsymbol{x} + \frac{1}{2}\boldsymbol{\xi}, t + \frac{1}{2}\tau)u^*(\boldsymbol{x} - \frac{1}{2}\boldsymbol{\xi}, t - \frac{1}{2}\tau), \quad (4)$$

where $\boldsymbol{\xi} = \{\xi_1, \xi_2\}$ and τ are space and time shift variables. The Wigner distribution of the signal is found by Fourier transformation of $R(\boldsymbol{x}, t; \boldsymbol{\xi}, \tau)$ with respect the space and time correlation variables $\boldsymbol{\xi}$ and τ, given by

$$W(\boldsymbol{x}, t; \boldsymbol{k}, f) = \int_{\boldsymbol{\xi} \in I\!R^2} \int_{\tau \in I\!R} \exp(\mathrm{j}2\pi(\boldsymbol{k}\boldsymbol{\xi} - f\tau)) R(\boldsymbol{x}, t; \boldsymbol{\xi}, \tau) \mathrm{d}\xi \mathrm{d}\tau, \quad (5)$$

where $\boldsymbol{k} = \{k_1, k_2\}$ is the spatial frequency vector and f is the temporal frequency. Invoking the relationship $\boldsymbol{k} = f\boldsymbol{p}$, as in Eq. (3), we can obtain a local angle-frequency representation $W(\boldsymbol{x}, t; \boldsymbol{p}, f)$ from the Wigner distribution, i.e.

$$W(\boldsymbol{x}, t; \boldsymbol{p}, f) = W(\boldsymbol{x}, t; f\boldsymbol{p}, f) = W(\boldsymbol{x}, t; \boldsymbol{k}, f), \; f \in I\!R^+. \quad (6)$$

The Wigner-Radon representation is obtained by inverse Fourier transformation of the local power spectrum $W(\boldsymbol{x}, t; \boldsymbol{\xi}, \tau)$ with respect to temporal frequency f, i.e.

$$\breve{S}(\boldsymbol{x}, t; \boldsymbol{p}, \tau) = \int_{f \in I\!R} \exp(\mathrm{j}2\pi f\tau)\chi_{I\!R^+}(f) W(\boldsymbol{x}, t; \boldsymbol{p}, f)\mathrm{d}f. \quad (7)$$

The local angle-frequency representation $W(\boldsymbol{x}, t; \boldsymbol{p}, f)$ can be computed by interpolation of a multi-dimensional Wigner distribution $W(\boldsymbol{x}, t; \boldsymbol{k}, f)$ onto a regular $(\boldsymbol{p}, f)$-grid. Because of this relation between the Wigner distribution and the Wigner-Radon representation, the kernel method can also be applied to derive a general class of Wigner-Radon representations [1],[6]. The sliding-window Radon transform and Wigner-radon transform both belong to this general class of local Radon power spectra. Similar as in the time-frequency case, the properties of the generalized Wigner-Radon representation are determined by the choice of kernel.

Further analysis (cf. Eqs. (1)) and (2) shows that $\breve{S}(\boldsymbol{x}, t; \boldsymbol{p}, \tau)$ is the Radon transformation of the local auto-correlation function with respect to the shift variables $\boldsymbol{\xi}$ and τ,

$$\breve{S}(\boldsymbol{x}, t; \boldsymbol{p}, \tau) = \int_{\boldsymbol{x} \in I\!R^2} R(\boldsymbol{x}, t; \boldsymbol{\xi}, \tau + \boldsymbol{p} \cdot \boldsymbol{\xi})\mathrm{d}\boldsymbol{x}. \quad (8)$$

