Vision on mirrors

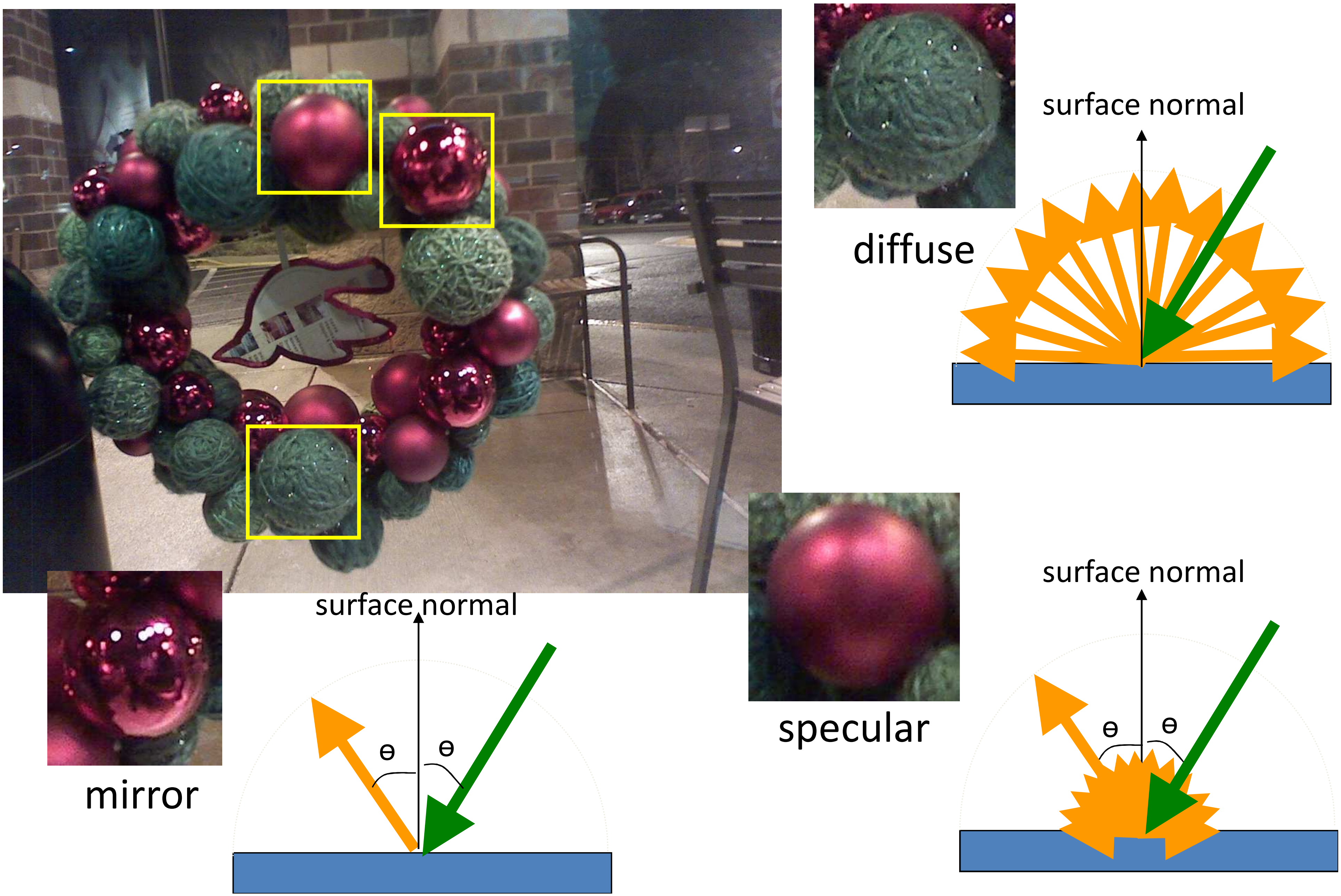
Aswin C Sankaranarayanan

Vision on mirrors

Joint work with

Ashok Veeraraghavan, Oncel Tuzel and Amit Agrawal

Object reflectance and image formation

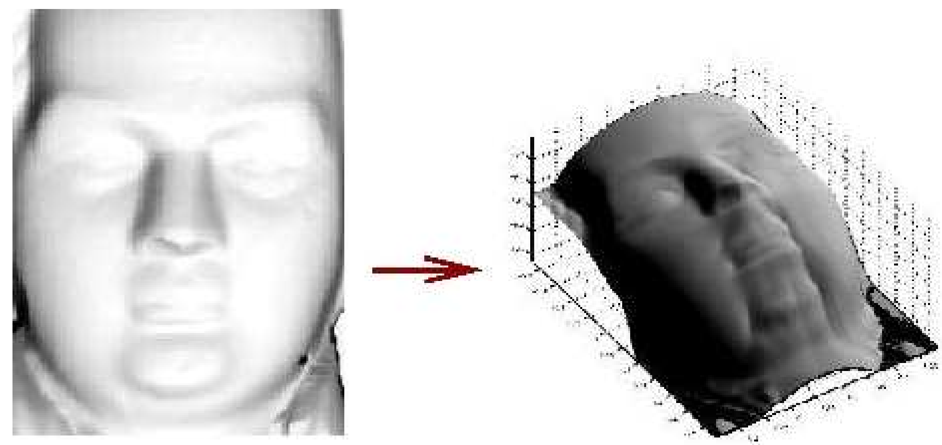


Lambertian Objects

- Our understanding of Lambertian (diffuse) objects is immense

$$I = \rho \max(n^T s, 0)$$

There has been significant advances in our ability to model and solve vision problems with Lambertian objects: Basri and Jacobs, Georghiades et al., Chen et al, …



a) Image b) 3D surface reconstructed from the single image a)

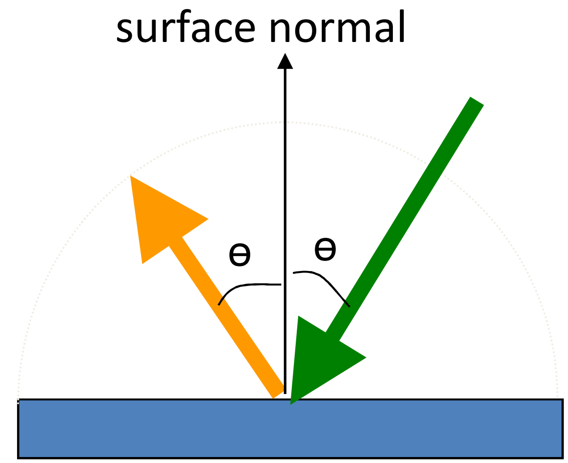
(Figure from Perception lab, INRIA)

For Lambertian objects, the photometric properties of the image depend strongly on the geometric features of the surface! This is key in a wide range of problems: correspondences, optical flow, tracking, …

Yet a large class of interesting objects are specular

We have limited understanding on how to solve computer vision problems on such objects

Mirrors: why are they hard ?



Simple physics

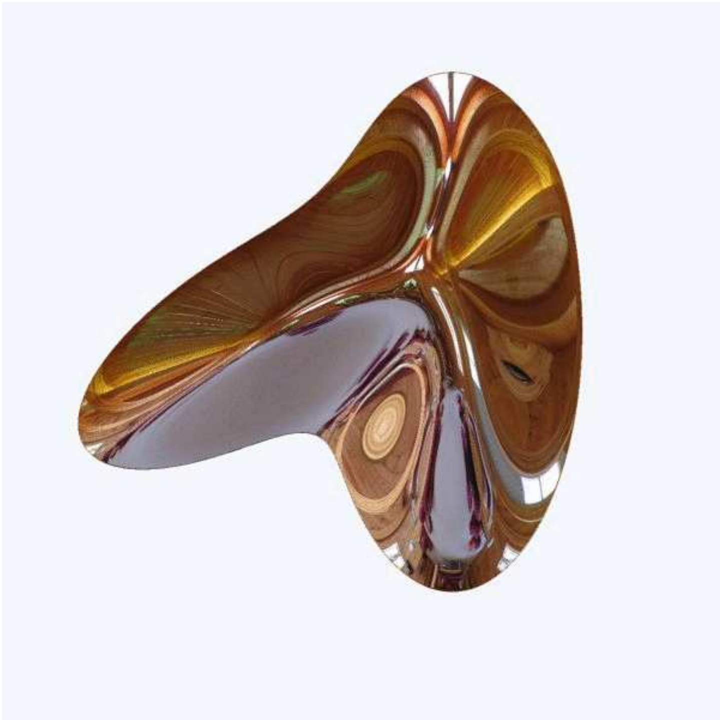
- Snell's laws of reflection
- What is imaged is a distorted copy of the surrounding scene

Mirrors have no appearance of their own

Without additional information (such as knowledge of its surface and/or detailed characterization of the scene), classic vision problems such as SfM, photometric stereo are extremely hard.

Mirrors: why are they hard?

- Lack of observable geometric features
  - Environment/mirror pairs can be designed to create arbitrary distortions and images.

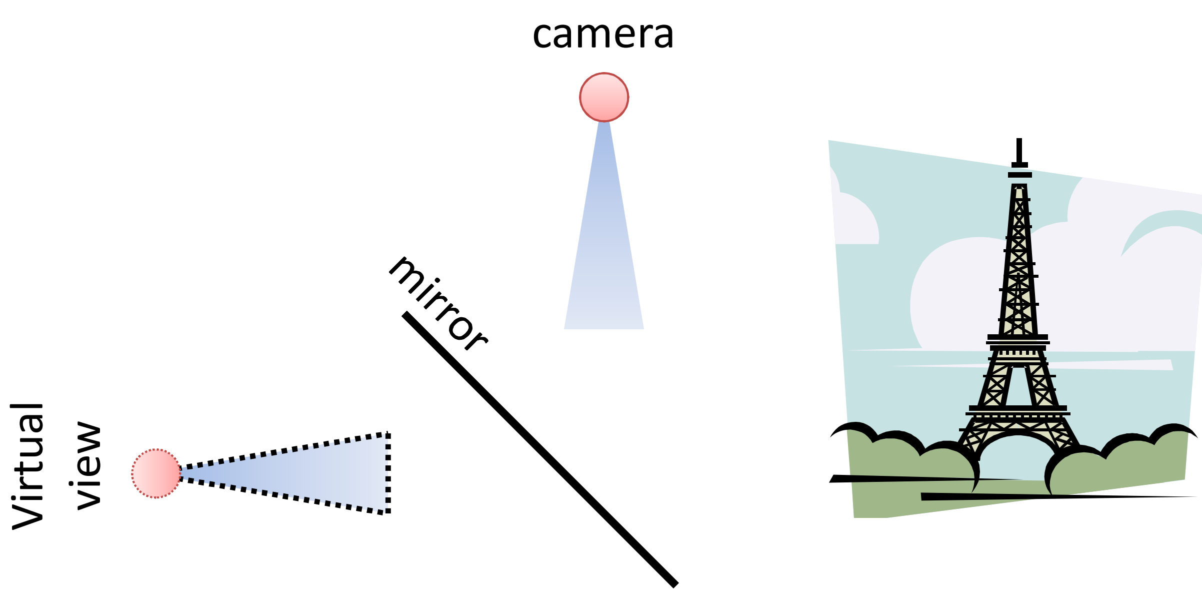


- Even under Euclidean motion, there is no consistent geometry of feature point trajectories
  - Optical flow and the projected motion flow can be completely incoherent
  - Triangulation is meaningless

Image invariants for smooth mirrors

Invariants ?

- Not without additional assumptions



Perspective camera looking at a planar mirror is the same as a perspective camera

As a consequence, the image can be anything!

Need additional assumptions to say anything meaningful.

why should there exist an invariant ?

- Human perception studies
  - We are capable of identifying mirror shapes even when we do not know the environment

- Existing literature comments on *“special”* behavior at certain points
  - **Ikeuchi**: inflexions at parabolic points
  - **Zickler and collaborates**: Parabolic points exhibit *infinite* flows

Analyzing image formation

Assumptions:

1. Perfect Mirror reflectance
2. Environment at Infinity
   - Reflection depends only on scene normal
   - No dependence on object location.
3. Orthographic Camera

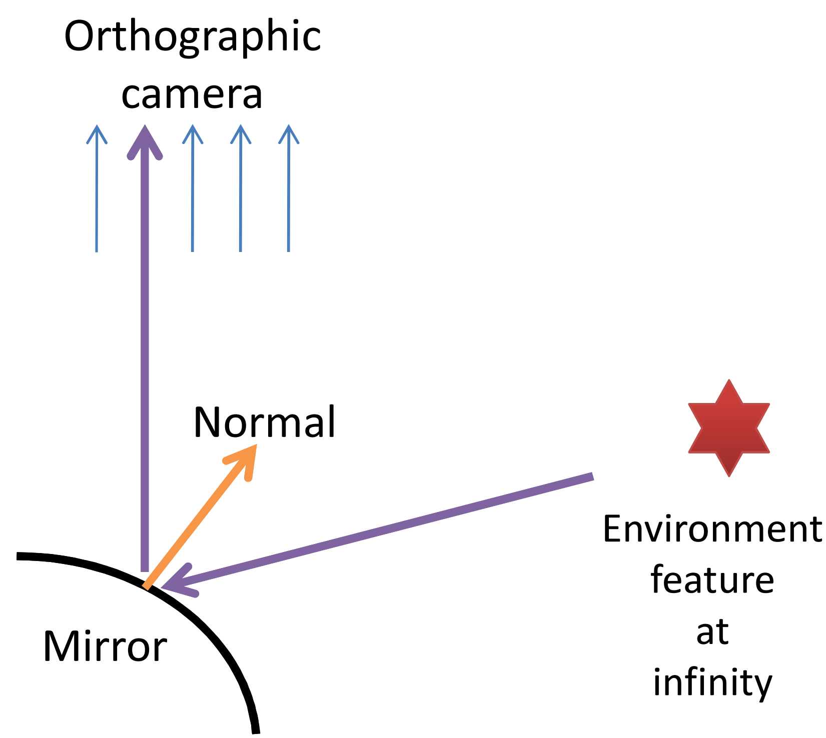


Image intensity at Pixel “x”

$$I(x) = E(\Theta(\nabla f(x)))$$

Environment reflected at pixel “x”

