Congestion Control and Channel Assignment in Multi-Radio Multi-Channel Wireless Mesh Networks

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Abstract—We address the problem of congestion control in multi-radio, multi-channel, wireless mesh networks. Compared to its single radio counterpart for which solutions exist, this problem is significantly more complex because it requires the radio channel assignments and the traffic allocations per channel be jointly optimized. We address the problem by introducing a formulation that allows its decomposition in two subproblems: A congestion control subproblem for traffic allocation to a fixed channel assignment over a node path and a discrete combinatorial channel assignment subproblem. We solve the conditional congestion control subproblem by mapping it to an optimization problem of traffic distribution to a set of radio paths. The solution provides channel congestion information that is utilized to address the channel assignment subproblem. This leads to an iterative procedure which guarantees successive increases to overall network utilization. Compared to existing work on multi-radio, multi-channel mesh networks, we show that our approach can yield significant gains both in terms of network utilization and establishing fairness.

I. INTRODUCTION

Mesh networks are being deployed with multiple radios operating in orthogonal channels in order to achieve higher speeds.1 In addition to the potential for interference mitigation via use of multiple channels, such architectures also introduce a yet unexplored flexibility to spatially allocate resources (radios and channels) to achieve fairness and congestion control objectives.

In this paper, we address the joint congestion control and channel assignment problem with an iterative, decomposition approach. Our joint optimization of rates and channel assignments incorporates that the latter is a discrete problem in nature, representing a significant departure from prior work. Our iterative, decomposition solution first determines a congestion-control driven channel assignment, and then for a given assignment, achieves the best distribution of traffic over the possible combinations of radios (i.e., logical paths). The iteration completes when we adapt channel assignments to ensure a higher total utility. In particular, by deriving feasibility conditions for the congestion control problem under a given channel assignment, we provide a significant “awareness” of the actual congestion limits of the multi-radio network, directly impacting the rate updates and the converged solution. Subsequently, by solving the channel assignment subproblem guided by the congestion control information of the previous problem solution, we derive guarantees that a new channel assignment yields an increase in the network fairness (utility) objectives. Our contributions are as follows.

First, for the congestion control subproblem, we account for the multi-radio multi-channel nature of the network by reducing the subproblem to one of distributing traffic to an appropriately selected set of radio paths. We construct this set in such a way that we also provide a solution to the traffic distribution problem at the different radios. When convergence takes place, the solution not only provides the transmission rates for each radio-to-radio link, but also determines which portion of the rate is designated for transmission in which radios (for all links along the multi-hop route), which can operate in different channels, under their own interference and congestion conditions.

Second, we solve the subproblem of channel assignment to radios by exploiting the congestion control information. We show that Lagrange multipliers, as an instance of the interaction of the two subproblems, can 1) locate the deficiencies of the previous iteration and motivate new changes 2) provide a local classification of the channels, a classification that apart from congestion, also reflects the impact of channel assignment on the global network fairness objective. We propose channel assignment algorithms that operate transparently to the notion of fairness, and provably guarantee successive increases to the network fairness objective. The results of the channel assignment subproblem in turn determine the new interference conditions and shape the congestion control problem of the next iteration.

We show that this interaction in solving the two subproblems is crucial to the performance of the joint solution. In comparison with existing work which addresses fairness issues in the multi-radio context, our approach can achieve significant gains of network utilization, while addressing a wide class of fairness objectives.

Related Work: Congestion control has been widely studied as a utility maximization problem in the context of wired networks, e.g., [13], [16], [17]. Studies for wireless networks using the utility maximization framework include congestion control design under asymmetries due to carrier sense [8], joint design of congestion control and power control [5], incorporation of clique-feasibility constraints [21], joint design

1See for example, Mesh Dynamics and BelAir Networks.
of congestion control and MAC [4],[23], and joint design of congestion control and scheduling [6],[14]. While joint optimization problems have been previously addressed, our joint problem is unique in that it is of a discrete, combinatorial nature. Moreover, none address the aforementioned challenges that arise in multi-radio multi-channel networks.

Utility maximization models have also been employed for multi-path routing in wired networks, e.g., [10], [12], [15], [22]. While multiple radios indeed provide multiple paths, [10], [12], [15], [22] do not incorporate the spatial resource allocation aspect of the problem that arises due to wireless channels.

Channel assignment in multi-radio networks has been studied with the objective of load-aware, interference-avoiding channel assignment [19],[20]. However, neither interference measures [19],[20], nor traffic load [20], are indicative of the network fairness objectives as considered here. Fairness objectives and channel assignment were taken into account in [2], together with throughput maximization and routing. However, in contrast to [2], we adopt a congestion-control oriented approach which results in improved incorporation of the congestion limits of the multi-radio multi-channel resources of the network. (We compare with [2] in Section V.) Finally, recent approaches employ exhaustive search for the channel assignment problem [18], yet are applicable only for networks of small size. To the best of our knowledge, we are the first to propose channel assignment algorithms jointly interacting with a multi-radio, multi-channel congestion controller.

