DETECTING PERIODIC BEHAVIOR IN NONSTATIONARY SIGNALS

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ABSTRACT

This paper presents results on the multiresolution analysis of nonstationary signals with the objective of detecting underlying periodic phenomena. Wavelet packet analysis with coefficient thresholding is the basis for the detection. The effectiveness of the method is illustrated by analyzing experimental data on sediment electrochemical redox potential in a tidal microcosm. The significance of the technique is that it can extract periodic phenomena from experimental data corrupted by catastrophic and random events, provide a signature of the basic periodic component, and give an estimate of the degree of deviation from periodic behavior. Consequently, it has potential applications in the analysis of quasi-periodic signals such as electrocardiograms (*ECG*s), where the determination of the extent of quasi-periodicity is of critical importance.

1. INTRODUCTION

In this paper, we aim to analyze experimental data relating to periodic phenomena which have been corrupted by catastrophic or random events, and formulate techniques to extract the periodic components from the data. The problem description and the solution we propose is explained by means of a case study using time series data [1] on sediment electrochemical (*Eh*) redox potential in a tidal simulation microcosm. The goal of the experiment is to isolate and identify the effect of diurnal tide cycles. The system is run for several weeks and measurements of redox potential are taken every hour. The microcosm is a carefully controlled environment with all reasonable experimental precautions and verifications in place. Nevertheless, for the experiment studied here, cold fronts caused significant temperature variations ($\sim 10^\circ C$) over the course of the run. Moreover, logs showed a power failure during a weekend. As a result, the experimental data showed variations that appear nonstationary in nature. Figure 1 displays the *Eh* data from the tide simulating microcosm at two electrodes, one at the surface, and the other at medium depth.

The problem was studied in some depth using time series and spectral analysis techniques ([1]). However, the conventional techniques failed to give satisfactory results. Figure 2 displays the spectral energy distributions corresponding

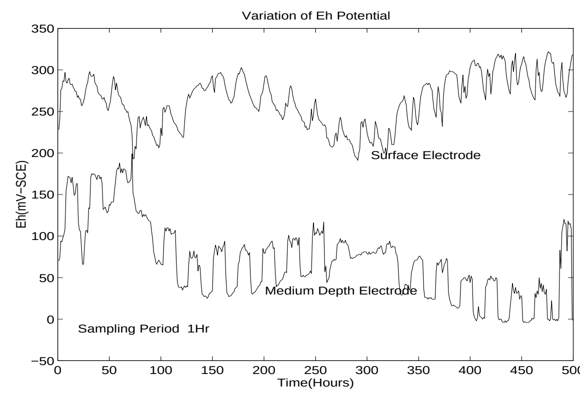

Figure 1: Electrochemical Potential

to the signals shown in Figure 1. A simple visual inspection of the spectral energy shows significant peaks at the fundamental frequency and harmonics, strongly supporting the presence of a periodic signal embedded in the data. The spectral data also indicates a strong component of very low frequency which points to long term variations. Our aim is to compensate for all the random and unwanted variations, and extract the basic underlying periodic signal. Problems of this nature have been studied in the past, and various solutions mainly involving conventional spectral analysis have been proposed ([2]). However,

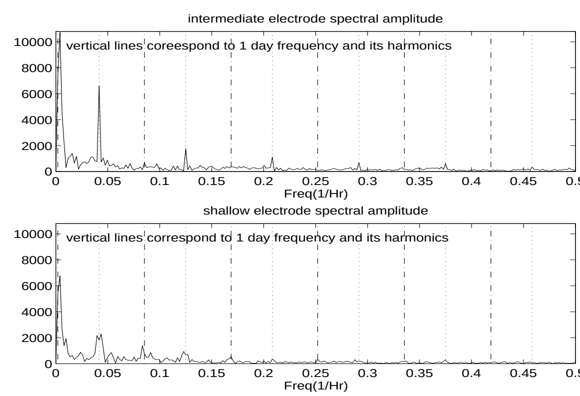

Figure 2: Spectral Energy Distribution

[1] We express our thanks to Dr. J. Catallo of the Laboratory for Ecological Chemistry at LSU who supplied the experimental data.