Forward imaging model depends *only* on surface normal of the point on the mirror

Imaging of a Mirror: Forward Model

Modeling the mirror as *z = f(x, y)*, the image gradients can be written as

$$\nabla_X I = \left[\frac{\partial^2 f}{\partial x^2}\right]\left[\frac{\partial \Theta}{\partial \nabla f}\right]^T \nabla_\Theta E$$

Image gradient | Hessian to the surface | Change of basis | Env. gradient

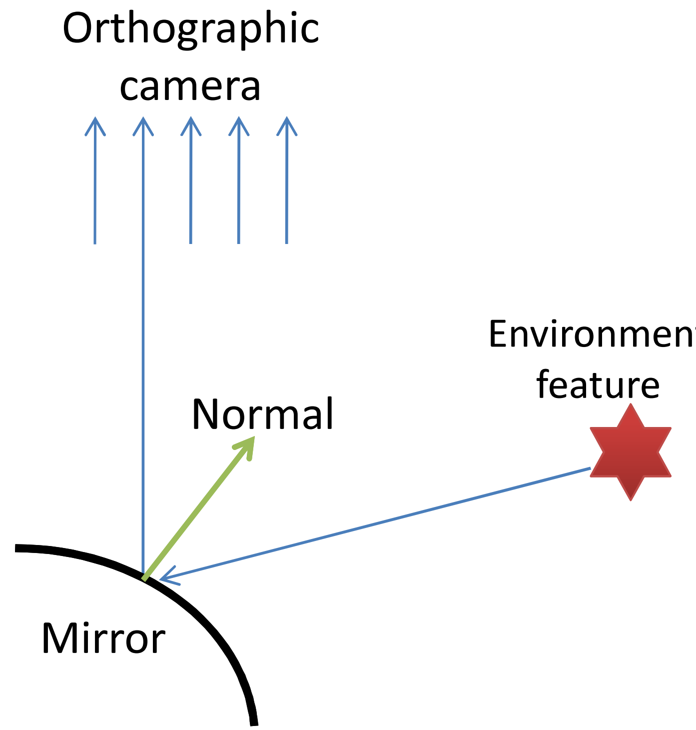


Image gradient are distorted versions of the environment texture gradients

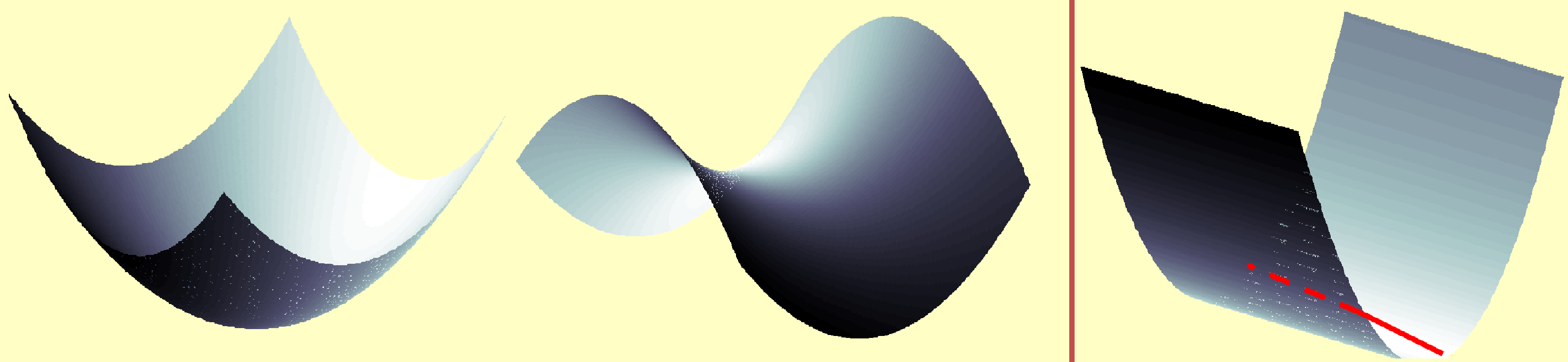
Distortion depends mainly on surface curvature!!!

A geometric feature!!!!!

Key idea:

If the hessian is degenerate, then the image gradient will be degenerate as well

Local surface properties



Elliptic	Hyperbolic	Parabolic
Surface bends similarly along different directions	**Surface bends in opposite directions**	**No bending along one direction**

Hessian is degenerate
It has rank 1

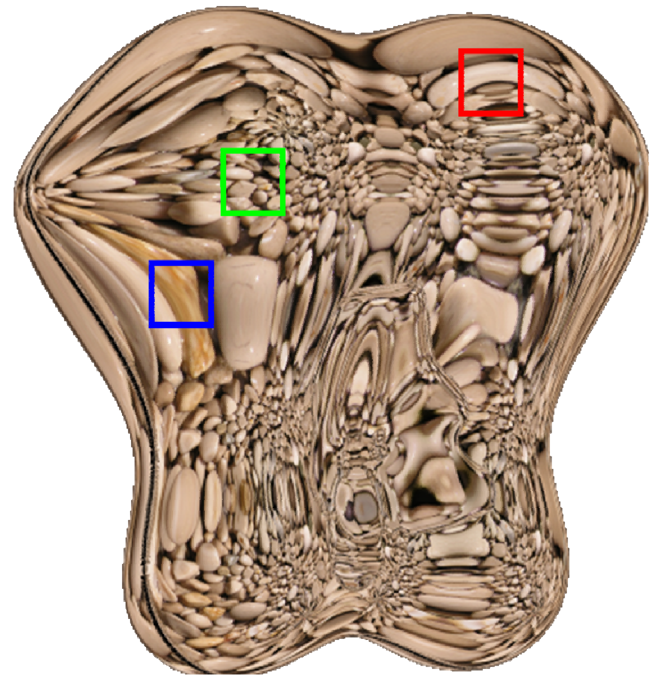
$$\nabla_X I = \left[\frac{\partial^2 f}{\partial x^2}\right]\left[\frac{\partial \Theta}{\partial \nabla f}\right]^T \nabla_\Theta E$$

crowning achievement

At a parabolic point, the image gradient is always parallel to the non-zero eigenvector of the hessian matrix

Its direction is *independent* of the environment!

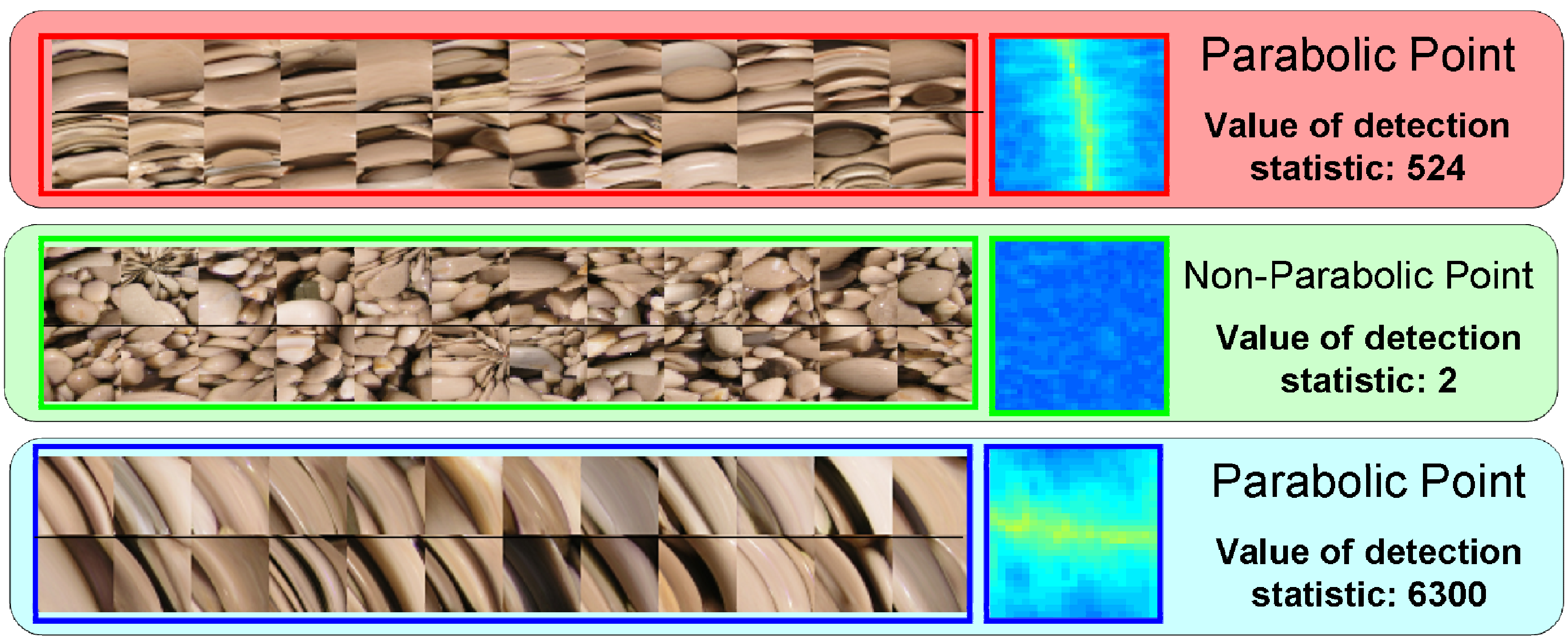
quick illustration



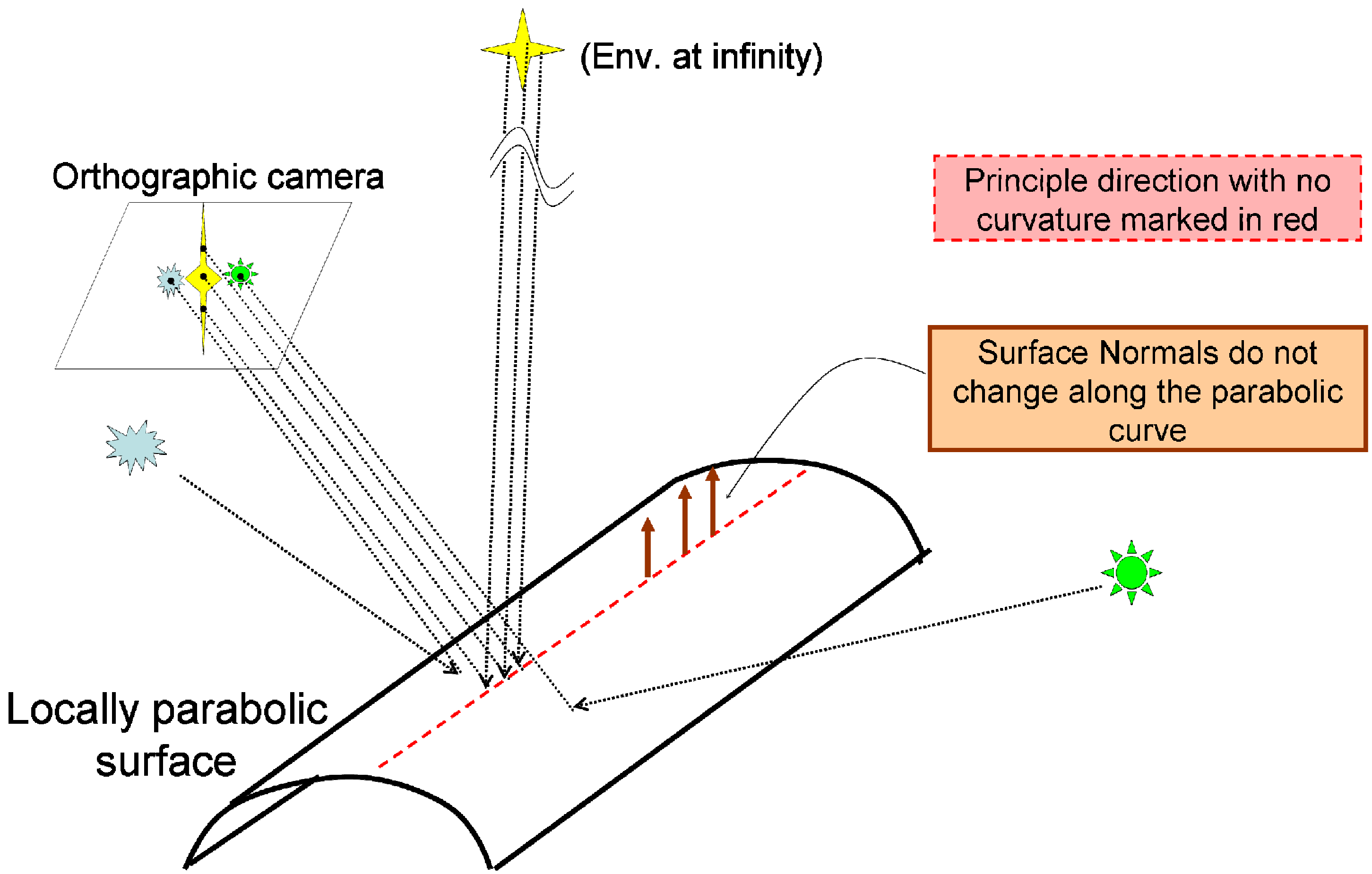
We image a mirror under a many different environments and look at small patches at various locations

At each pixel, compute $M = \sum_E \nabla I_X(E) \nabla I_X(E)^T$

If M is rank 1, then the pixel is at a parabolic point



A geometric perspective



Surface normals do not change along a parabolic curve. Therefore, the same environment feature is imaged as we traverse along the parabolic curve. Hence, no image gradient in that direction (the direction of the zero eigenvector)