The remainder of this paper is organized as follows: In Section II we describe our network model, the joint problem and the generation of the set of radio-paths. Section III addresses the problem of congestion control for a given channel assignment and Section IV describes the proposed channel assignment algorithm. Section V provides simulation results and Section VI concludes.

II. NETWORK MODEL

We consider a static wireless mesh network with a set of nodes denoted by \( \mathcal{N} \), with \( |\mathcal{N}| = N \). Each node \( n \in \mathcal{N} \) is equipped with \( M_n \) identical radios. Ability for successful transmission between nodes within wireless range is denoted by a set of logical node-to-node links \( \mathcal{E} \), with \( |\mathcal{E}| = E \). The graph \( (\mathcal{N}, \mathcal{E}) \) is referred as the network graph.

Each link \( e \in \mathcal{E} \) consists of one or more radio-to-radio logical links \( l \), formed between the radios of \( e \)'s endpoints. The set of all radio-to-radio logical links in the mesh network is denoted by \( \mathcal{L} \), with \( |\mathcal{L}| = L \). For each radio-to-radio link \( l \), we assume that data is transmitted at a constant rate \( c_l \). We assume stationary channel conditions and low mobility so that connectivity and transmission capabilities remain fixed.

The network operates with \( K \) orthogonal channels of equal bandwidth. An instance of a channel assignment to the radios of the network is denoted by: \( \pi = \{ k_{n,i}, i = 1, \ldots, M_n, n = 1, \ldots, N \} \), where the radio \( i \) of node \( n \) is operating at channel \( k_{n,i} \). We consider a slot-synchronized system with a periodic frame consisting of multiple slots. A node cannot transmit or receive on the same radio at the same slot and simultaneous operation of different radios of the same node at the same slot is permitted only if they operate at different channels. In addition, if the transmitter node of one link is within range of the receiver node of another link, then the links can transmit at the same slot only on radios of these nodes that have been assigned to different channels. Finally, incoming traffic to a radio of a node can be immediately forwarded for transmission to a different radio (and channel) of the same node.

We consider a set of sources \( S \), with \( |S| = S \), originating from network nodes and share the mesh network. The utility of a source transmitting at average rate \( x_s \) is expressed by a well-known family of utility functions:

\[
U(x_s) = \begin{cases} 
  w_n x_s^{1 - \alpha}, & \text{if } \alpha \neq 1 \\
  w_n \log x_s, & \text{otherwise}
\end{cases}
\]

where \( U(\cdot) \) is a strictly concave, non-decreasing, twice differentiable function. Finding a source rate vector that maximizes aggregate utility can lead to realization of various fairness objectives. The fairness region depends on the priority parameters \( w = \{ w_s, s = 1, \ldots, S \} \) and parameter \( \alpha \). For example, \( \alpha = 0 \) leads to throughput maximization, \( \alpha = 1 \) to proportional fairness, \( \alpha = 2 \) harmonic mean fairness and \( \alpha = \infty \), max-min fairness.

Each source \( s \) is associated with an origin-destination node pair denoted by \( (h_s, d_s) \). We consider fixed node-to-node

<table>
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<tr>
<td>( N )</td>
<td>Number of Nodes</td>
</tr>
<tr>
<td>( M_i )</td>
<td>Number of node ( i )'s radios</td>
</tr>
<tr>
<td>( E )</td>
<td>Number of links between nodes</td>
</tr>
<tr>
<td>( L )</td>
<td>Number of links between radios</td>
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<tr>
<td>( K )</td>
<td>Number of channels</td>
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<td>( \pi )</td>
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<tr>
<td>( S )</td>
<td>Number of traffic sources</td>
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<tr>
<td>( x_s )</td>
<td>Transmission rate of source ( s )</td>
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<tr>
<td>( H_{l,s,p} )</td>
<td>Binary variable denoting routing of traffic from path ( p ) of source ( s ) through link ( l )</td>
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<tr>
<td>( D )</td>
<td>Maximum route size in the network</td>
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<td>Radio paths for traffic distribution of source ( s )</td>
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<td>Transmission rate of source ( s ) at path ( p )</td>
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<tr>
<td>( C_j )</td>
<td>Capacity of clique ( j )</td>
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<td>( \lambda_j )</td>
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<td>( \bar{P}_{i,j} )</td>
<td>Binary variable indicating whether link ( i ) between two radios belongs in a clique ( j )</td>
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<tr>
<td>Add/ Rmv</td>
<td>Set of links between radios that are created/broken due to a modification of the channel assignment</td>
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<tr>
<td>( H_l )</td>
<td>Set of links between the endpoints of ( l ), operating in different channel from ( l )</td>
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Fig. 1. Notation Table
routing, expressed by binary variables $R_{l,s}$ that are equal to one if the route of source $s$ is using link $l$ and zero otherwise. Although node-to-node routes are fixed, it is possible to split traffic across radios and channels of each path. We denote by $D$ the maximum route size in the network. In the case of mesh networks where traffic is routed to wired gateways, $D$ does not typically exceed 6 hops.