for the data presented, though the conventional spectral analysis reveals strong harmonic components with period close to 24 hrs (Figure 2), the signals reconstructed using these harmonics show poor matches with the experimental data. Figure 3 displays the reconstructed signal obtained using the first four harmonics. For comparison, we include the original signal with very long term variations removed. It is clear that the spectral analysis technique leaves a sig-

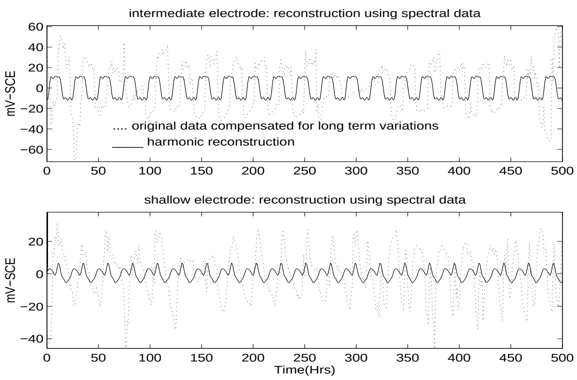

Figure 3: Harmonic Reconstruction With Four Terms

nificant residual attributable to the noise signal and inaccuracies in the computation of the period. Moreover, this technique has a very serious drawback in that it completely disregards *local temporal variations*, such as those occuring near the 90 hour mark and the 300 hour mark.
It is apparent that the problem of detecting buried periodic phenomena is not limited to the microcosm experimental set-up. Any similar experiment over such an extended period is likely to display random effects. It is thus necessary to develop a processing algorithm that can compensate for such random variations and enhance the features that are being sought. The effect of noise can be reduced with additional observations. However, the inability to describe local variations is a known limitation of spectral analysis. The Short Time Fourier Transform ($STFT$) was developed as a partial solution to overcome this limitation. Unfortunately, the $STFT$ can only provide a fixed time resolution determined by the size of the selected window. However, a technique based on wavelets and multiresolution representations permits a description of the data in a "time-frequency" space using variable resolution in both time and frequency. Thus, we use the wavelet packet analysis as the basis for our detection.

2. MULTIRESOLUTION APPROACH

The processing algorithm is based on a multiresolution analysis of the experimental data. Data was processed with a perfect reconstruction filter bank creating a complete wavelet packet decomposition. Each component of the packet gives a representation of signal details in a specific resolution level. The wavelet used was $Db10$, Daubechies' compact support orthogonal wavelet with 20 taps ([3]). It is apparent from the data in Figure 1, that there exist some long term variations, probably caused by weather changes (two cold fronts passed through during the experiment). These slow variations can be eliminated by subtracting an orthogonal component with very low resolution. For this purpose, we used a low resolution component containing details with periods longer than 64 hours. Figure 4 shows first the

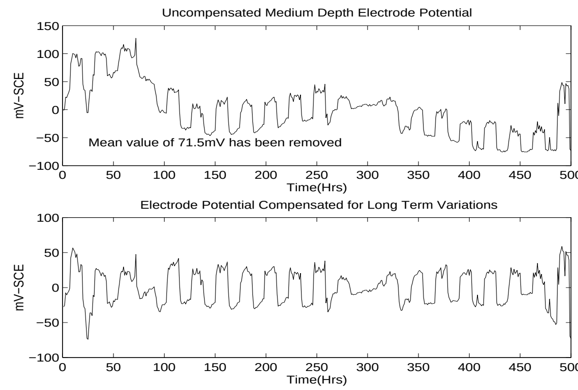

Figure 4: Compensation of Long Term Variations in Submerged Electrode

variation of the potential in the submerged electrode and then the same signal after subtracting the low resolution orthogonal component. Observe that the long term variation compensation produces no loss of fine features. A similar result was obtained for the surface electrode. In the computations and displays, the mean value has been removed from the original uncompensated Eh readings.
A cursory examination of the compensated data shows what appear to be significant periodic features corrupted by other effects (noise). To verify the existence of such periodic features, we use the wavelet packets to create *low resolution representations* of the signal. The assumption is that at low resolution only the most significant effects will appear. Thus, we expect the low resolution representations to enhance the periodic behavior, while suppressing the noise, and other random effects. Figure 5 displays the orthogo-