We can recover parabolic curvature points by

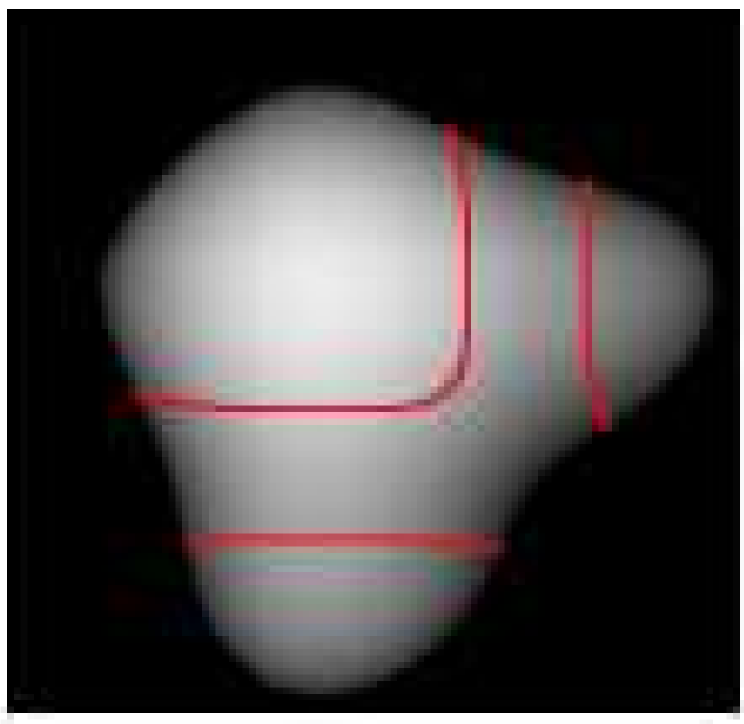
Changing the environment texture

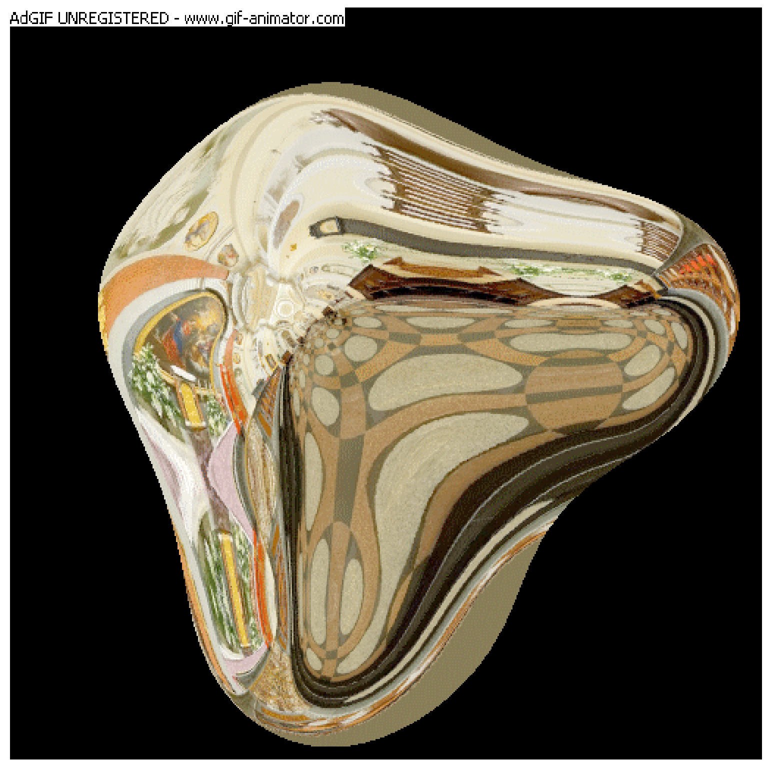
projector, active illumination

Rotating the scenes or equivalently, the camera-object pair

creates “novel” environments

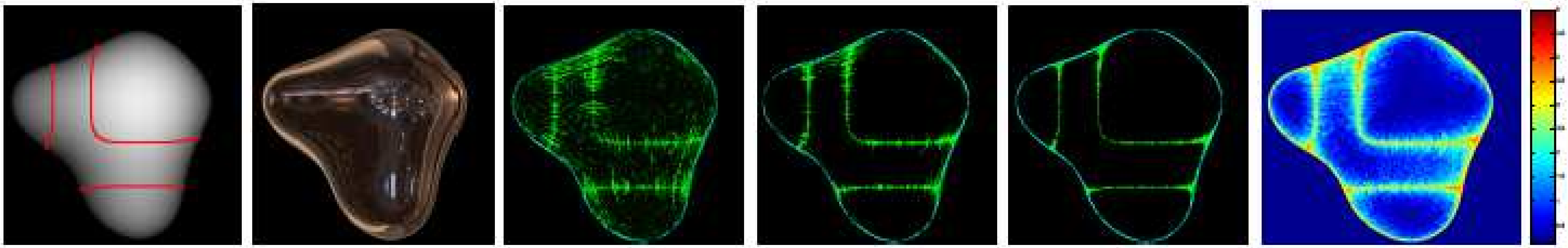
Rotating the mirror about camera axis with known motion



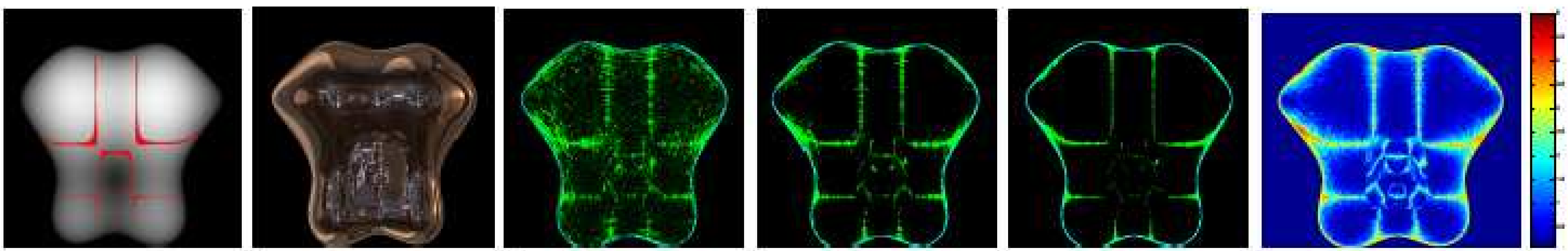


Detecting Parabolic curvature points

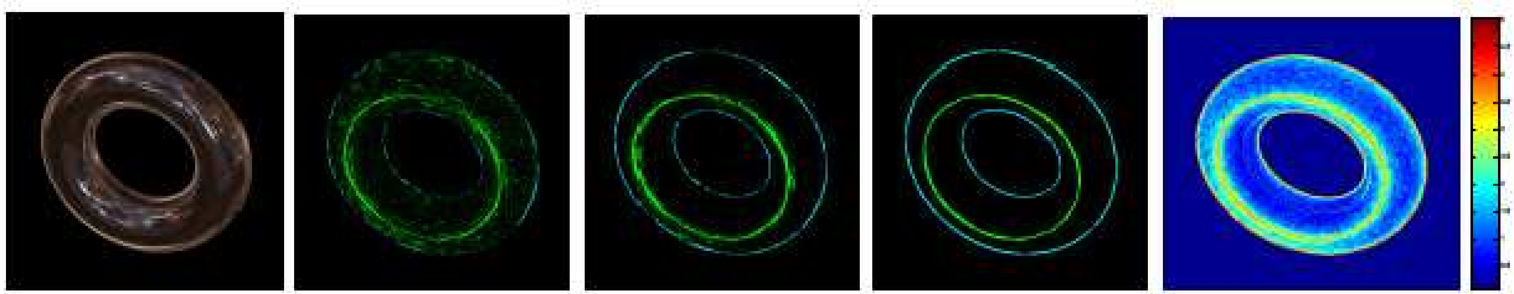
2 Images **5 Images** **25 Images**



(a) $z = \sqrt{4 - x^2 - y^2 - cos(2x - 2) - sin(2y)}$



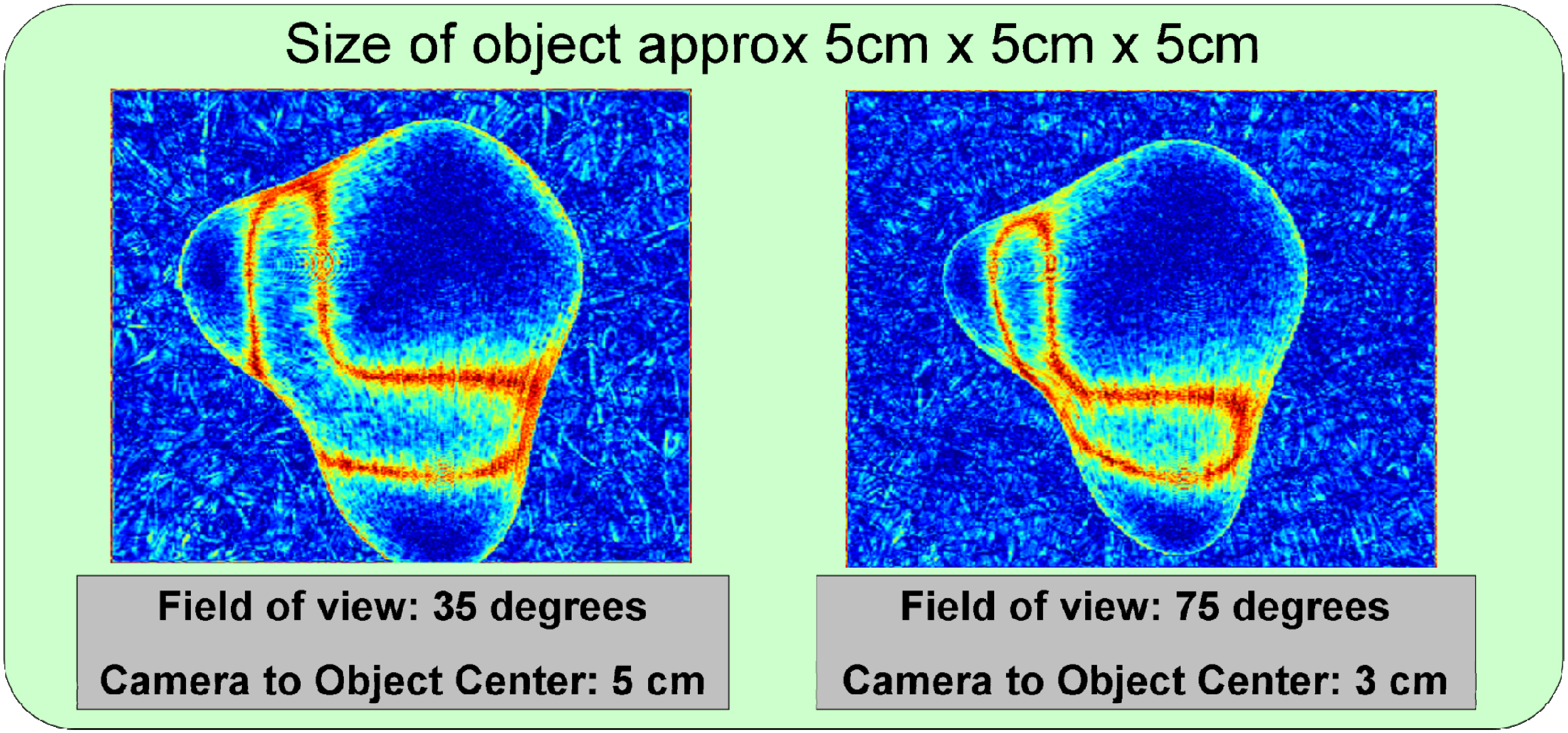
(b) $z = \sqrt{4 - x^2 - y^2 - cos(3x - 6) - 2sin(2y)}$



(c) Rotated Torus

Departure from assumptions:

from orthographic to perspective

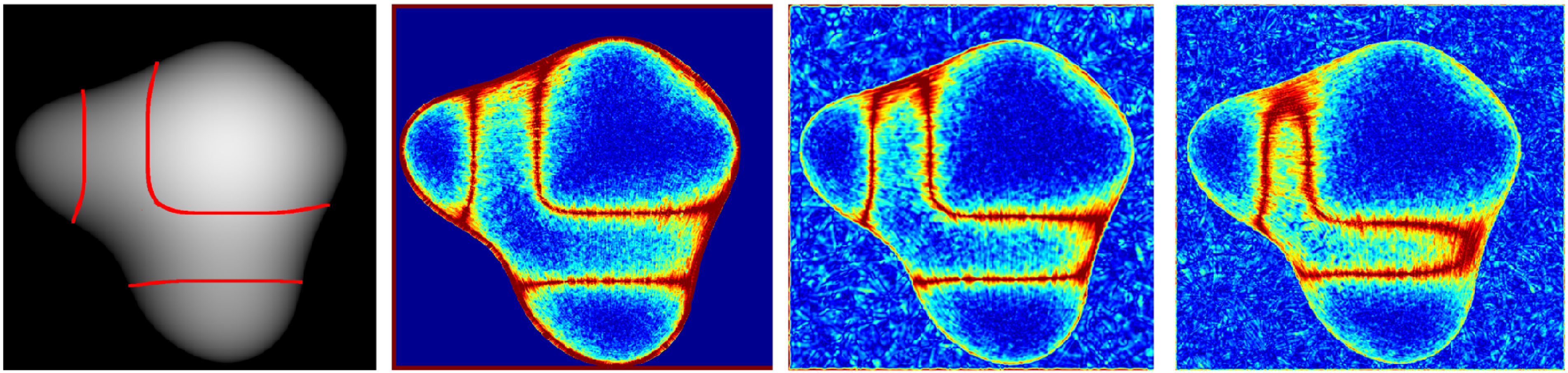


Imaging by moving camera extremely close to the mirror.

Note that the drift in view point is partially explained by the change in camera position

Departure from assumptions:

from scene from infinity to finite distance



(a) Depth map and true parabolic curves. The depth of the surface varies by 5 cm.

