Problem [1]. Consider the dynamical system described by \( \dot{x} = Ax + Bu, \ y = Cx, \) where
\[
A = \begin{bmatrix}
0 & 1 & 3 \\
-1 & -2 & -2 \\
-1 & 1 & -3
\end{bmatrix}, \quad B = \begin{bmatrix}
1 \\
1 \\
-1
\end{bmatrix}, \quad C = [a \ b \ c].
\]
Compute the eigenvalue decomposition of \( A = V \Lambda V^{-1}; \) is \( A \) diagonalizable? Explain. Hence compute the matrix exponential \( e^{At}; \) finally compute the real constants \( a,b,c \) such that the impulse response of this system contains a single pure exponential (i.e. no exponentials times \( t \) are allowed). How many different solutions are there and how are they related to eigenvectors of \( A? \)

Problem [2]. Aircraft AFTI-16 lateral motion. The state equations are given by \( \dot{x} = Ax + Bu, \) where
\[
A = \begin{bmatrix}
-0.746 & 0.006 & -0.999 & 0.0369 \\
-12.9 & -0.746 & 0.387 & 0 \\
4.31 & 0.024 & -0.174 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0.0012 & 0.0092 \\
6.05 & 0.952 \\
-0.416 & -1.76 \\
0 & 0
\end{bmatrix}
\]
The state is \( x = [\beta \ \dot{\phi} \ r \ \phi]^T, \) where \( \beta \) is the slip-angle, \( \dot{\phi} \) is the roll rate, \( r \) is the yaw rate and \( \phi \) is the roll angle. The control inputs are \( u = [u_1 \ u_2]^T, \) where \( u_1 \) is the aileron deflection and \( u_2 \) is the rudder deflection. Find the eigenvalues of \( A \) and hence determine stability. Moreover, find all initial conditions which will not excite the oscillatory modes of the aircraft. Compute the steady-state values of the states for \( u_1 = 1, \ u_2 = 0 \) and \( u_1 = 0, \ u_2 = 1 \) (unit step). Find the impulse response \( h \) from the rudder deflection \( u_2 \) to the roll rate, i.e. \( y = \dot{\phi}. \) Identify the transient and state state parts of \( h. \)

Problem [3]. Consider the RLC system given in the Case Studies with the following values of the parameters:
\( C_1 = C_2 = 1, \ R = R_1 = R_2 = 1, \ L_1 = L_2 = 1. \)
(a) Given the states as shown in the handout, write state and output equations.
(b) Is the system stable?
(c) Compute and sketch the impulse response \( h(t). \) Hence compute the transfer function \( H(s) \) of this system.
(d) From \( H(s) \) compute the differential equation relating the input \( u_1 \) and the output \( x_2. \) Using the procedure discussed in class, construct a state for this differential equation, determine the associated matrices \( A, B, C, \) and compute the transfer function. Is the answer what you expect?

Problem [4]. Extra Credit 20%. In problem [3] above, let the input be \( u_1 \) and the output be \( x_2. \) Let also the state space system, with states as shown in the handout, be given in terms of the matrices \( (A_o, B_o, C_o) \), while the state and output matrices obtained from \( H(s) \) using the construction-of-state procedure described in class, are denoted by \( (A_n, B_n, C_n) \) (where the subscripts ‘o’ and ‘n’ refer to ‘old’ and ‘new’ states, respectively). Define the matrices
\[
R(A_o, B_o) = [B_o, \ A_o B_o, \ A_o^2 B_o, \ A_o^3 B_o], \quad R(A_n, B_n) = [B_n, \ A_n B_n, \ A_n^2 B_n, \ A_n^3 B_n] \in \mathbb{R}^{4 \times 4},
\]
and define \( T = R(A_n, B_n)[R(A_o, B_o)]^{-1} \in \mathbb{R}^{4 \times 4}. \) Show that the relations
\[
A_n = T A_o T^{-1}, \quad B_n = T B_o, \quad C_n = C_o T^{-1},
\]
hold. Therefore the new and old states are related by means of the transformation \( T, \) i.e. \( x_n = T x_o. \) For this particular example, write down explicitly how the old state variables \( x_1, x_2, x_3, x_4 \) relate to the new ones.