Given the node-to-node routing, we exclude from the set $E$ those node-to-node links that are not included in the route of any source and their corresponding radio-links from $L$. We also denote by $\Pi$, the space of all acceptable channel assignments, in the sense that they establish the necessary connectivity, i.e. at least one common channel exists between two nodes $(n,n') \in \mathcal{E}$.

**Problem statement:** We first discuss the multi-radio congestion control problem by casting it in an abstract manner. We will provide rigorous formulation when proceeding to its solution in the next sections. Let $P$ denote the traffic distribution options in the network for a given channel assignment and $\sigma$, the available capacity of resource $j$ (capacity minus service demand). The joint congestion control and channel assignment problem can be described as follows:

$$\begin{align*}
\max_{x_s(P), \pi \in \Pi} & \quad \sum_{s=1}^{S} U(x_s(P(\pi))) \\
\text{s.t.} & \quad \sigma_j(\pi, x, P(\pi)) \geq 0, \quad \forall j
\end{align*}$$

The difficulty in solving problem MRMC-CC-CA stems from the discrete combinatorial nature of the channel assignment decisions which can be formulated as integer variables and determine the channel participation of radios and links. Simpler combinatorial problems for multi-radio networks such as channel assignment that realizes a given set of rate demands have been shown to be NP-hard [20] while in our case the optimal source rates of the globally solution are not known. We adopt a decomposition approach which divides the problem in a congestion control subproblem and a channel assignment subproblem. The congestion control subproblem is subject to a fixed channel assignment and accounts for conditions that help in distributing traffic up to the congestion limits of the actual multi-radio, multi-channel, network resources. Based on the computed source rates, the channel assignment subproblem utilizes information such as the Lagrange multipliers and link utilization from the congestion control subproblem solution to re-optimize the radio channel assignments in a beneficial direction for the network. The two subproblems are sequentially and iteratively solved until termination.

### III. Congestion Control

In this section we address the congestion control subproblem subject to a fixed channel assignment $\pi$. The main goal for this non-linear programming problem is to derive rate adaptation updates whose converged solution accounts and exploits transmissions from multiple radios, traffic distribution options, and channel-dependent interference. This is achieved by constructing a set of radio paths for traffic distribution and by deriving an appropriate set of feasibility conditions for traffic to share channels and time slots.

**Path Construction:** Multiple radios empower channel assignment decisions to spatially distribute the network capacity. While the advantage of exploiting multiple transmission options through different radios and channels is obvious, the exact way traffic should be distributed in the most efficient way is not clear. The total incoming traffic at each intermediate node is arriving from multiple radios and should be split to each of the outgoing links which might also be incident to different radios. In addition, the per source distribution of incoming/outgoing traffic for each radio has to be determined in conjunction with the resultant aggregate source rate and its fairness requirements while also accounting for the impact of the induced interference on other flows within transmission range.

We construct a set of radio-to-radio paths for each node-to-node route $s$ as follows. Starting from the link of the route $e$ that is adjacent to the source $s$, we create one path $p$ for each common channel between its two end nodes $tr(e), recv(e)$. For subsequent links of the route $e'$, for each common channel between its end nodes, we append the corresponding radio-to-radio link to all the paths constructed for the previous link. This incremental path construction procedure iterates for all links until the end of the node-to-node route and results in a set of paths $P_s$ for each source $s$ (Fig. 2).

Our congestion control design distributes the traffic of each source $s$ across its set of paths $P_s$. Based on the construction procedure, the number of paths $P_s = |P_s|$ of each route $s$ is equal to the product of common channels at each link along the route. This number is loosely upper-bounded by $(\max_{i \in N} M_i)^{D}$ where $D$ is the maximum route length in the network. For example, in Figure 2 route $s$ has $P_s = 3 \times 1 \times 2 = 6$ radio paths as opposed to the upper bound of 27. One consideration would be that a large $P_s$ would cause a very slowly converging congestion control algorithm to an extent of not being implementable. For example, in a 10-hop route of 2-radio nodes, $P_s$ is in the order of 1024 paths. Here, the mesh architecture hypothesis is crucial. Every node is within $D$ hops of a gateway and in typical deployments $D$ is rarely
greater than 5-6 hops. Hence, we consider that for multi-hop networks of mesh type, such an approach can be viable.

A source $s$ perceives utility $U(x_s)$ when data are transmitted from $h(s)$ to $d(s)$ at a total rate of $x_s$. Rate $x_s$ is the aggregate traffic achieved by transmission to each radio path $p \in P_s$ with rate $x_{s,p}$, hence $x_s = \sum_{p=1}^{P_s} x_{s,p}$.

We denote by $x_s = \{x_{s,p}, \ p = 1, \ldots, P_s\}$ the source-distribution vector, and by $X = \{x_s, \ s = 1, \ldots, S\}$ the network-distribution vector. In addition we use the binary routing variables $R_{l,s,p}$ indicate if radio-to-radio link $l$ is used by the path $p$ of source $s$ or not.