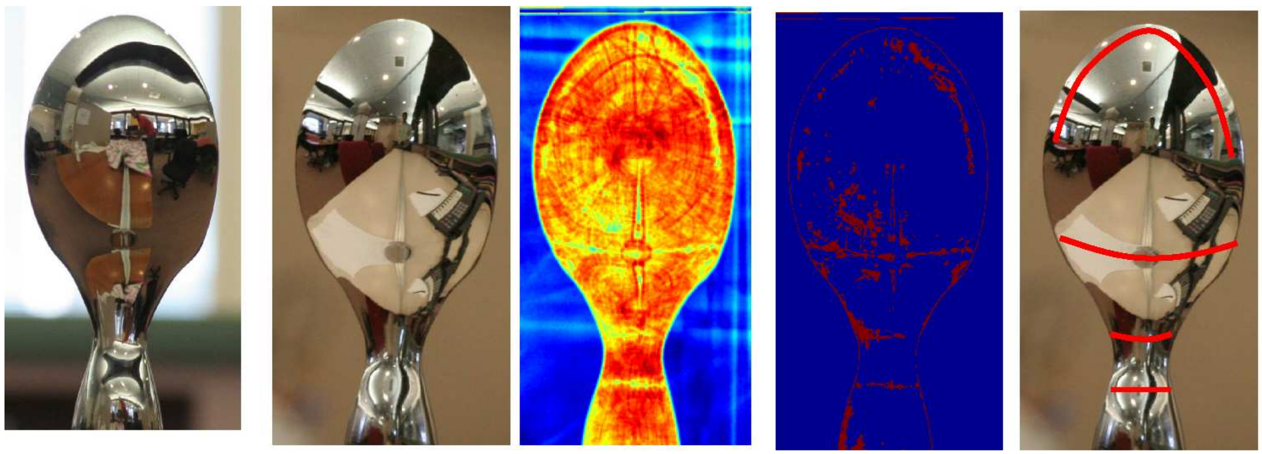
(b) Parabolic curve decision statistic when the environment map is at infinity.

(c) Parabolic curve decision statistic when the environment is 11 cm away.

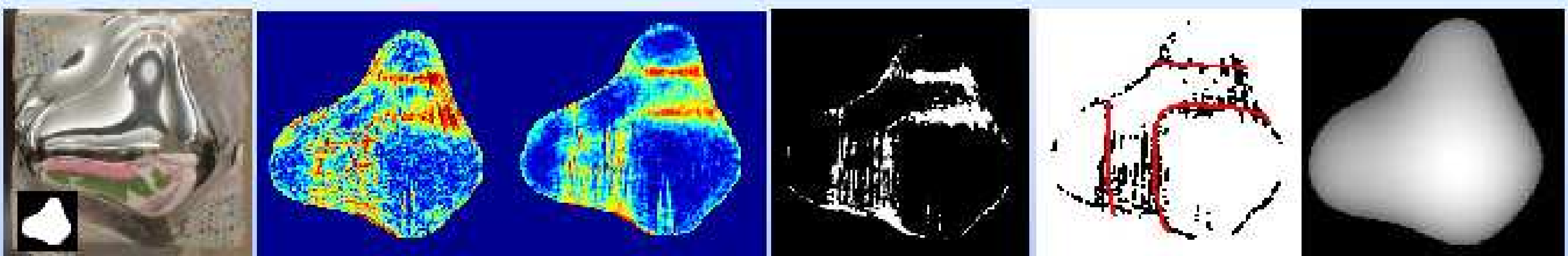
(b) Parabolic curve decision statistic when the environment is 5 cm away.

Parabolic point detection is stable when the assumptions of orthography and scene at infinity are relaxed.

Detecting points of parabolic curvature

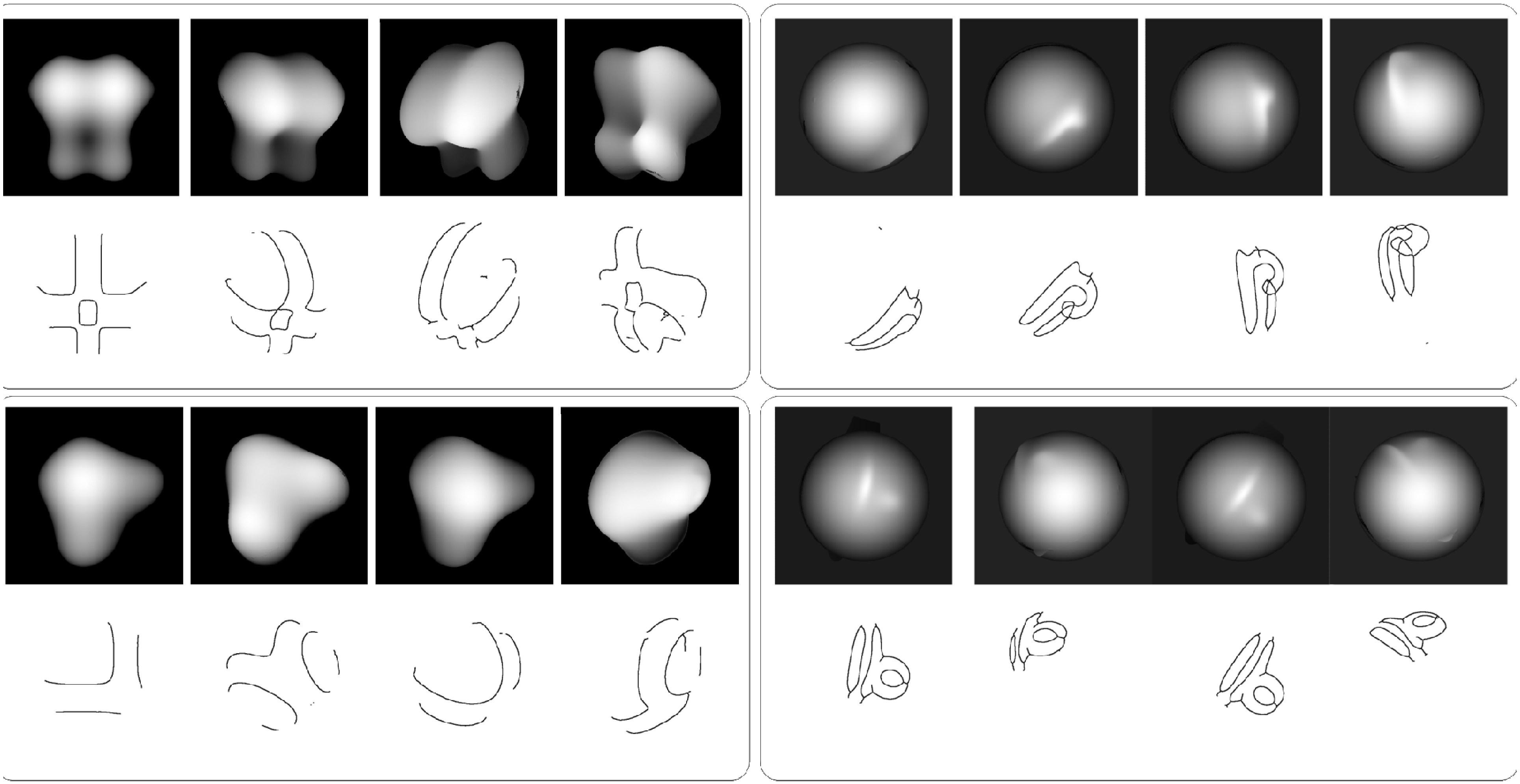


Input Images | Estimated detection statistic | Detection Results | Manually marked ground truth



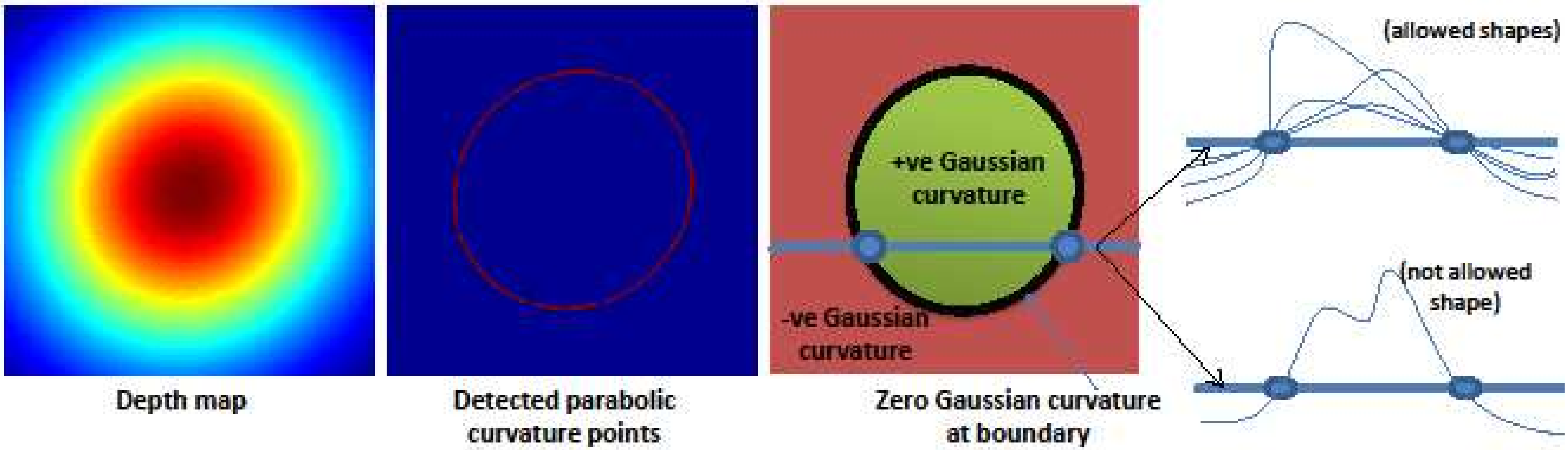
An input image. Inset is the segmentation mask. | Decision statistic (log scale) with 10 (left) and 25 (right) images of the mirror. | Detected parabolic points with 25 images | (Left) pose matching results and (right) mirror synthesized in estimated pose.

Parabolic points/curves are geometric features



Parabolic points are discriminative and can be used for pose estimation as well as classification!