Seismic attribute extraction

Seismic attribute analysis aims at the extraction of signal characteristics that bring forward geological relevant information from the data. The volume-dip is defined as the angle with respect to the (x_1, x_2)-plane and is given by $|\boldsymbol{p}| = \sqrt{p_1^2 + p_2^2}$. The angle within this plane is the azimuth, which is defined as $\alpha = \tan^{-1}\{p_1/p_2\}$. For geological surface analysis, it often more convenient to work with these angles instead of the Cartesian vector $\boldsymbol{p}$. We can obtain the local dip-azimuth representation interpolation of the (p_1, p_2)-grid to cylindrical coordinates $(|\boldsymbol{p}|, \alpha)$, where we have $\{|\boldsymbol{p}|\cos\alpha, |\boldsymbol{p}|\sin\alpha, t\} = \{p_1, p_2, t\}$. The mean volume-dip and mean azimuth of each subsurface sample point $(\boldsymbol{x}, t)$ is estimated by computation of the relative first order moment with respect to $|\boldsymbol{p}|$ and α of the local dip-azimuth spectrum $\breve{S}(\boldsymbol{x}, t; |\boldsymbol{p}|, \alpha, \tau = 0)$. For instance, the mean volume-dip is found as

$$\langle|\boldsymbol{p}|\rangle(\boldsymbol{x}, t) = \frac{\int_0^{2\pi}\int_{|\boldsymbol{p}| \in I\!R} |\boldsymbol{p}|\breve{S}(\boldsymbol{x}, t; |\boldsymbol{p}|, \alpha)\mathrm{d}|\boldsymbol{p}|\mathrm{d}\alpha}{\int_0^{2\pi}\int_{|\boldsymbol{p}| \in I\!R} \breve{S}(\boldsymbol{x}, t; |\boldsymbol{p}|, \alpha)\mathrm{d}|\boldsymbol{p}|\mathrm{d}\alpha}. \quad (9)$$

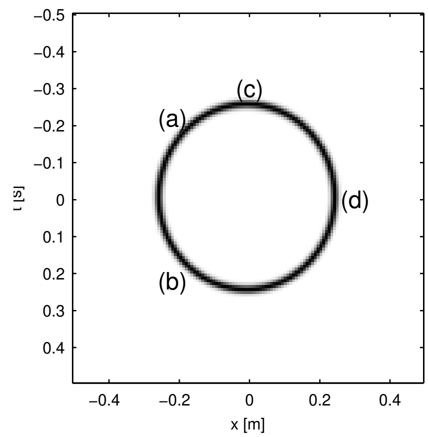

Figure 1: Ring-shaped intensity profile, (a)-(d) give the positions of the Wigner-Radon representations of Fig. 2.

3. COMPUTATION AND EXAMPLES

Figure 2 shows four local Radon (p, τ) power spectra of a ring-shaped intensity profile (Fig. 1). The pseudo-Wigner distribution was used to compute the Wigner-Radon representations at each of the locations (x_i, t_i) .

For the computation of the pseudo Wigner-Radon representation first the two-dimensional pseudo Wigner distribution $W(x_i, t_i; k, f)$ is computed by Fourier transformation of a windowed local autocorrelation. This local two-dimensional power spectrum is then interpolated onto a rectangular (p, f)-grid. After inverse Fourier transformation of $W(x_i, t_i; p, f)$ over temporal frequency f, we obtain the Wigner-Radon representation $\breve{S}(x_i, t_i; p, \tau)$. The Wigner-Radon representation results in a sharp localization of energy in the (τ, p)-plane. Consequently, this representation will be very well suited to extract directional information from a multi-dimensional signal.

Because of the noisy character of seismic data, cross term suppression in the Wigner distribution is important for robust seismic attribute extraction. Another important issue is computational complexity and efficient data I/O. Since the size of a typical 3-D seismic data set is several 100 megabytes, most of the standard algorithms for time-frequency analysis cannot be simply scaled up to three dimensions. Several algorithms have been designed to compute smoothed Wigner distributions of sliding data cubes. The most efficient algorithms first compute a 3-D sliding-window Fourier transform, either with an FFT of a small data cube or an 'update and subtract' algorithm. Better space-time localization is then obtained by correlation over frequency in the 3-D local Fourier transform [5], [6].

In Fig. 3 the mean volume-dip of a seismic data set along a slice of constant time is shown. The local Radon power spectrum was computed by interpolation of the 3-D smoothed pseudo Wigner distribution. The resulting local (p_1, p_2) power spectra where then interpolated onto a dip-azimuth $(|\boldsymbol{p}|, \alpha)$ grid by a bilinear interpolation. The mean volume-dip of the signal was then estimated at each sample location using Eq. (9).