The rate of a radio-to-radio link equals the sum of the individual rates of all the paths of the network crossing this link. Those individual rates also indicate the portion of the aggregate traffic of the link that should be routed in each radio path.

**Feasibility Conditions:** We use a generic contention graph $CG_0$ to describe interference relationships in the network graph. Each vertex in $CG_0$ corresponds to a node-to-node link in the network graph and each edge corresponds to a potentially interfering links in the network graph (transmitter node of one link is within range of the receiver node of the other link) assuming there is only a single channel in the network.

Let $N^{cl}_0$ be the total number of maximal cliques in $CG_0$. Since it is possible for all links of each maximal clique to be used in all channels, we replicate each maximal clique $K$ times, resulting in a total of $N_{cl} = K \cdot N^{cl}_0$ maximal cliques, viewed as potential resources shared by the radio-to-radio links. Since each clique maps to a single channel, the feasibility conditions for any network distribution vector $X$ should impose the normalized aggregate load on each clique $\Phi_j$ not to exceed a normalized capacity $C_j$:

$$\sum_{s=1}^{S} \sum_{p=1}^{P_s} \sum_{l=1}^{L} R_{l,s,p} F_{l,j} \frac{x_{s,p}}{C_l} \leq C_j, \quad \forall j = 1, \ldots, N_{cl} \tag{1}$$

where the binary variables $F_{l,j}$ depend on the radio channel assignments and indicate whether radio link $l = 1, \ldots, L$ belongs to clique $j, j = 1, \ldots, N_{cl}$. The clique capacities $C_j$ should guarantee the existence of a time slot schedule realizing the radio link loads induced by $X$. Our clique formulation implies that cliques in each channel will be scheduled independently in the time domain. This allows leveraging results of single channel systems to determine capacities that ensure schedulability. More specifically, it has been shown in [9] that setting $C_j = 0.46$ for each clique ensures such sufficient feasibility conditions. On the other hand, if $CG_0$ is a perfect graph, then a maximum utilization factor $C_j = 1$ yields both sufficient and necessary conditions (i.e. the constraints can capture all feasible allocations $X$)[11].

Our clique-based formulation introduces the complexity of computing all maximal cliques in a graph which in general is an NP-complete problem and a time-consuming computation in practice. However, this is a one-time computation, which we deem reasonable for the static mesh network setting. In Section V, we enumerate all maximal cliques for fairly large networks that may arise in practice. Alternatively, we could utilize a set of link-based constraints where the aggregate normalized traffic of each link and all its interfering links in each channel is less than unity [2]. This set of constraints does not require the clique computation step. However, since not all interfering links of link $l$ are within range of each other this set of constraints can be overly conservative. We show in Section V that it can lead to severe network under-utilization.

Given radio paths and feasibility conditions we formulate the congestion control subproblem as the following utility-maximization problem:

**MRMC-CC:**

$$\max \sum_{s=1}^{S} \sum_{p=1}^{P_s} P_s U(x_{s,p})$$

s.t. $$\sum_{s=1}^{S} \sum_{p=1}^{P_s} \sum_{l=1}^{L} R_{l,s,p} F_{l,j} \frac{x_{s,p}}{C_l} \leq C_j, \quad j = 1, \ldots, N_{cl}$$

Each source $s$ is physically located at a given node and perceives satisfaction from individual transmissions over the multiple combinations of traffic distribution in its path set $P_s$. Hence $U(\cdot)$ is a strictly concave function of $x_s$ but not the variables $x_{s,p}$. From a mathematical standpoint, the lack of strict concavity with respect to the variables $x_{s,p}$ prohibits optimization solutions used in previous single-channel clique-based models (e.g., [7],[4],[8]). Even when congestion prices converge the source updates would cause oscillations and the original problem will never be solved.

Our radio-path based formulation enables convergence to a unique solution with a technique previously used to address multi-path routing problems in wireline networks [15],[22],[13]. The objective function is modified, with small penalty quadratic terms: $-\delta \sum_{s=1}^{S} \sum_{p=1}^{P_s} (x_{s,p} - z_{s,p})^2$, where $\delta$ is a small positive constant. These terms cause a small deviation from the optimal solution of MRMC-CC, however the objective function becomes strictly concave with respect to each $x_{s,p}$. However, only smooth convergence within a certain deviation from the optimal solution of MRMC-CC is guaranteed so far. **Exact** solutions to the original problem MRMC-CC can be achieved, with the use of Proximal Optimization Theory [3], similarly with existing approaches for wired networks [15]. According to these approaches, additional outer loops are iteratively used for eliminating the deviation effect that is due to the quadratic terms. More precisely, an additional variable $z_{s,p}$ is associated with each $x_{s,p}$ and the transformed problem is as follows:

**MRMC-CC exact:**

$$\max \sum_{s=1}^{S} \sum_{p=1}^{P_s} P_s U(x_{s,p}) - \delta \sum_{s=1}^{S} \sum_{p=1}^{P_s} (x_{s,p} - z_{s,p})^2$$

$$\sum_{s=1}^{S} \sum_{p=1}^{P_s} \sum_{l=1}^{L} R_{l,s,p} F_{l,j} \frac{x_{s,p}}{C_l} \leq C_j, \quad j = 1, \ldots, N_{cl} \tag{2}$$
The solution to MRMC-CC exact can be found following analogous steps as in [15] by considering the Lagrange multipliers (congestion prices) \( \lambda \) associated with each constraint. The procedure can be summarized as follows (see [15] for detailed description).