Applications: Surface priors

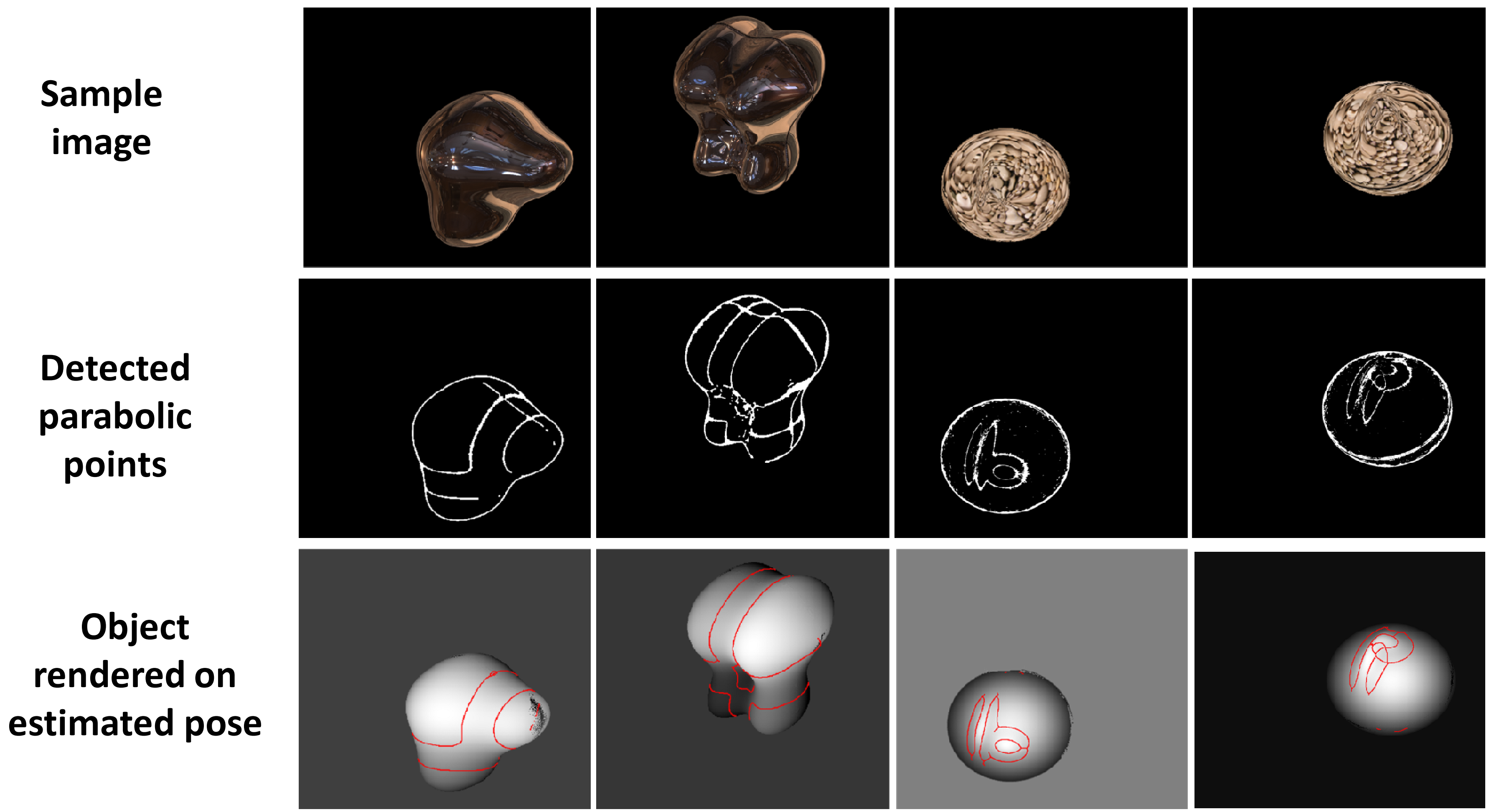


Parabolic points provide the following priors

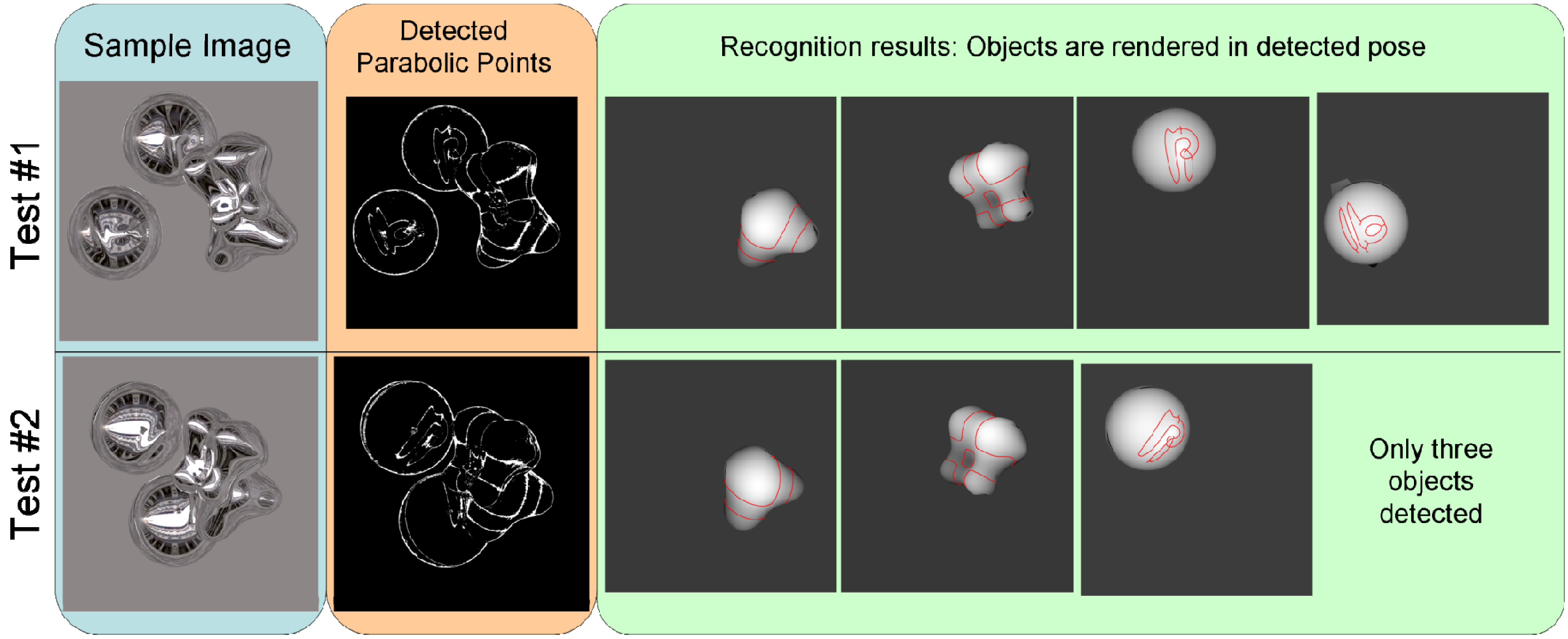
1. Along parabolic curves, the surface curvature is zero.

2. For simple surfaces, closed parabolic curves segment object into regions of elliptic and hyperbolic curvatures!

Pose Estimation Results



Classification and pose estimation

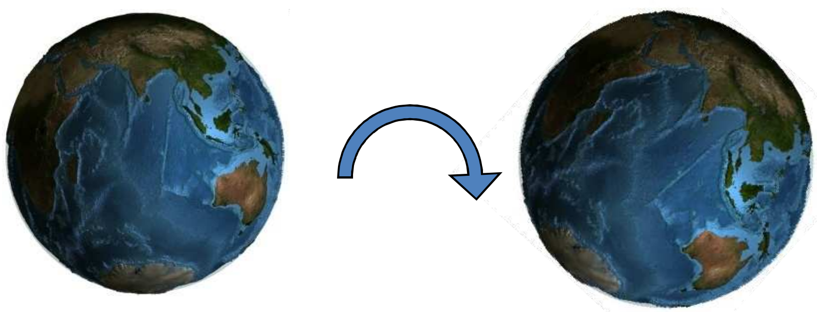


Structure from Motion for mirrors

What is Specular Flow

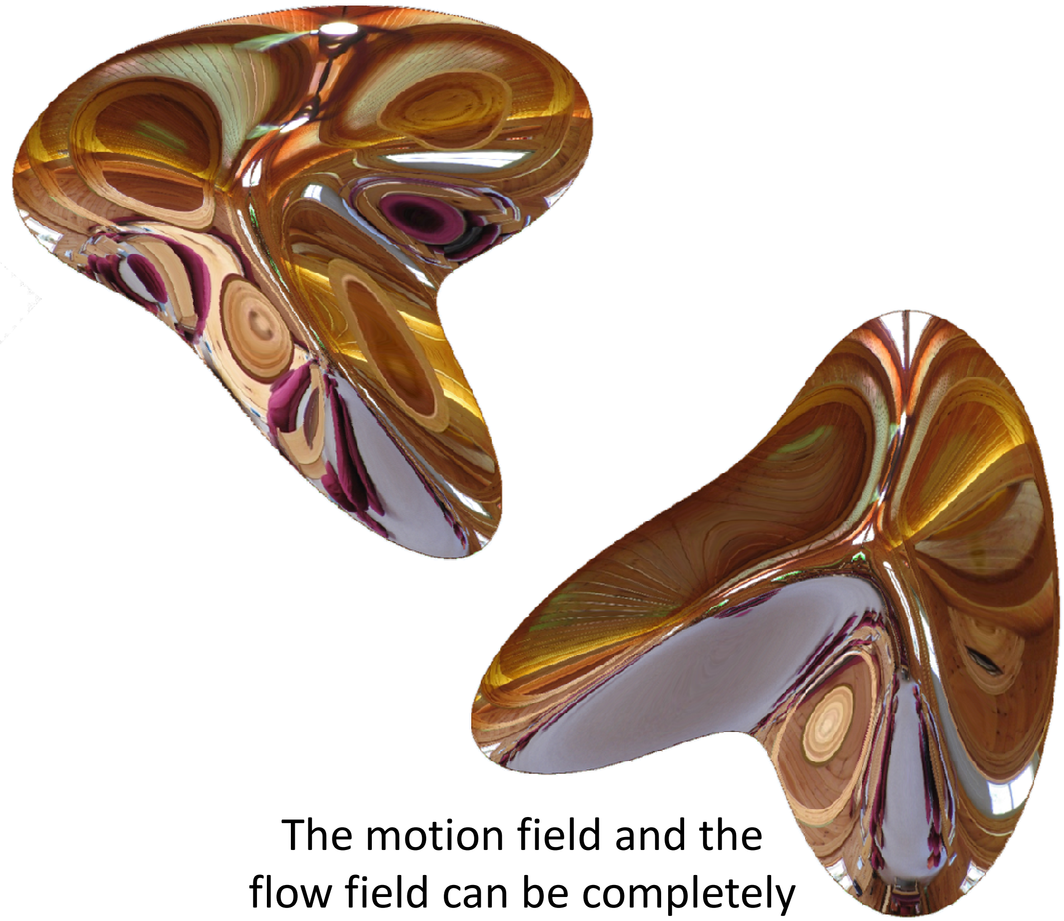
When an object is rotated, how does its appearance change ?

Diffuse



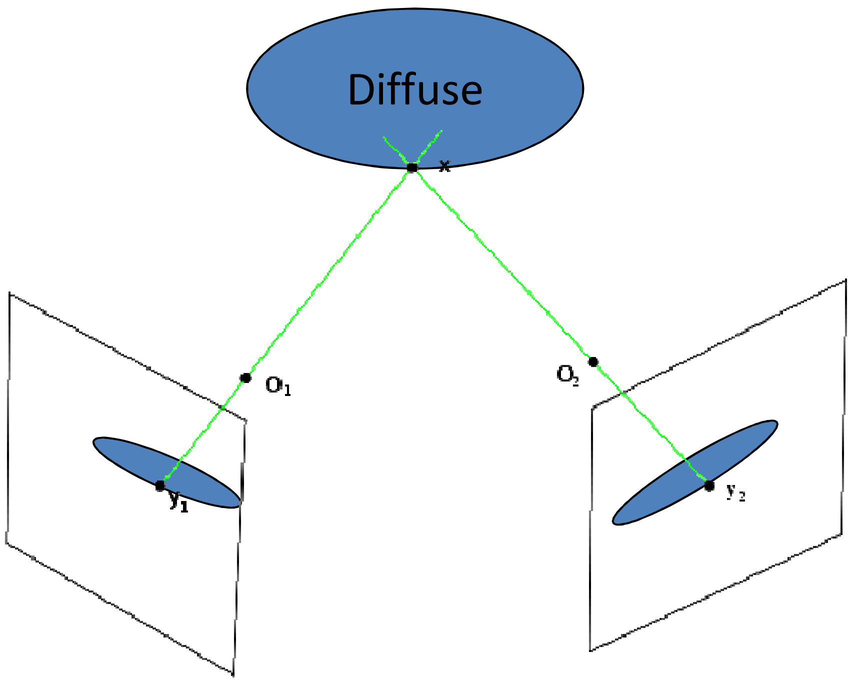
For a diffuse object, the optical flow of the image features is a projection of the motion field

Mirror

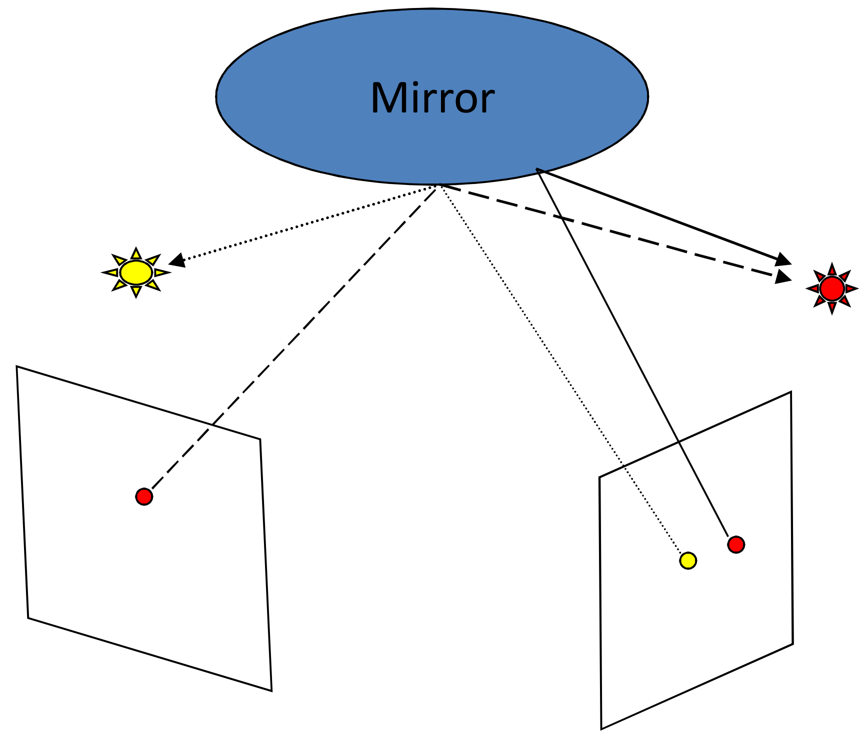


The motion field and the flow field can be completely incoherent

Diffuse vs Mirror Objects

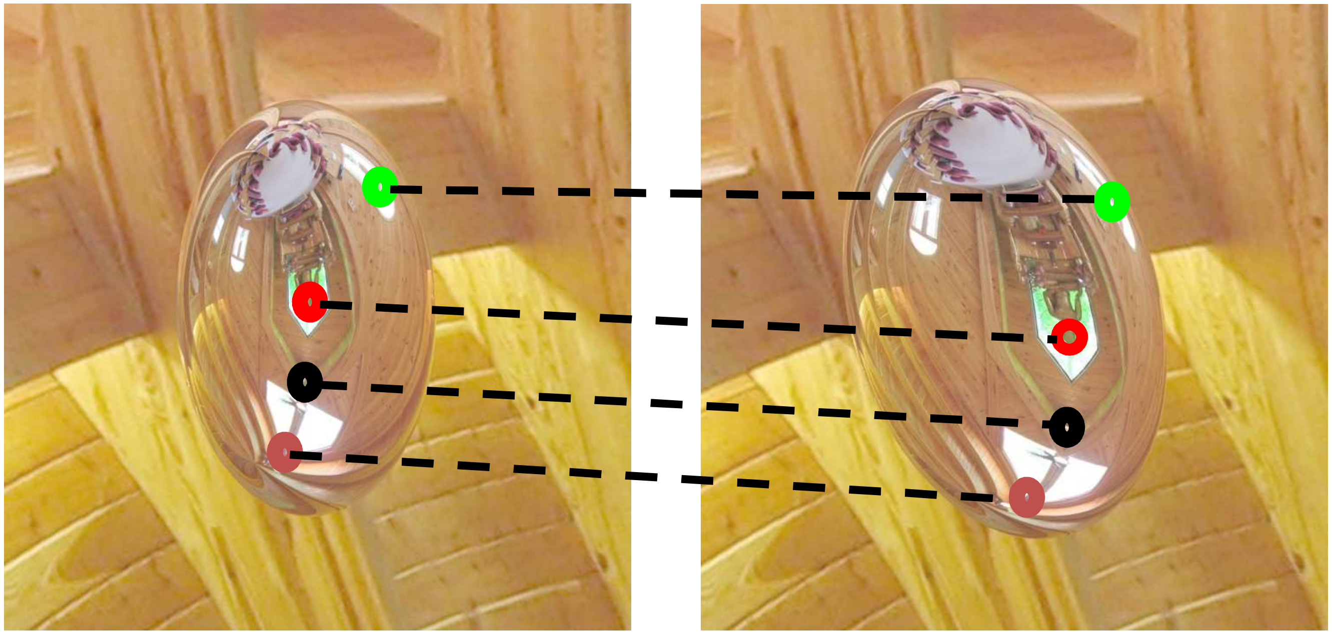


Triangulation is the basic step behind localizing when point correspondences are known



For mirror objects, the “appearance” of a surface point is not fixed. The object reflects the environment onto the camera.