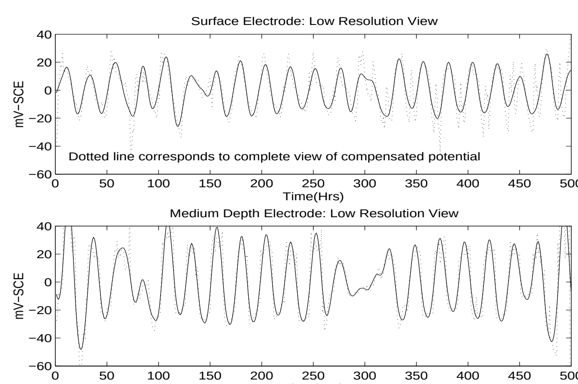

Figure 5: Low Resolution Orthogonal Components

nal components of the compensated electrode potential with components of periods larger than 16 hours. For clarity, the compensated signal is also shown in dotted lines.

It can be seen that the signals obtained at low resolution do indeed display very regular periodic behavior. In order to obtain a measure of periodicity, we determined the maxima and minima in the low resolution components using a peak detector. Table 1 presents the data for the submerged electrode. A similar table is available for the surface electrode. From the tabulated data we removed the data corresponding to the maxima and minima in the interval $300 - 330$ hours, where the experimental data showed a clearly anomalous behavior, and determined that the maxima and minima were spaced in the average by 23.78 hours, with a standard deviation of 2.1 hours, making a good case for diurnal variations. This gives us an estimate of the average period, and also a measure of the deviation. Note that in addition to depicting the periodic components, the low resolution representation also highlights details which were masked in the original data, like the catastrophic changes occurring in the neighborhood of 300 hours (when logs indicated a power failure), which are enhanced in the low resolution view.

Table 1: Data On Maxima And Minima Of The Submerged Electrode

Minima	Separation	Maxima	Separation
4		14	
26	22	38	24
49	23	66	28
79	30	86	20
98	19	110	24
122	24	133	23
146	24	158	25
170	24	182	24
193	23	205	23
217	24	229	24
241	24	254	25
264	23	277	23
289	25	298	21
303	14	313	15
316	13	325	12
338	22	350	25
362	24	374	24
387	25	398	24
410	23	422	24
433	23	446	24
458	25	469	23
482	24	494	25

3. ENHANCEMENT OF PERIODIC BEHAVIOR

The determination of the period was carried out using a limited resolution representation. In order to capture all the nuances of the periodic phenomena, we assume that the non-periodic phenomena will have no preferred location either in time or frequency and that their energy will be spread uniformly in the time-frequency domain. The energy contribution of non-periodic phenomena is finite. Hence, the energy in each component of the wavelet packet should be approximately the same, and decrease with the number of components. On the other hand, the periodic phenomena are expected to have energy only in some components in the wavelet packet decomposition. Therefore, those components showing periodic behavior will have an improved signal to noise ratio.

Figure 6 shows the electrode potential, one orthogonal component with clearly defined periodic features and one component that does not display significant periodic behavior. The component showing periodic behavior contains harmonics with periods between 8 and 16 hours. The component without clear periodic features has components with periods ranging from 2.7 hours to 3.2 hours. If we seek to add details to the periodic variations, we should consider the features with periods between eight and 16 hours and safely ignore the others, as these would most likely be high frequency noise. A very significant fact is that the high resolution features also preserve the temporal information. Where the periodic signal was destroyed, the details are also missing. Hence, if desired, one can perform signal enhancing using only relevant information. Each of the components can be analyzed in the same manner. In this study, we took a simplistic approach and we either used an orthogonal component or rejected it.