At each iteration \( k \),

1) The distribution of the traffic of source \( s \), to the radios of the path \( p \) is given by:

\[
x_{s,p}(k+1) = \text{arg max} \left\{ U \left( \sum_{p=1}^{P_s} x_{s,p} \right) - \sum_{j=1}^{N_c} \{ \lambda_j \times R_{l,s,p} F_{l,j} \frac{x_{s,p}}{c_l} \} - \delta \sum_{i=1}^{P_s} (x_{s,p} - z_{s,p})^2 \right\},
\]

\[
x_{s}(k+1) = \sum_{p=1}^{P_s} x_{s,p}
\]

2) For \( j = 1, \ldots, N_{cl} \) the Lagrange multipliers are updated according to:

\[
\lambda_j(k+1) = \left[ \lambda_j(k) + \gamma \left( \sum_{s=1}^{S} \sum_{p=1}^{P_s} \sum_{l=1}^{L} R_{l,s,p} F_{l,j} \frac{x_{s,p}}{c_l} - C_j \right) \right]^+
\]

where \( \gamma \) is a sufficiently small step size. After convergence, the variables \( z_{s,p} \) take the converged value \( x_{s,p} \) of the corresponding variables \( x_{s,p} \) and the entire process above is repeated until convergence.

Problems related to multi-path routing have been addressed before in the wired networks literature. Our contribution does not lie in showing how to solve multi-path routing problems but in formulating and reducing the multi-radio congestion control subproblem with the path construction technique.

As the congestion controller distributes traffic in a clique, it converges to some rates for each of its radio-to-radio links. Those rates, being the sum of multiple individual paths that express different radio transmission combinations, also indicate the portion of the traffic that should be distributed to each of the paths. All along the multi-hop route, traffic of that link will be carried to radios that operate in under distinct interference and coexistence with other flows. However, we want to highlight that the congestion controller will converge to such a solution in the rate distribution problem to the different radios and channels, that will be ‘optimal’ in the sense of better meeting the fairness objectives, as expressed by the network aggregate utility.

### IV. CHANNEL ASSIGNMENT

The congestion control sub-problem yields optimal radio paths and source rates given a channel assignment. Except for very small networks, reaching the optimal solution of the joint problem through exhaustive search is not feasible due to the large number of channel assignments (order of \( \sum_{n=1}^{N} (M_n^K) \)).

We propose an approach where congestion control and channel assignment sub-problems are solved sequentially and iteratively. Given a solution \( X \) of the congestion control subproblem that results from channel assignment \( \pi \), we seek a new channel assignment \( \pi' \) that will yield a congestion subproblem solution \( X' \) of higher aggregate network utility in the next iteration.

In principle, the new channel assignment \( \pi' \) should remove traffic from highly congested resources or add bandwidth to highly congested resources if possible. In a multi-radio mesh network these two actions translate to channel modifications that result in deletion of radio links from highly congested channels and cliques or addition of radio links on other channels, respectively. An intuitive attempt to realize this high-level goal would be to formulate and run a global channel allocation optimization problem that minimizes network-wide interference subject to traffic vector \( X \). However, this approach would not necessarily yield a channel assignment that results in increase of the network utility function.

We propose a heuristic for the channel assignment sub-problem which uses congestion control information and guarantees successive increases in network utility. The key idea of our channel assignment algorithm is to use the Lagrange multipliers of the congestion control subproblem to identify the most congested cliques as local areas of highest priority. This approach focuses the algorithm search at a level local to a clique. Within these cliques local channel modifications are sought that result in links deleted from the congested clique or by radio links added on other channels for reinforcement.

From the optimization problem viewpoint, the channel modifications result in modifications of the discrete constraint coefficients \( F_{l,j} \) thus producing a new set of constraints for the congestion control sub-problem of the next iteration. A channel modification of even a single radio link not only modifies the constraints of the cliques it belongs but also the constraints of other cliques in different channels. The challenge is to find the channel modifications that will guarantee an increase in the network utility function without solving the congestion control sub-problem for each potential modification. Our algorithm identifies such channel modifications by utilizing the traffic vector \( X \) of the congestion control sub-problem.

#### A. Local channel modifications

We first identify the minimal channel modifications that result in radio link deletions or additions. We then identify a set of conditions that need to be satisfied in order for such channel modifications to yield higher aggregate network utility of the congestion control sub-problem of the next iteration. Finally we introduce a channel assignment algorithm that incorporates the channel modifications and conditions.