Reflection Correspondences



Reflection Correspondences are pairs of points on images of a mirror that reflect the same environment feature

Image formation (same as before)

Assumptions:

1. Perfect Mirror reflectance
2. Environment at Infinity
   - Reflection depends only on scene normal
   - No dependence on object location.
3. Orthographic Camera

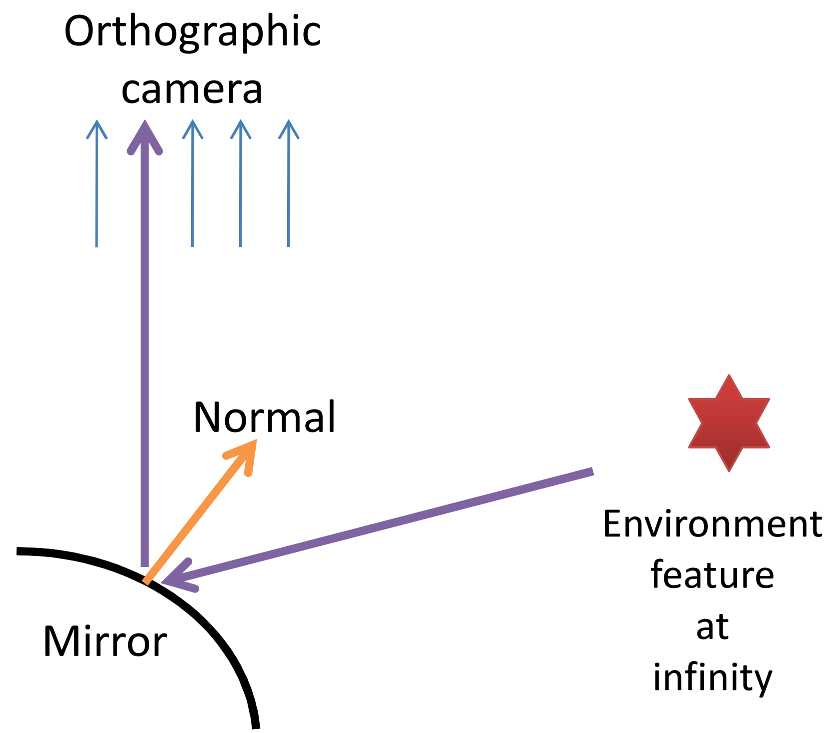


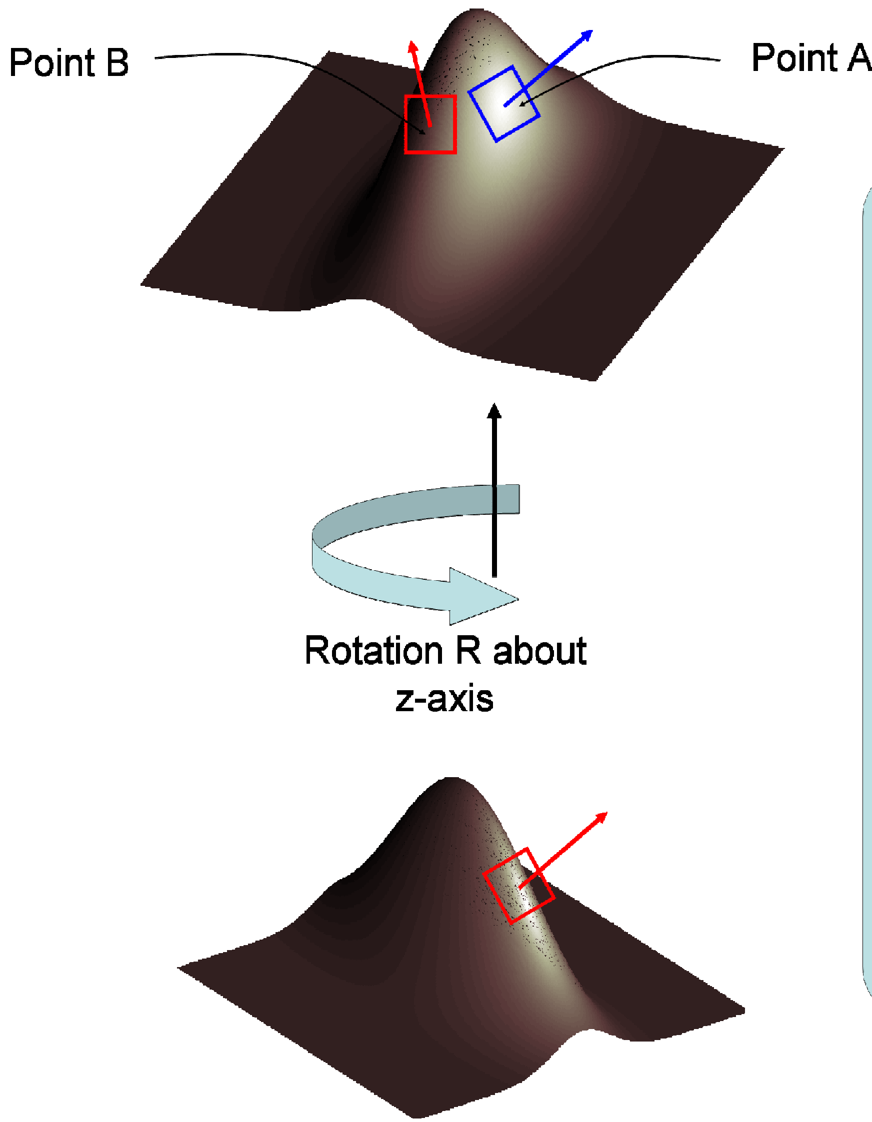
Image intensity at Pixel "x"

$$I(x) = E(\Theta(\nabla f(x)))$$

Environment reflected at pixel "x"

Forward imaging model depends *only* on surface normal of the point on the mirror

Constraints from Reflection Correspondences



Point x_A is marked in blue

Point x_B is marked in *red*

Under rotation, x_B moves to x'_B such that $x'_B = Rx_B$

Under rotation, the normal at B undergoes a rotation as well.

$$\nabla f_R(x'_B) = R\nabla f(x_B)$$

Point B after rotation sees the same scene feature as Point A before rotation

$$\nabla f(x_A) = \nabla f(x'_B)$$

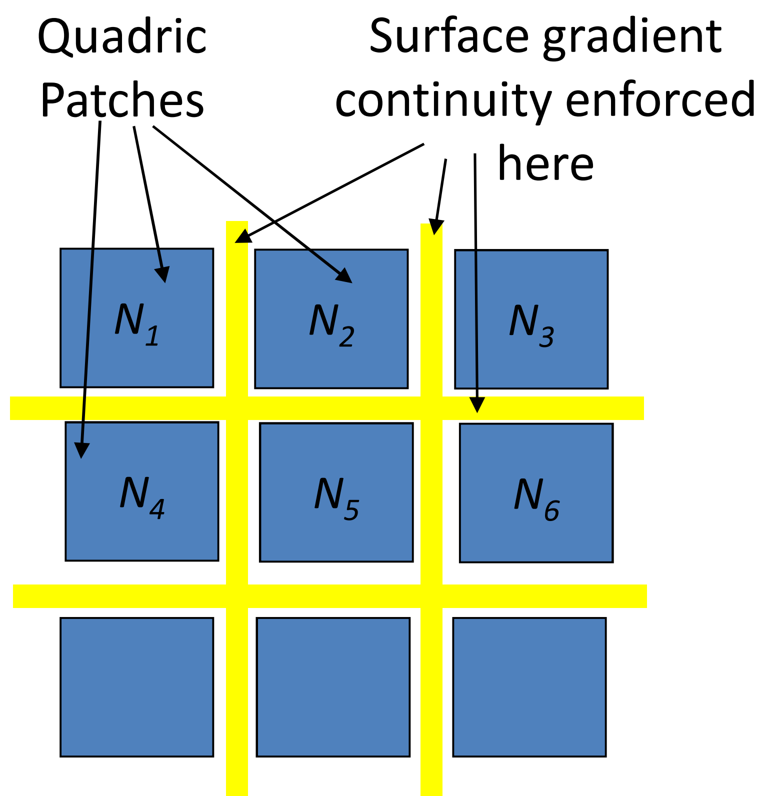
Surface priors

Reflection correspondences are spatially sparse. Not enough information to reconstruct surface!

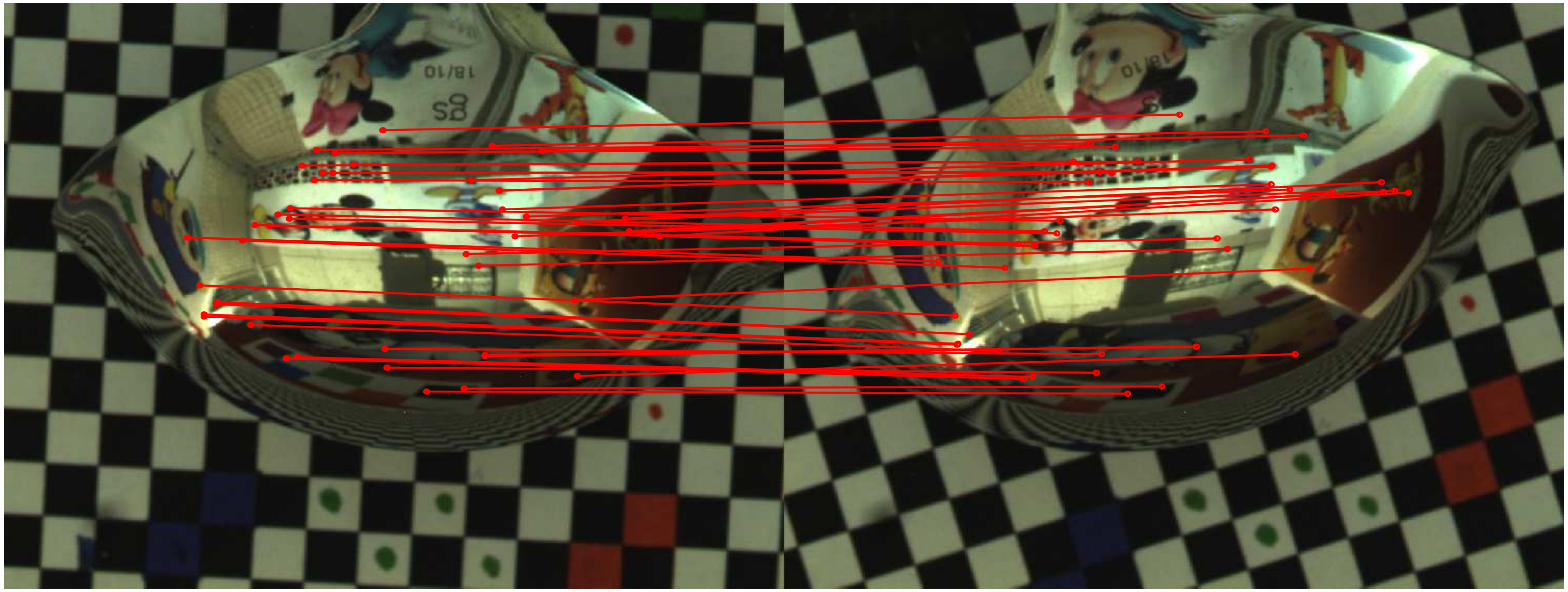
Use a patchwise quadric model on the surface to reduce number of degrees of freedom

Quadric as the RCs induce linear equations on the unknown surface parameters

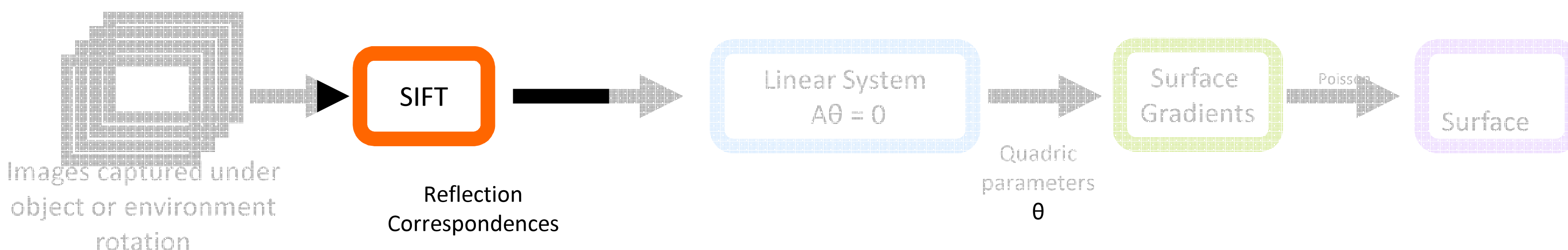
Surface smoothness enables us to enforce continuity of surface gradients at the boundary of quadric patches