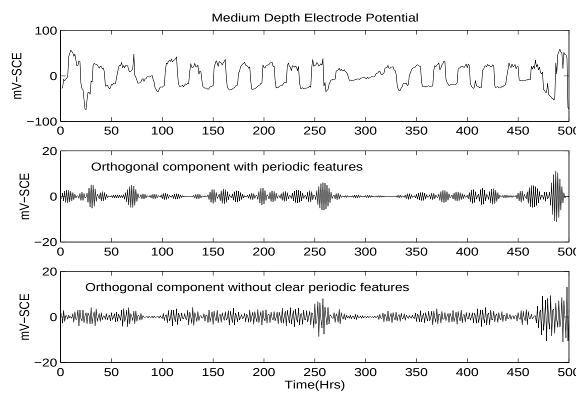

Figure 6: Orthogonal Component With And Without Periodic Features

The last step in the processing is the accounting of catastrophic events, such as the power failure and other random effects. For this processing, we used the representation with enhanced periodic behavior and defined "daily vectors," starting at the first minima and containing data for one 24 hour period. The collection of "daily vectors" was then analyzed to discard outliers, thus eliminating the distortion due to catastrophic events. The remaining vectors were averaged to create the final signature of one cycle. Essentially, we determined the mean value of the "daily vectors" set, and obtained the correlation coefficients of this mean vector with each vector in the set. By fixing a suitable threshold (.8 in our case), we could identify all those elements of the set which had a high correlation with the mean value. These were then used to determine the periodic signature for the compensated signal. Figures 7 and 8 show the results of processing the surface and medium depth signal using the $Db10$ wavelet. For each case, the compensated

signal is shown along with the signal obtained using only the peridic signature, and they are displayed using the same scale for better clarity. Note the significant reduction in the amplitude of the noise corrupting the original signals, in the enhanced signals. It is important to highlight that our approach determines **one typical period of the signal**. We concatenate periods for display purpose only.
It is reasonable to suppose that the simple averaging procedure outlined above, worked very well in our case because the signals were inherently highly periodic. The technique though might not be suitable for highly irregular signals. For such signals, we would need to formulate more sophisticated processing techniques.

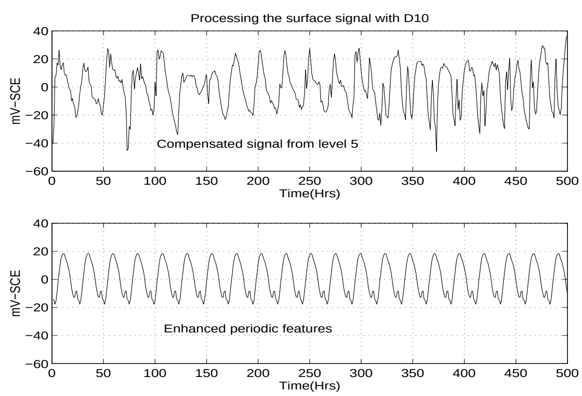

Figure 7: Enhanced Periodic Features Of The Surface Signal

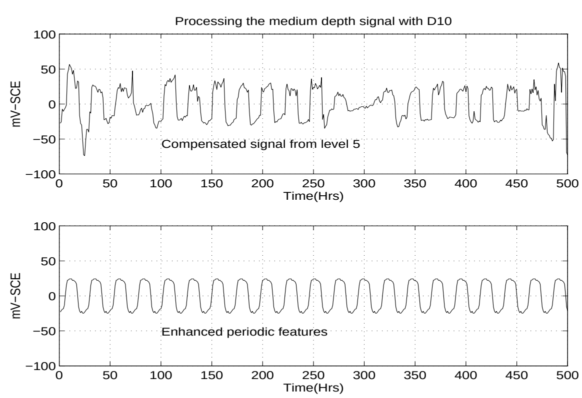

Figure 8: Enhanced Periodic Features Of The Medium Depth Signal

4. CONCLUSIONS

We have demonstrated how the wavelet packet decomposition can be used to look at a nonstationary signal at different frequency resolutions and time instances. As the case study shows, this property has very interesting and useful applications. The signal enhancing approach we used in this work is naive, but the technique is economical and easy to implement. The results show the tremendous capablities of the wavelet based multiresolution technique. The methodology presented in this work appears to be an effective tool to compensate for random effects and to enhance periodic behavior embedded in the data corresponding to quasi-periodic signals.

5. REFERENCES

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