Consider a congested clique \( j \) that operates in channel \( k_j \). We seek minimal local channel modifications that either delete radio links from clique \( j \) and channel \( k_j \) or reinforce clique \( j \) by adding radio links to other channels that share common node endpoints with the links of clique \( j \). The minimal
channel modifications can be link-based or radio-based. Link-based modifications involve switching both radios of a radio link to a different channel. Radio-based modifications involve switching only a single radio to a different channel. Link-based modifications are more drastic because they result in more links switching channels. For ease of illustration in the following we describe radio-based channel modifications for link deletion and link reinforcement at clique $j$. Link-based channel modifications are performed in a similar manner.

**Link deletion:** A radio of link $l$ in clique $j$ switches from channel $k_l(s = k_j)$ to channel $k'$. The new channel $k'$ should be different than the channels assigned to the other radio links of node-to-node link $e_l$ where radio link $l$ belongs. This modification results in deletion of radio link $l$ and all adjacent links of its switched radio on channel $k_l$. At the same time this modification may result in addition of new links adjacent to the radio if there exist other radios within transmission range in channel $k'$.

**Link reinforcement:** Let $e_l'$ be the node-to-node link where a radio link $l'$ of clique $j$ belongs. A radio of a “parallel” radio link $l$ that also belongs to $e_l'$, switches from its channel $k_l$ to channel $k'$. This modification results in deletion of links adjacent to this radio in channel $k_l$ and addition of links adjacent to this radio in channel $k'$. The new channel $k'$ should be different than the channels assigned to the other radio links of $e_l'$. Also, to ensure reinforcement, channel $k'$ should be such that more “parallel” radio links to the links of clique $j$ are added for channel $k'$ than deleted from channel $k_l$.

**B. Eligibility conditions**

We now derive the conditions under which the above modifications result in a new channel allocation $\pi'$ that results in an increase of the aggregate network utility function, given channel assignment $\pi$ and the traffic distribution $X$ of the congestion control sub-problem.

Let $x_l$ be the load of each radio link $l$, and $\sigma_j$ be the available bandwidth of each clique $j$:

$$x_l = \sum_{s=1}^{S} \sum_{p=1}^{P} R_{lsp} \frac{x_{lsp}}{c_l}, \quad l = 1, \ldots, L$$

$$\sigma_j = C_j - \sum_{l=1}^{L} F_{lj} x_l, \quad j = 1, \ldots, N_{cl}$$

Consider a channel modification (either link deletion or link reinforcement). Let $Rmv$ be the set of deleted radio links from channel $k_l$ and $Add$ be the set of new radio links in channel $k'$. For each deleted link $l \in Rmv$, denote $H_l \subseteq \pi_l$ the set of radio links that have common node endpoints with link $l$ and operate on different channels than $k_l$ according to channel assignment $\pi$. Also denote by $Add_l$ a link in $Add$ that has common node endpoints with link $l$ in $Rmv$ set and operates in channel $k'$. For each link $l \in Rmv$, assume that we (independently) load each radio link $l'$ in sets $H_l$ and $A_l$ with the load of link $l$, i.e. $x_{l'} = x_l$. The following conditions guarantee that the load can be supported by all channels (other than $k_l$) and all their cliques after the local channel modification from $k_l$ to $k'$:

$$\sum_{l' \in \Phi_j} x_{l'} \leq \sigma_j, \quad j = 1, \ldots, N_{cl}$$

where $\Phi_j = \bigcup_{l \in Rmv} (H_l \cup Add_l) \cap j$ is the set of all added or existing radio links that shared common node endpoints with deleted links that belong to clique $j$.

**Theorem 1:** Let $U(X, \pi)$ denote the aggregate utility of the solution $X$ under channel assignment $\pi$. For every minimal channel modification $\pi \rightarrow \pi'$ that obeys conditions (8), the solution $X'$ of the new congestion control sub-problem will yield $U(X', \pi') \geq U(X, \pi).

**Proof:** Modifying a channel assignment signifies displacement of one or more radio links to different constraints of the original congestion control sub-problem, as well as deletion or addition of radio links in the constraints. If the local channel modification satisfies the conditions (8), all constraints in the new congestion control sub-problem under $\pi'$ will be able to carry the traffic carried under $\pi$. This is due to the fact that the rate of all deleted links in set $Rmv$ can be certainly served by other links without causing a decrease to the rate of any other flow in the network. Hence, it follows that the new aggregate converged rates for each of the radio-to-radio links will either be increased - if the congestion controller of the next iteration decides that this is of benefit - or in the worst case remain unchanged. The same argument holds for the converged rates of the paths and the sources as well, and by the increasing property of the utility functions, the result follows.

**Example.** Fig. 3 provides an example for the derivation of eligibility conditions (8) for a local channel modification where the radio of node B tuned to channel 1, switches to channel 2. This results in radio links BF(1),AB(1),BE(1) to be removed from channel 1 ($Rmv$ set) and radio link BF(2) to be created on channel 2 ($Add$ set). To derive the conditions we focus on channels 2 and 3. In the new channel assignment, the load of each radio link in the $Rmv$ set should be carried by its corresponding existing links (the $H_l$ sets) and/or added links ($Add_l$ sets) in the other channels (channels 2 and 3). The eligibility conditions are derived by (i) “loading” the links from all sets $H_l \cup Add_l$ with the loads of $Rmv$ set and (ii) finding the intersecting links of sets $H_l \cup Add_l$ and each of the six cliques in channels 2 and 3.