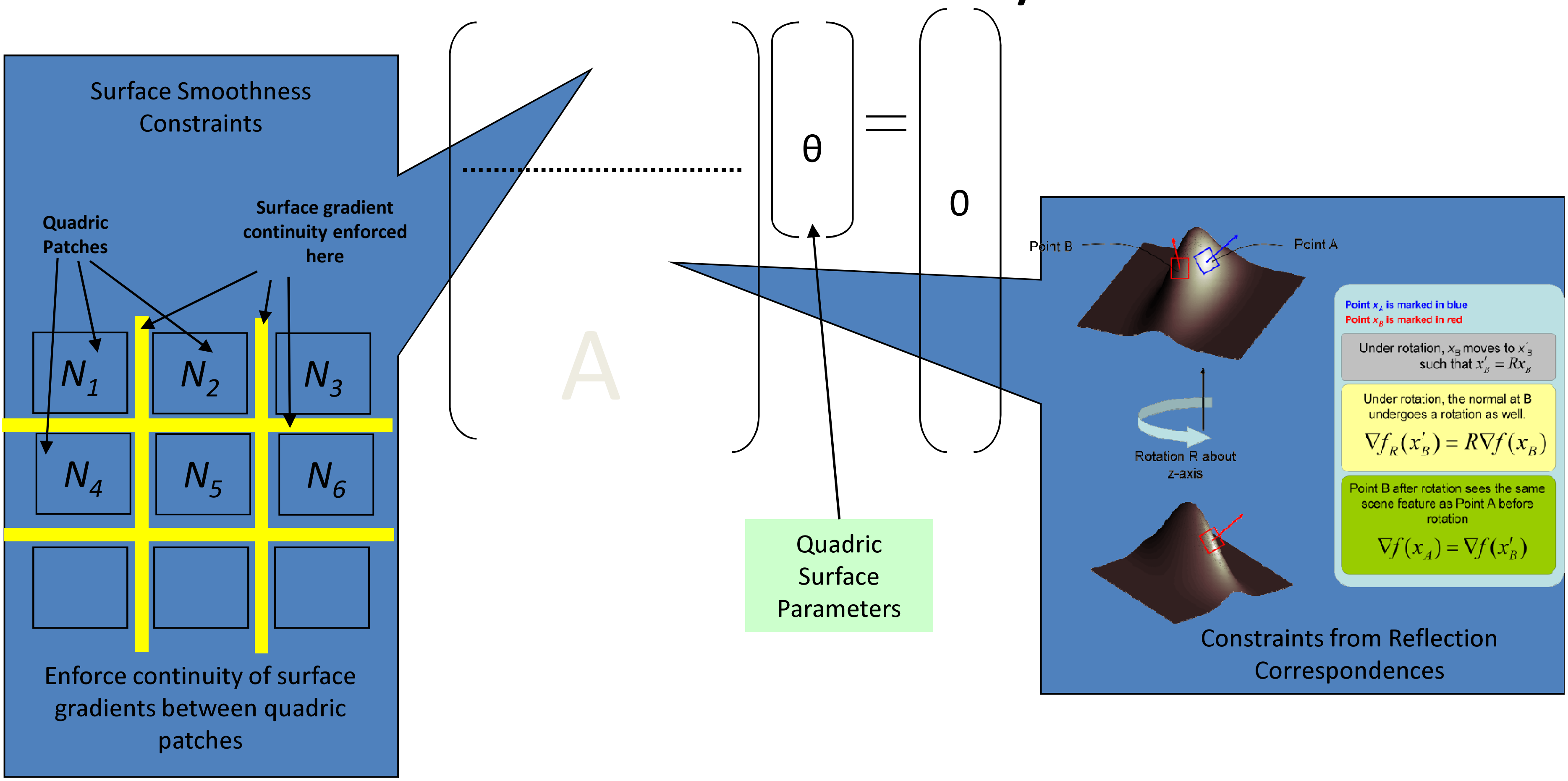
Reflection Correspondences from SIFT Matching

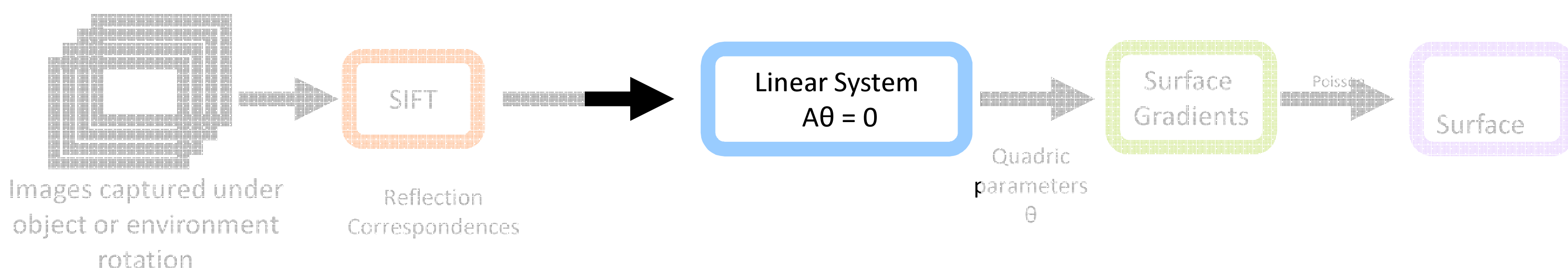


Each RC pair is a constraint relating surface normals at two different locations



Formulation of Linear System



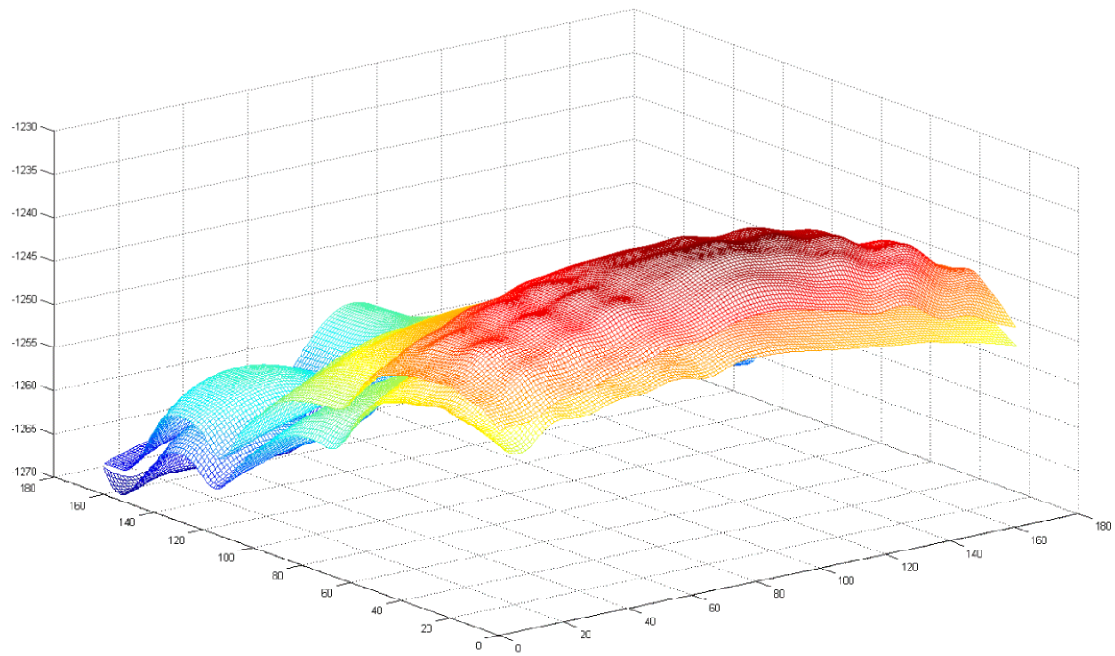


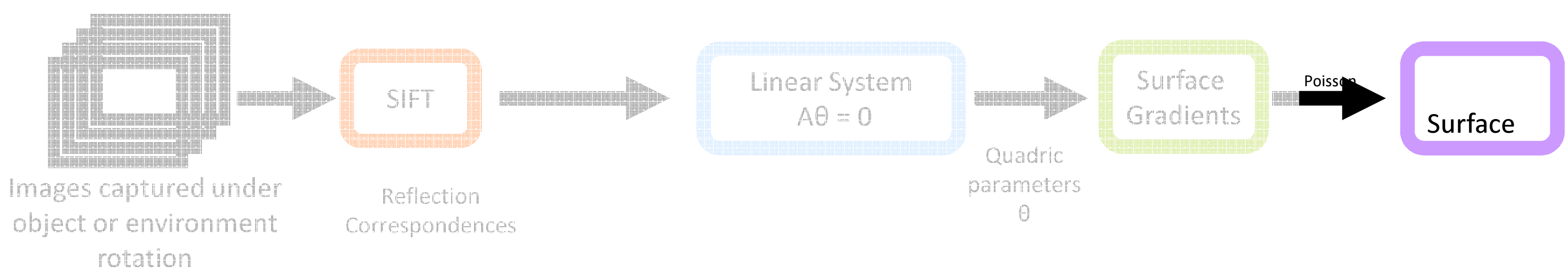
Surface Reconstruction Results

Reconstruction vs Ground Truth (top)

(Vertically displaced for visualization)

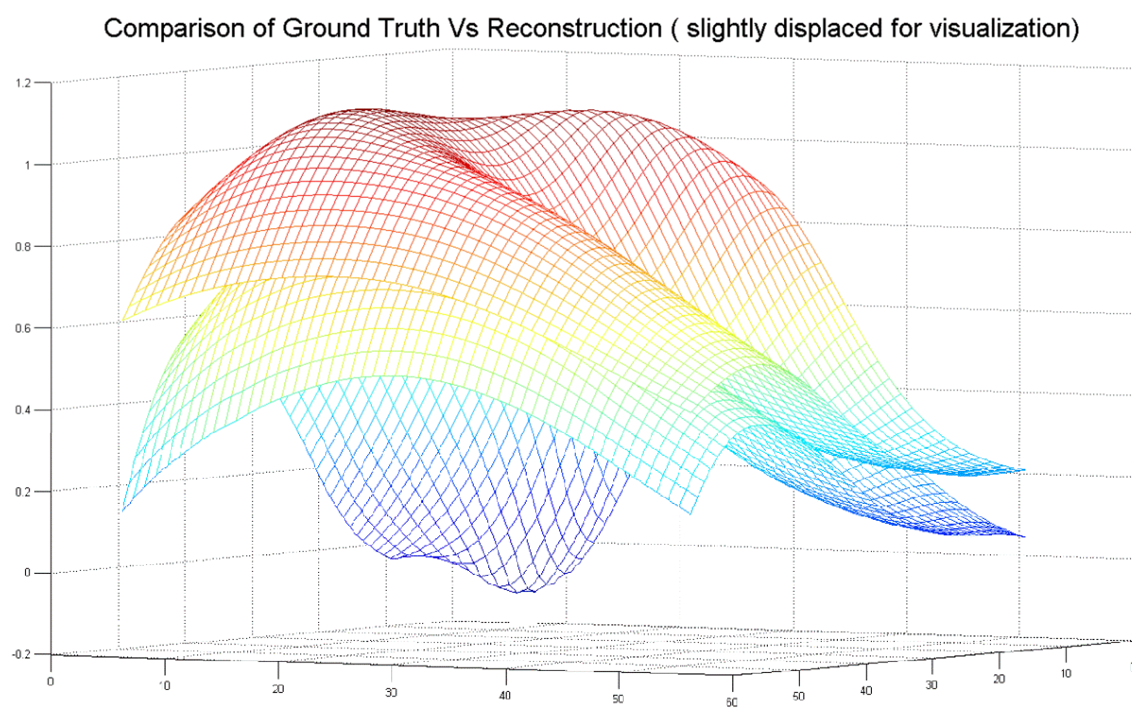
ANIMATION IN THIS SLIDE. Pl USE SLIDESHOW MODE

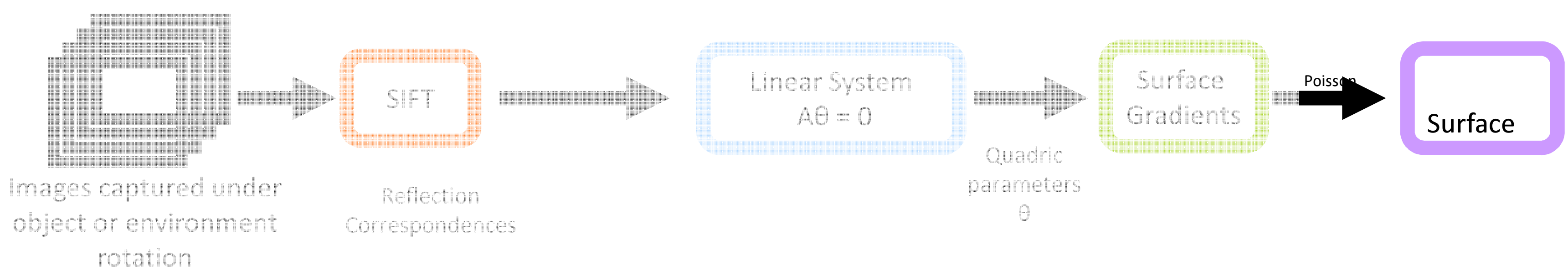




Surface Reconstruction Results

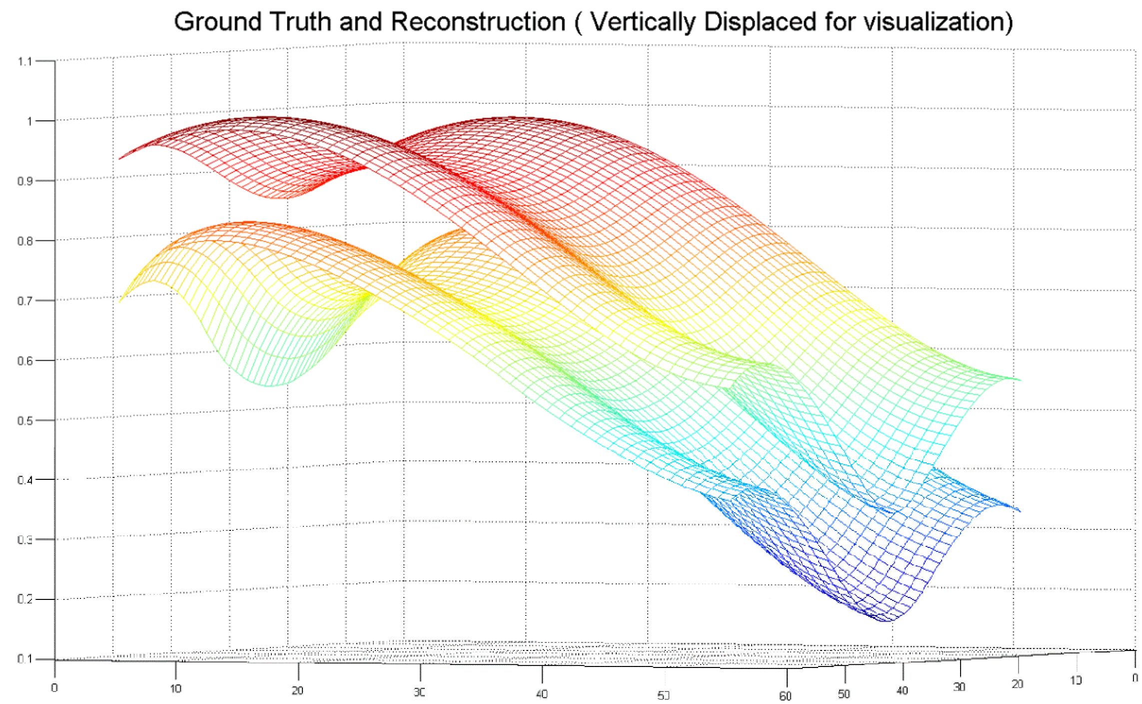
ANIMATION IN THIS SLIDE. Pl USE SLIDESHOW MODE