The volume-dip attribute brings forward discontinuities in the reflection surfaces. The steeply dipping features indicate geological discontinuities, such as faults. The signal discontinuities, which generally delineate important geological features, can be detected with more accuracy in the attribute image than in the original data.

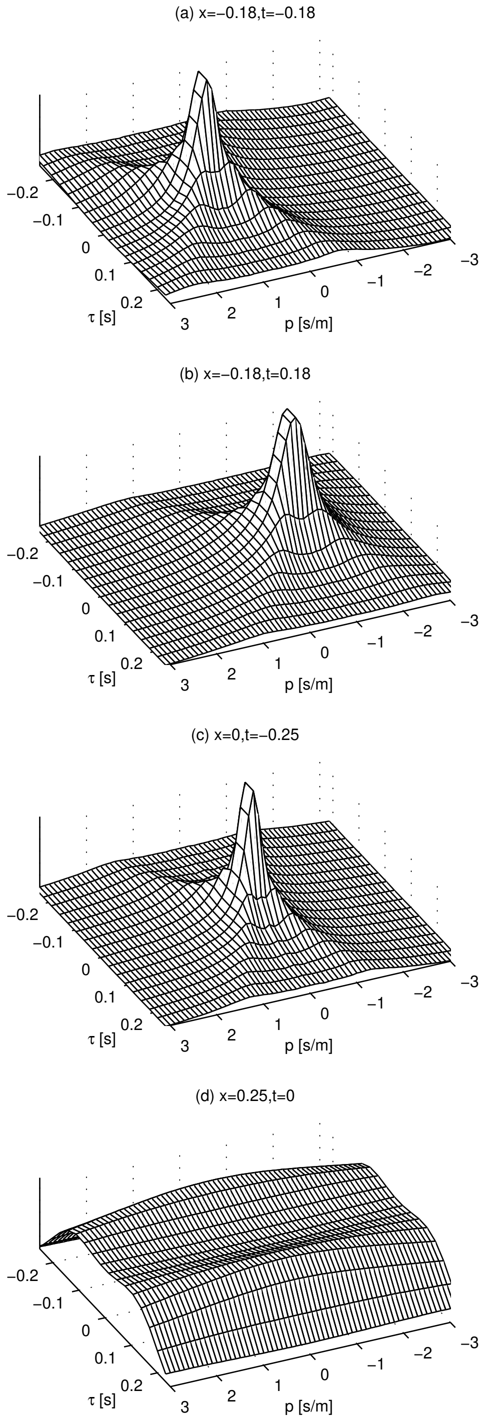

Figure 2: Wigner-Radon representations at four locations on the ring-shaped intensity profile of Fig. 1. The modulus $|\check{S}(x_i, t_i; p, \tau)|$ is shown.

4. CONCLUSIONS

A Wigner-type local Radon power spectrum has been derived. This Wigner-Radon representation is very well suited for the extraction of geometrical properties of seismic reflection surfaces. The close relation between the Wigner-Radon representation and the multi-dimensional Wigner distribution permits the extension of many of the techniques that have been developed for one-dimensional time-frequency analysis to multi-dimensional local Radon power spectra. The increased complexity of 3-D analysis and the size of seismic data sets poses limits on this extension. However, efficient algorithms have been designed that have made routine application of 3-D local frequency analysis of seismic reflection data feasible.