**C. Channel assignment algorithm**

We describe the algorithm that selects the channel re-assignment at each iteration of the congestion control/channel assignment loop. Its input is the solution $X$ (or link loads $x_l$) and clique Lagrange multipliers of the congestion control sub-problem and the current channel assignment $\pi$. Its output is a new channel assignment $\pi'$ that aims at higher aggregate network utility in the next iteration.

The algorithm visits all cliques $j$ in descending order of their Lagrange multipliers $\lambda_j$. For each clique $j$, the algorithm
interference-aware greedy heuristic for link channel assignment such as [19], [20] can be used for initialization. We have implemented a greedy heuristic that assigns channels to radios instead of node-to-node links. In contrast to previous approaches this allows traffic to be transferred simultaneously over different pairs of radios on the same link.

However, in the beginning not only congestion control information is not available, but more important, the cliques are unknown. Algorithms that examine possible generations of cliques would be of extremely high complexity. Hence we believe that initially the best that can be done with low complexity is an interference avoiding assignment that gives priority to links that seem more important.

We consider a routing-constraint graph as expressed by the node-to-node routing requirements (figure 4). The procedure takes place in two parts:

Step#1: In the first step each node $n$ has $M_n - 1$ radios available for assignment, without any connectivity concerns. The main idea is to assign less congested frequencies to more important / congested links. Consequently, at each iteration $k$

For each node-to-node link $l$, we define a weight $u_l$ that is equal to the sum of the weights of the sources that are using this link, and a local congestion indicator $r(j, l, k)$ for channel $j$ as perceived in link $l$ at iteration $k$:

$$u_l(k) = \sum_{s \in l} w_s, \quad r(j, l, k) = \sum_{s \in l, m} w_s$$

respectively, where by $u_l$ we denote the possible number of radio-to-radio links for link $l$. Notation $s : l$ means that source $s$ is using this node-to-node link and $\exists m, s : m$ means that there exists a link $m$ within interference range of $l$ so that is used by source $s$. At each iteration the sender & receiver radios of the link with the biggest weight $u_l$ are greedily assigned to the less congested (minimum $r$) frequency $j$. The procedure iterates until no other assignments are possible.

Of course some radios of a node might be unmatched (apart from the reserved radio), with all the radios of the neighbors being assigned to a frequency. In this case, the unmatched radio of node $i$, follows the same frequency $f$ with a radio of that neighbor $j$, $l = (i, j)$ , that seems to incur the more benefit, with a view to generated interference as well. Hence,

$$j = \arg \max_j \frac{\sum_{s \in l} w_s}{r(f, l)}$$

Procedure iterates until only the reserved radios are not assigned.

Step#2: In the second step, in the worst case all reserved radios operate in the same frequency and necessary connectivity is ensured. However, with the connection of some nodes already having taken place in the first step, then the connection graph might be separated to subgraphs each group of which can operate in a separate frequency. Then all the radios of the subgraph are assigned to the frequency $j$ so that:

$$j = \arg \min_j \max_l \left\{ \sum_{s \in l} w_s + r(j, l) \right\}$$

searches over all local channel modifications in terms of link deletion or link reinforcement and identifies a set $I_j$ of eligible modifications using conditions (8). If no eligible modification is found the algorithm proceeds to the next clique. If multiple eligible modifications are found, the algorithm terminates with a NULL output; this also signals no further improvements can be guaranteed by channel assignment.

**Complexity analysis.** We provide a worst-case analysis of the channel assignment algorithm in terms of the maximum node (and radio) degree $\Delta$ in the network graph and the number of cliques $N_{cl}$, maximum clique size $\phi_i$. The analysis also uses the fact that the maximum number of radios $\phi_n$ that have a radio link in a given clique equals $2\phi_i$.

Sorting the cliques in descending order of congestion price is of $O(N_{cl} \log(N_{cl}))$ complexity. While visiting a clique, at most $\phi_n$ radios will be examined. For each radio, the algorithm examines switching the channel of the clique in one of the remaining $K - 1$ channels. The size of the sets $Add$ and $Rmv$ are bounded above by $\Delta$. Thus each clique visit yields complexity of $O(K\Delta \phi_n)$. Since $\phi_n < 2\phi_i$ both link deletions and link reinforcements for each clique will be of complexity $O(K\Delta \phi_i)$. In the worst case the algorithm will visit all of the $N_{cl}$ cliques. Taking into account the initial sorting complexity, the complexity of the channel assignment algorithm is $O(N_{cl}K\Delta \phi_i + N_{cl} \log N_{cl})$.