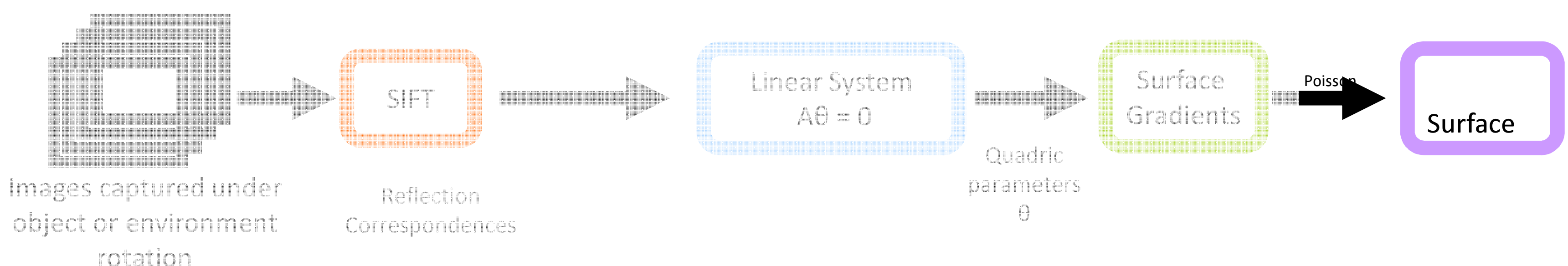


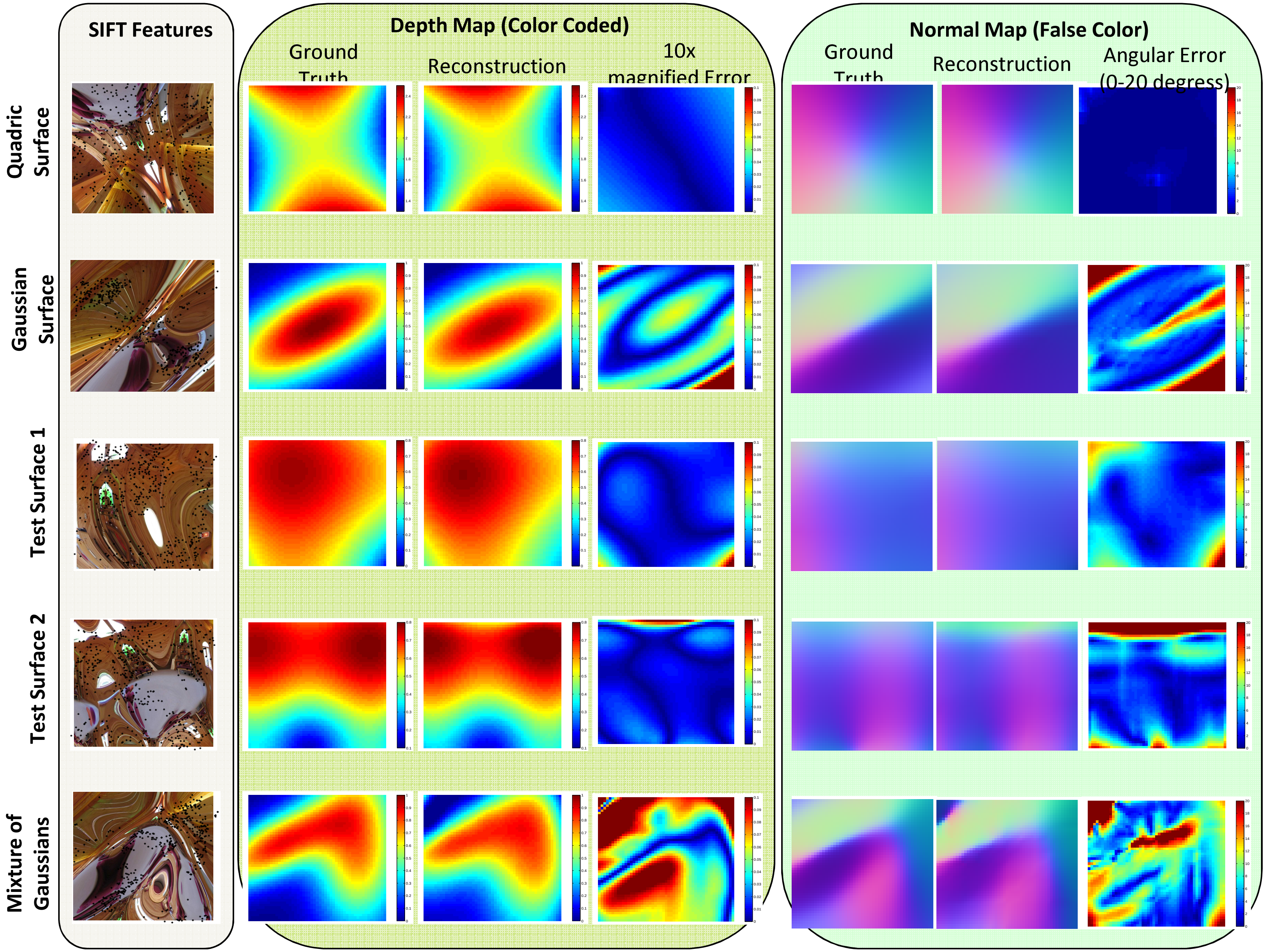


Surface Reconstruction Results

ANIMATION IN THIS SLIDE. Pl USE SLIDESHOW MODE

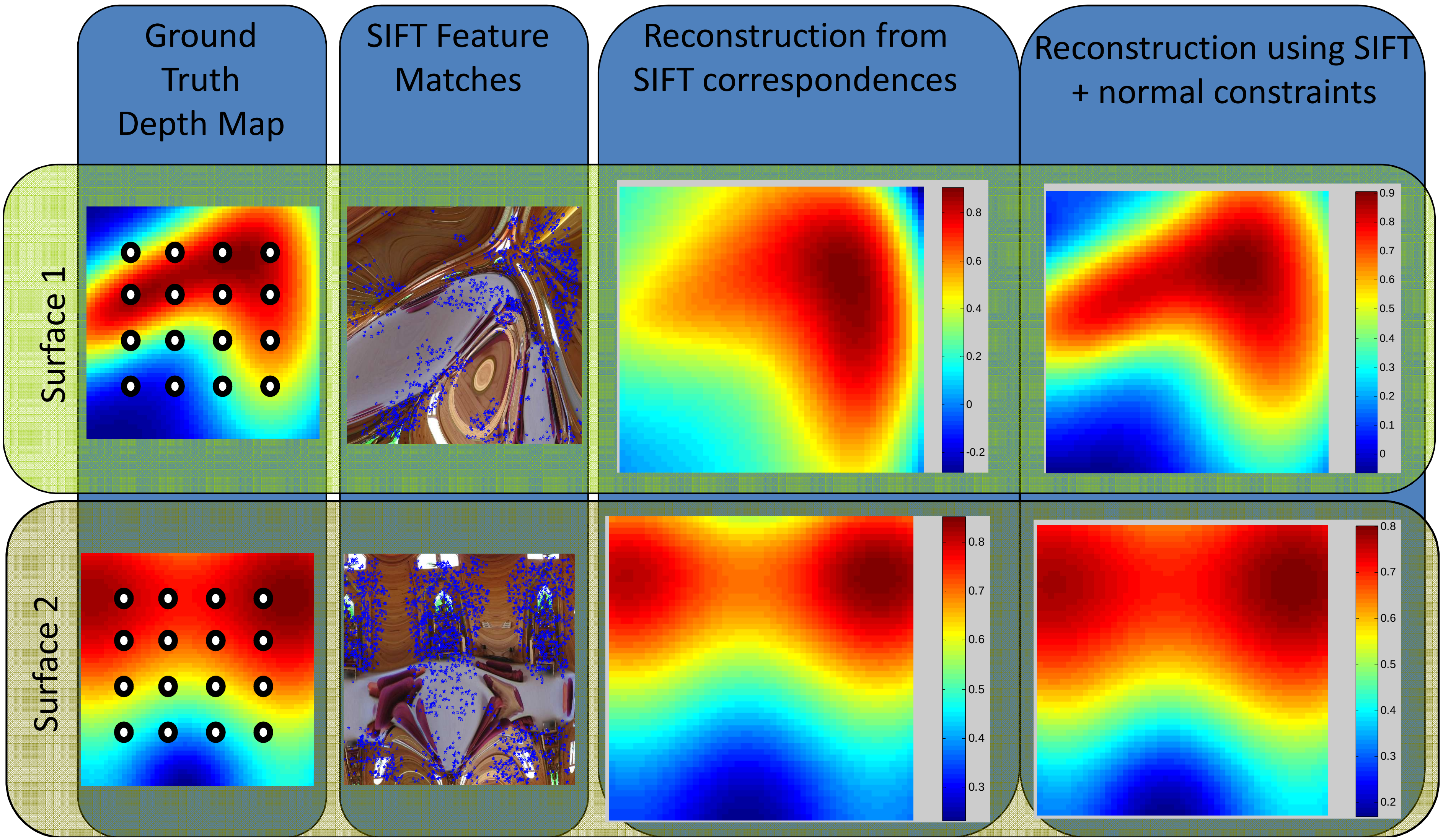




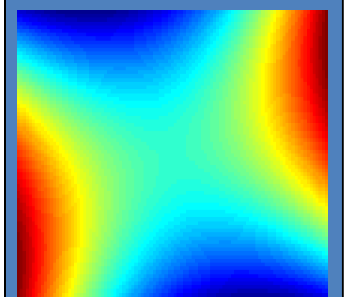


Reconstruction results on surfaces of varying complexities using SIFT correspondences modeled as RC.

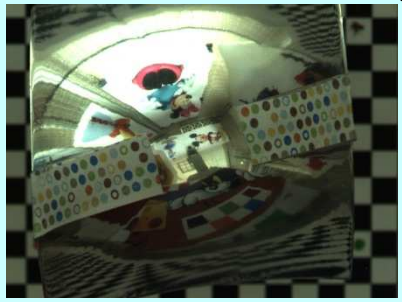
Surface Reconstruction Results under knowledge of a few surface normals



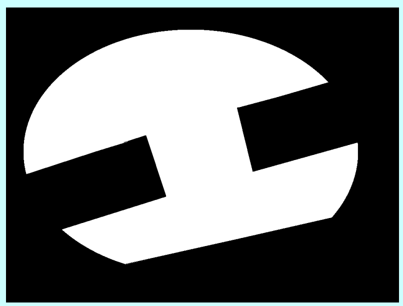
Experiment Results on Real Data



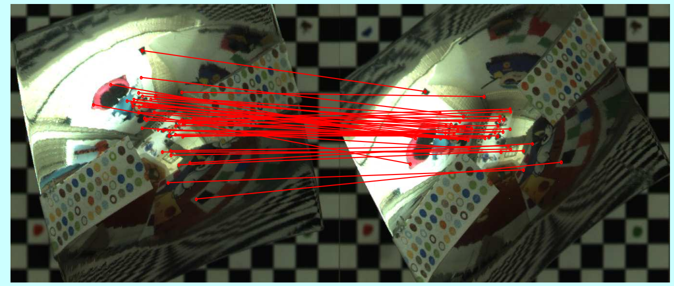
Ground Truth



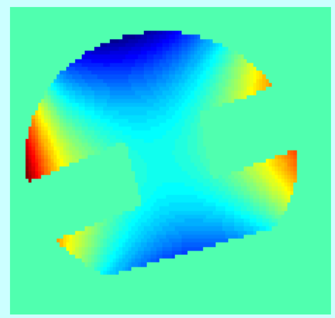
Sample Image



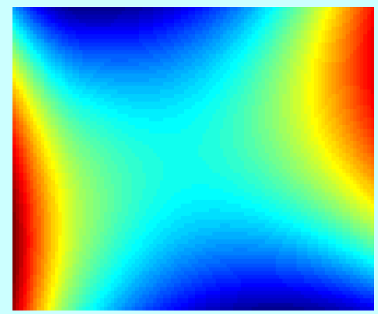
Manual mask for mirror



SIFT Matches



Reconstructed Mirror



Reconstructed Mirror (Quadric interpolation)

Reconstruction SNR: 2 mm over a 27 mm depth variation of test the object

Summary

- Small (but important first steps) towards vision for mirrors and eventually, vision for arbitrary surface reflectance
- Two Ideas
  - Recovery of geometric (shape) properties
  - Understanding what correspondences give us

Mirrors have invariants!

This should completely change the way you look at mirrors!