5. REFERENCES

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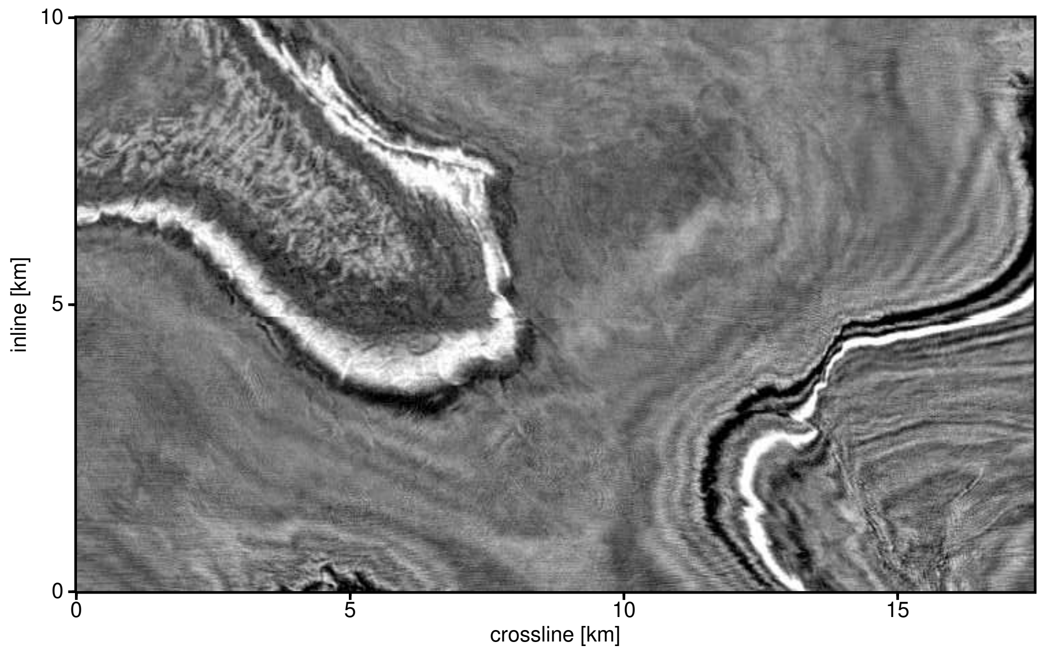

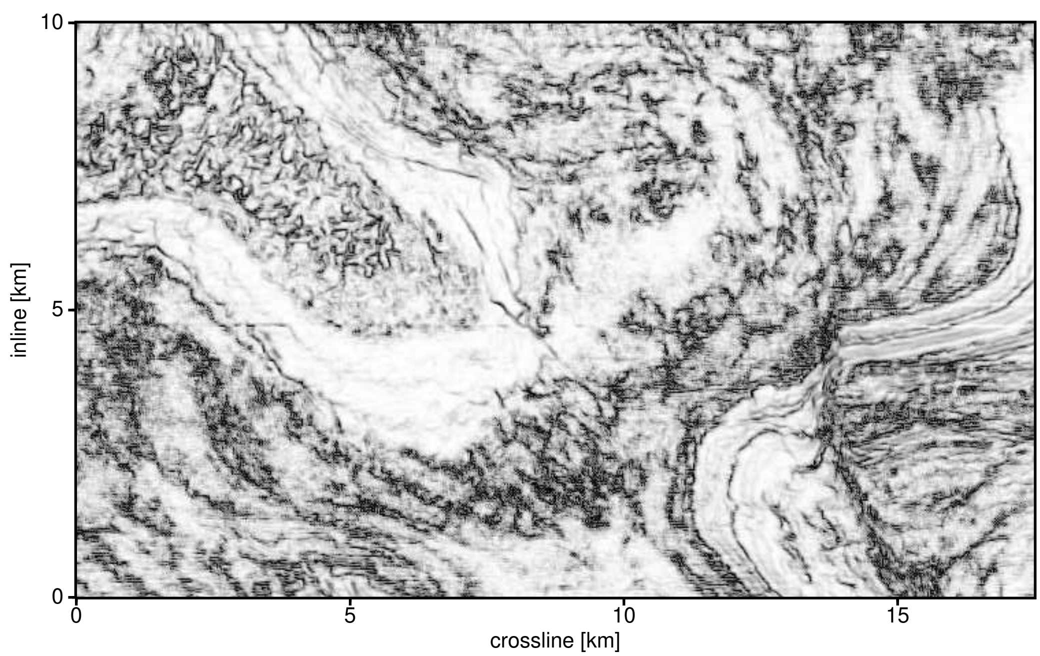

Figure 3: (top) Constant time slice at 1.3 [s] through a 3-D seismic data set and (below) corresponding volume-dip attribute slice. A 4x4x4 sliding analysis cube was used to compute the Wigner-Radon representation from which the mean volume-dip was estimated. Steep dips are given in black, zero local dip is white.