**Initial channel assignment algorithm.** The iterative procedure begins using an initial channel assignment that is not aware of the congestion control information. In principle, any
which means that the whole path follows the frequency that causes the smallest interference imbalance in the network, as examined for each link $l$. Certainly connectivity is ensured. Topology significantly affects the performance of this heuristic, however it is easy to see that undesirable performance (assignment of all the reserved radios in the same frequency) is conducted in the case of dense ad-hoc networks. However, in mesh networks with a tree-like structure, it is easy to see that the resulting subgraphs can be of very low size (can be typically 1-2 radios).

Unmatched radios after this procedure, follow the previously mentioned rule.

V. SIMULATION RESULTS

In this section we evaluate the performance of our joint channel assignment and congestion control algorithm. We compare our approach to the work in [2], a paper that addresses throughput maximization in multi-radio networks subject to fairness constraints. Even though our methodology is different we consider a comparison that also involves the problem formulations for addressing the above-mentioned goals which we share. We solved the linear problems in [2] for various topologies, using CPLEX [1]. For the same topologies, we implemented and solved the proposed congestion control and channel assignment algorithms with the use of a simulator describing the wireless channel by the clique constraints discussed in Sec. III. In Figure 5(b), we consider a grid topology as in [2] with exactly the same parameter setup (number of nodes and gateways, link capacities, etc) as in [2]. For a variety of $(M, K)$ values, we plot the network utilization as expressed by the average source rate $\sum_{s=1}^{S} \frac{x_s}{\lambda}$, for the final outcome of our joint scheme, for the cases of $\alpha = 0$ and $\alpha = 1$, i.e. pure throughput maximization and proportional fairness. In [2], throughput was maximized subject to the following fairness constraint: Each source $s$ has a demand $w_s$, and all sources attain a rate $x_s$ equally scaled from their demands, i.e. $x_s = \lambda^* w_s$. In the examples in [2], Fig. 5(b), Fig. 6(b), all nodes have equal demands. Of course, for $\alpha = 0$, the average source rate is very high, without any bounds in the discrepancies between the rates. We also plot the value of the minimum rate $\min_s x_s$, attained for our solution for a simple scenario where: $\alpha = 1$ and $w_s = 1, \forall s$, i.e. proportional fairness for equal weights for each utility function. While the average network utilization is significantly higher, the minimum rate $\min_s x_s$ for this scenario is of the same order with the solution in [2]. Hence, our statement is as follows: By following a joint congestion control and channel assignment approach, while addressing a wide range of fairness classes, rates and channels can be assigned in such a way that high network utilization is always ensured and appropriate selection of $\alpha$, and $w_s$, can yield minimum rates that are mostly limited from the bottlenecks of the network, rather than induced unfairness from the joint operation of the algorithm. While the previous topology had a relatively regular node density (and hence interference) distribution, the importance of our statement can be observed in the following example. We consider the topology of a city-wide deployed network in Chaska, MN (Fig. 6(b)). In this example, 24 access points (out of 194) are chosen as gateway nodes. Realistic as it is, node density can be far from uniform, i.e. observe the south part of the network in Fig. 6(b). According to [2], all demands should be equally scaled down to such an extend that all transmissions through bottlenecks are feasible. Hence, such a bottleneck would be a sufficient cause for severe rate degradation in the remaining of the network. The underutilization is partially based on the fairness constraint that forces all flows to have equal rates even in heterogeneous topology scenarios as well as the usage of link-based constraints which are overly conservative compared to clique-based constraints. The resources that we can actually exploit are much more, see Fig. 6(b). Our explanation arises from the operation of the congestion control algorithm, itself. On the first hand, rate adaptations take congestion prices into account, which leads to a greater ‘awareness’ of the congestion limits of the network. More importantly, the described path construction, arms possible traffic distribution options with a wiser view for the usage of the shared, additional multiple-radio and channel resources, something that can be reinforced by observing the increasing gap for greater values of $(M, K)$.

Finally, for illustration purposes, in Fig 7 we demonstrate an evolution of the iterative algorithm by plotting both the outcome of congestion control solution and the network utilization $\sum_{s=1}^{S} x_s$ $(\alpha = 1$, with $(M, K) = (2, 10))$ for each channel assignment in every iteration, until termination. We also simply note that our simulations show that when the ratio $\frac{K}{M}$ is low, the number of iterations before termination is relatively low as well.

VI. CONCLUSIONS

In this paper we addressed a new problem of congestion control and channel assignment algorithms in multi-radio multi-channel networks. We proposed a formulation that relies on distributing traffic to a set of paths that characterize the different and multiple radio/channel transmission capabilities of the network. We showed that with such an approach, the congestion control algorithm can explicitly exploit these features and obtain convergence to a significantly higher
network utilization than achievable with existing techniques. Moreover, we showed how channel assignment decisions are crucial to the operation of congestion control, yielding a different channel assignment problem that emerges from the congestion control problem itself. Thus, we proposed novel channel assignment algorithms that combine congestion control information with channel allocation to solve the joint problem.

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Fig. 7. Evolution of channel assignment algorithm iterations (M=2, K=10, 60 nodes, 8 gateways)